

Question 15 Of 25 Given Any Triangle ABC With Corresponding Side Lengths A, B, And C, The Law Of Cosines

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Understanding the properties and calculations involved in triangles is fundamental in geometry. Among the various tools used to analyze triangles, the Law of Cosines stands out for its versatility, especially when dealing with non-right triangles. This article aims to provide a comprehensive understanding of the Law of Cosines, its derivation, applications, and how it can be used to solve problems involving any triangle ABC with sides A, B, and C.

Introduction to the Law of Cosines

The Law of Cosines is a fundamental theorem in trigonometry that relates the lengths of the sides of a triangle to the cosine of one of its angles. It generalizes the Pythagorean theorem, which applies only to right-angled triangles, to any triangle, whether acute, obtuse, or right-angled.

Statement of the Law of Cosines:

For any triangle ABC with sides of lengths A, B, and C, opposite angles A, B, and C respectively, the Law of Cosines states that:

- $C^2 = A^2 + B^2 - 2AB \cos C$
- $B^2 = A^2 + C^2 - 2AC \cos B$
- $A^2 = B^2 + C^2 - 2BC \cos A$

This formula allows us to find an unknown side when two sides and the included angle are known or to find an angle when all three sides are known.

Derivation of the Law of Cosines

Understanding the derivation of the Law of Cosines helps in grasping its geometric meaning.

Step-by-step derivation:

1. Consider triangle ABC:

- Sides: A, B, C
- Angles: A, B, C

2. Place the triangle in a coordinate plane:

- Position point A at the origin $((0,0))$.
- Place point C along the x-axis at $((c, 0))$.
- Let point B be at $((x, y))$.

3. Express coordinates:

- Since side AB has length (B) , and side AC has length (A) ,
- Coordinates of B: $((x, y))$,
- Coordinates of C: $((c, 0))$,
- Coordinates of A: $((0, 0))$.

4. Use distance formulas:

- $(AB = \sqrt{(x - 0)^2 + (y - 0)^2} = B)$,
- $(AC = c)$,
- $(BC = \sqrt{(x - c)^2 + y^2} = A)$.

5. Express side lengths:

- From the above, $(x = B \cos C)$,
- $(y = B \sin C)$.

6. Apply the Law of Cosines:

- The length (A) , which is side BC, can be written as:

$$\begin{aligned} & \sqrt{ \\ A^2 &= (x - c)^2 + y^2 \\ } \end{aligned}$$

- Substitute $(x = B \cos C)$, $(y = B \sin C)$:

$$\begin{aligned} & \sqrt{ \\ A^2 &= (B \cos C - c)^2 + (B \sin C)^2 \\ } \end{aligned}$$

- Expand:

$$\begin{aligned} & \sqrt{ \\ A^2 &= B^2 \cos^2 C - 2 c B \cos C + c^2 + B^2 \sin^2 C \\ } \end{aligned}$$

- Simplify using $(\cos^2 C + \sin^2 C = 1)$:

$$A^2 = B^2 - 2 c B \cos C + c^2$$

- Rearranged, we obtain:

$$c^2 = A^2 + 2 c B \cos C - B^2$$

- Recognizing that (c) is the side opposite angle (C) , and relabeling accordingly, this leads to the Law of Cosines formula.

Conclusion:

The derivation confirms that the Law of Cosines relates side lengths with the cosine of the included angle, providing the foundation for solving various triangle problems.

Applications of the Law of Cosines

The Law of Cosines is versatile and applies in numerous scenarios:

- Finding a side when two sides and the included angle are known:

$$c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

- Finding an angle when all sides are known:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- Solving non-right triangles:

It extends the Pythagorean theorem to oblique triangles, facilitating the calculation of missing sides or angles.

Solving Triangle ABC Using the Law of Cosines

Suppose you are given triangle ABC with sides A, B, and C, and need to find a specific side

or angle.

Example 1: Find side C given sides A, B, and angle C

Given:

- $(A = 7)$,
- $(B = 10)$,
- $(\angle C = 60^\circ)$,

Find side (C) .

Solution:

$$C^2 = A^2 + B^2 - 2AB \cos C$$

$$C^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 60^\circ$$

$$C^2 = 49 + 100 - 140 \times 0.5$$

$$C^2 = 149 - 70 = 79$$

$$C = \sqrt{79} \approx 8.89$$

Example 2: Find angle C given sides A, B, and C

Given:

- $(A = 8)$,
- $(B = 6)$,
- $(C = 10)$,

Find $(\angle C)$.

Solution:

$$\cos C = \frac{A^2 + B^2 - C^2}{2AB}$$

$$\cos C = \frac{8^2 + 6^2 - 10^2}{2 \times 8 \times 6}$$

$$\cos C = \frac{8^2 + 6^2 - 10^2}{2 \times 8 \times 6}$$

$$\cos C = \frac{64 + 36 - 100}{96} = \frac{0}{96} = 0$$

$$\angle C = \arccos 0 = 90^\circ$$

Comparison with the Pythagorean Theorem

While the Pythagorean theorem is a special case applicable only to right-angled triangles:

$$c^2 = a^2 + b^2$$

the Law of Cosines reduces to the Pythagorean theorem when $\cos C = 0$ (i.e., $\angle C = 90^\circ$). This illustrates the Law of Cosines's role as a more general formula that encompasses right triangles and extends to all triangles.

Important Tips for Using the Law of Cosines

- When solving for an unknown side, ensure you identify the correct sides and the included angle.
- When calculating an angle, verify that all side lengths are positive and consistent.
- Use a calculator set to the correct angle mode (degrees or radians) depending on the given data.
- Remember that the Law of Cosines can also be used to find the area of a triangle:

$$\text{Area} = \frac{1}{2}ab \sin C$$

and since $\sin C = \sqrt{1 - \cos^2 C}$, you can combine the Law of Cosines with sine calculations.

Common Mistakes to Avoid

- Mixing degrees and radians during calculations.
- Confusing sides and angles; always check which side is opposite which angle.
- Forgetting to square the side lengths when applying the formula.
- Not verifying the triangle's validity before calculations (triangle inequality).

Real-World Applications of the Law of Cosines

The Law of Cosines is applied in various fields:

- Navigation and surveying: Calculating distances between points when only angles and some side lengths are known.
- Astronomy: Determining distances to celestial objects.
- Engineering: Structural analysis involving non-right-angled components.
- Physics: Vector calculations where components form non-orthogonal angles.

Conclusion

The Law of Cosines is an essential theorem in geometry that bridges the gap between simple Pythagoras and more complex triangle calculations. Its ability to relate sides and angles makes it invaluable for solving triangles where right-angle assumptions do not hold. Whether working on academic problems, engineering designs, or navigation tasks, mastering the Law of Cosines expands your mathematical toolkit and enhances your understanding of triangle properties.

By understanding its derivation, applications, and potential pitfalls, you can confidently apply the Law of Cosines to a wide range of geometric

Frequently Asked Questions

What is the Law of Cosines in the context of triangle ABC with sides A, B, and C?

The Law of Cosines relates the lengths of the sides of a triangle to the cosine of one of its angles. For triangle ABC, it states that $C^2 = A^2 + B^2 - 2AB \cos(C)$, and similarly for the other angles.

How can the Law of Cosines be used to find an unknown side in triangle ABC?

If two sides and the included angle are known, the Law of Cosines can be rearranged to solve for the unknown side: for example, $A = \sqrt{B^2 + C^2 - 2BC \cos(A)}$.

How is the Law of Cosines related to the Pythagorean theorem?

The Law of Cosines generalizes the Pythagorean theorem for non-right triangles. When the angle is 90° , $\cos(90^\circ) = 0$, and the formula reduces to $A^2 = B^2 + C^2$.

Can the Law of Cosines be used to find an angle in triangle ABC?

Yes, by rearranging the Law of Cosines, you can solve for an angle. For example, $\cos(C) = (A^2 + B^2 - C^2) / (2AB)$.

What are the steps to apply the Law of Cosines in problem-solving?

First, identify the given sides and angles, then select the appropriate formula. Plug in the known values, solve for the unknown side or angle, and verify the results.

Is the Law of Cosines applicable to all types of triangles?

Yes, it applies to all triangles, whether acute, right, or obtuse, as it relates sides and angles in any triangle.

How does the Law of Cosines assist in solving SAS (Side-Angle-Side) problems?

In SAS problems, where two sides and the included angle are known, the Law of Cosines helps find the third side by substituting the known values into the formula.

What is the significance of the Law of Cosines in real-world applications?

It is used in navigation, engineering, and physics to determine distances and angles when direct measurement isn't possible, such as triangulating locations.

Can the Law of Cosines be derived from the Law of

Sines?

While both relate sides and angles, the Law of Cosines can be derived from the Law of Sines combined with the Pythagorean theorem and geometric reasoning, but they are distinct formulas.

What is an example calculation using the Law of Cosines in triangle ABC?

Suppose sides $B=7$, $C=10$, and angle $A=60^\circ$. To find side A , use: $A^2 = B^2 + C^2 - 2BC \cos(A)$
 $= 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cos(60^\circ) = 49 + 100 - 140 \cdot 0.5 = 149 - 70 = 79$. So, $A \approx \sqrt{79} \approx 8.89$.

Additional Resources

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Understanding the Law of Cosines is essential for anyone studying geometry, especially when dealing with non-right triangles. This fundamental rule extends the Pythagorean theorem to oblique triangles—those that are neither right-angled nor necessarily isosceles. Whether you're solving for an unknown side or an angle, mastering the Law of Cosines unlocks a powerful tool for navigation through triangle problems. In this article, we will explore the Law of Cosines in detail, connect it to the context of triangle ABC with sides A, B, and C, and demonstrate its applications with practical examples.

Introduction to the Law of Cosines

In any triangle ABC, the sides are typically labeled as a , b , and c , opposite the angles A , B , and C , respectively. The Law of Cosines relates these sides and angles in a formula that generalizes the Pythagorean theorem:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Similarly, for the other sides:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

This relationship allows us to find an unknown side when two sides and the included angle are known, or to find an angle when all three sides are known.

Significance of the Law of Cosines in Triangle ABC

Suppose you are given a triangle ABC with side lengths A , B , and C . Here, it's important to

clarify the notation:

- Side a is opposite angle A
- Side b is opposite angle B
- Side c is opposite angle C

In some contexts, the sides are labeled as uppercase letters (A, B, C), but for clarity, we often use lowercase for sides and uppercase for angles.

The Law of Cosines becomes particularly useful for:

- Calculating an unknown side when two sides and the included angle are known
- Finding an angle when all three sides are known
- Solving oblique triangles where no right angle exists

Deriving the Law of Cosines

The Law of Cosines can be derived from basic geometric principles, especially the Pythagorean theorem, by dropping a perpendicular from one vertex and considering the resulting right triangles.

Derivation Sketch:

1. Consider triangle ABC with side lengths a, b, c.
2. Drop a perpendicular from vertex C to side c, creating a right triangle.
3. Use the definitions of cosine and sine in the right triangle to relate side lengths and angles.
4. Derive the formula for side a in terms of sides b, c, and angle A.

This derivation reveals that the Law of Cosines is essentially an extension of the Pythagorean theorem for non-right triangles, with the cosine term adjusting for the lack of a 90° angle.

Applying the Law of Cosines in Triangle ABC

Let's explore how the Law of Cosines operates with triangle ABC, where side a is opposite angle A, and so forth.

When You Know Two Sides and the Included Angle

Suppose you know sides b and c, and the included angle A:

To find side a:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Example:

Given:

- $b = 7$ units
- $c = 10$ units
- $A = 60^\circ$

Calculate a :

$$a^2 = 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cos(60^\circ)$$

$$a^2 = 49 + 100 - 2 \cdot 7 \cdot 10 \cdot 0.5$$

$$a^2 = 149 - 70$$

$$a^2 = 79$$

$$a = \sqrt{79} \approx 8.89 \text{ units}$$

When You Know All Three Sides

Suppose you know sides a , b , and c :

To find angle A :

$$\cos A = (b^2 + c^2 - a^2) / (2bc)$$

Example:

Sides:

- $a = 8$ units
- $b = 9$ units
- $c = 10$ units

Calculate angle A :

$$\cos A = (9^2 + 10^2 - 8^2) / (2 \cdot 9 \cdot 10)$$

$$\cos A = (81 + 100 - 64) / 180$$

$$\cos A = (117) / 180 \approx 0.65$$

$$A \approx \cos^{-1}(0.65) \approx 49.46^\circ$$

Practical Examples and Problem-Solving Strategies

Example 1: Finding an Unknown Side

Given triangle ABC with:

- $B = 45^\circ$
- $C = 60^\circ$
- Side $a = 8$ units (opposite A)

Find side b :

Using Law of Cosines:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

But first, we need c or b . If c is unknown, and a and B are known, then we'd need additional information. Alternatively, if we know a , b , and C , we could find c .

Example 2: Finding an Unknown Angle

Given:

- $a = 7$ units
- $b = 9$ units
- $c = 10$ units

Find angle C :

$$\cos C = (a^2 + b^2 - c^2) / (2ab)$$

$$\cos C = (49 + 81 - 100) / (2 \cdot 7 \cdot 9)$$

$$\cos C = (30) / (126) \approx 0.238$$

$$C \approx \cos^{-1}(0.238) \approx 76.2^\circ$$

Special Considerations and Limitations

- The Law of Cosines is most effective when you have enough information about sides and angles to avoid ambiguity.
- In ambiguous cases, especially when solving for angles given sides, there may be two possible solutions (the "ambiguous case").
- Always verify your calculations, especially when dealing with inverse cosine functions, as multiple angles can share the same cosine value.

Connecting Law of Cosines to Other Triangular Rules

The Law of Cosines is part of a broader family of formulas used to solve triangles:

- Law of Sines: Relates sides and angles in a different way, useful when given ASA or AAS configurations.
- Pythagorean Theorem: Special case of the Law of Cosines when the triangle is right-

angled.

Understanding how these formulas complement each other enables more flexible and comprehensive approaches to triangle problems.

Summary and Final Thoughts

The Law of Cosines is a cornerstone of triangle geometry, providing a crucial link between sides and angles in non-right triangles. Whether you're looking to find an unknown side when two sides and an included angle are known, or determining an angle when all sides are given, this law offers a straightforward and reliable method.

In triangle ABC, with sides A, B, and C, mastering the Law of Cosines empowers you to solve complex geometric problems, analyze real-world measurements, and deepen your understanding of the relationships within triangles.

Key Takeaways:

- The Law of Cosines generalizes the Pythagorean theorem.
- It can be written as: $a^2 = b^2 + c^2 - 2bc \cos A$ (and similarly for other sides).
- It is essential for solving oblique triangles where no right angle exists.
- It complements other triangle laws like the Law of Sines.

By practicing problems and understanding its derivation, you'll become proficient in applying the Law of Cosines to a variety of geometric challenges involving triangle ABC and beyond.

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