R(t) = (2 Cos 6t)i + (2 Sin 6t) J + (5t)k. Find The Curvature,where Does The Normal Unit Vector N Point.

R(t) = (2 Cos 6t)i + (2 Sin 6t) J + (5t)k. Find The Curvature, where Does The Normal Unit Vector N Point.

Understanding the behavior of space curves, especially their curvature and associated vectors, is fundamental in fields such as differential geometry, physics, and engineering. The given vector function:

$$R(t) = (2 \cos 6t) \mathbf{i} + (2 \sin 6t) \mathbf{j} + (5t) \mathbf{k}$$

represents a space curve that combines circular motion in the xy-plane with linear motion along the z-axis. This article aims to provide a comprehensive, step-by-step derivation of the curvature of this curve and clarify the direction of the normal unit vector $\mathbf{N}$.

Understanding the Vector Function $R(t)$

Before delving into the calculations, it’s important to interpret the components of $R(t)$:

- X-component: $x(t) = 2 \cos 6t$
- Y-component: $y(t) = 2 \sin 6t$
- Z-component: $z(t) = 5t$

This describes a helical path with a radius of 2 in the xy-plane, revolving at an angular speed of 6 radians per unit time, while simultaneously ascending along the z-axis at a rate of 5 units per time.

Step 1: Compute the First Derivative $R'(t)$

The first derivative of $R(t)$ with respect to t gives the velocity vector:

$$
R'(t) = \frac{d}{dt} \left(2 \cos 6t,\, 2 \sin 6t,\, 5t \right)
$$

Calculating component-wise:

- $\frac{d}{dt} [2 \cos 6t] = -2 \times 6 \sin 6t = -12 \sin 6t$
- $\frac{d}{dt} [2 \sin 6t] = 2 \times 6 \cos 6t = 12 \cos 6t$
- $\frac{d}{dt} [5t] = 5$

Therefore,

$$
R'(t) = \mathbf{v}(t) = \left(-12 \sin 6t,\, 12 \cos 6t,\, 5 \right)
$$

Step 2: Compute the Second Derivative $R''(t)$

Calculating the acceleration vector:

$$
R''(t) = \frac{d}{dt} R'(t)
$$

Component-wise:

- $\frac{d}{dt} [-12 \sin 6t] = -12 \times 6 \cos 6t = -72 \cos 6t$
- $\frac{d}{dt} [12 \cos 6t] = -12 \times 6 \sin 6t = -72 \sin 6t$
- $\frac{d}{dt} [5] = 0$

Thus,

$$
R''(t) = \left(-72 \cos 6t,\, -72 \sin 6t,\, 0 \right)
$$

Step 3: Compute the Curvature $\kappa(t)$

The curvature measures how sharply a curve bends at a point. The formula for curvature in space curves is:

$$
\kappa(t) = \frac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|^3}
$$

\]

where:

- \(\mathbf{v}(t) = R'(t) \)
- \(\mathbf{a}(t) = R''(t) \)

Step 3.1: Compute \(|\mathbf{v}(t)| \)

\[
|\mathbf{v}(t)| = \sqrt{(-12 \sin 6t)^2 + (12 \cos 6t)^2 + 5^2}
\]

Calculations:

\[
= \sqrt{144 \sin^2 6t + 144 \cos^2 6t + 25}
\]
\[
= \sqrt{144 (\sin^2 6t + \cos^2 6t) + 25}
\]
\[
= \sqrt{144 \times 1 + 25} = \sqrt{169} = 13
\]

Step 3.2: Compute \(\mathbf{v}(t) \times \mathbf{a}(t) \)

Using the cross product formula:

\[
\mathbf{v}(t) \times \mathbf{a}(t) =

\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

-12 \sin 6t & 12 \cos 6t & 5 \\

-72 \cos 6t & -72 \sin 6t & 0

\end{vmatrix}

\]

Calculating the determinant:

- \(\mathbf{i} \)-component:

\[

(12 \cos 6t)(0) - (5)(-72 \sin 6t) = 0 + 360 \sin 6t

\]

- \(\mathbf{j} \)-component:

\[

- \left[(-12 \sin 6t)(0) - (5)(-72 \cos 6t) \right] = - (0 + 360 \cos 6t) = -360 \cos 6t

\]

- \(\mathbf{k} \)-component:

\[

(-12 \sin 6t)(-72 \sin 6t) - (12 \cos 6t)(-72 \cos 6t) = 864 \sin^2 6t + 864 \cos^2 6t

\]

\[

= 864 (\sin^2 6t + \cos^2 6t) = 864

\]

Thus,

$$
\mathbf{v}(t) \times \mathbf{a}(t) = \left(360 \sin 6t,\, -360 \cos 6t,\, 864 \right)
$$

Step 3.3: Find the magnitude of the cross product

$$
|\mathbf{v} \times \mathbf{a}| = \sqrt{(360 \sin 6t)^2 + (-360 \cos 6t)^2 + 864^2}
$$

$$
= \sqrt{129600 \sin^2 6t + 129600 \cos^2 6t + 746496}
$$

$$
= \sqrt{129600 (\sin^2 6t + \cos^2 6t) + 746496}
$$

$$
= \sqrt{129600 \times 1 + 746496} = \sqrt{876096}
$$

Expressed as:

$$
|\mathbf{v} \times \mathbf{a}| = \sqrt{876096}
$$

Step 3.4: Final calculation of curvature

Recall:

$$
\kappa(t) = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}
$$

\]

Given \(|\mathbf{v}| = 13 \), so:

\[
|\mathbf{v}|^3 = 13^3 = 2197
\]

Therefore,

\[
\boxed{
\kappa(t) = \frac{\sqrt{876096}}{2197}
}
\]

This provides the curvature at any point \(t \) along the curve. To simplify the numerator, note that:

\[
876096 = 864^2
\]

since

\[
864^2 = (860 + 4)^2 = 860^2 + 2 \times 860 \times 4 + 4^2 = 739600 + 6880 + 16 = 746496 + 6880 + 16 \neq 876096
\]

Let's verify:

- \(864^2 = (800 + 64)^2 = 800^2 + 2 \times 800 \times 64 + 64^2 = 640000 + 102400 + 4096 = 746496 \)

Since \(864^2 = 746496 \), our earlier calculation of \(|\mathbf{v} \times \mathbf{a}| \) was 864, not \(\sqrt{876096} \). Let's double-check the previous step.

Rechecking the Cross Product Magnitude

From previous calculations:

\[
|\mathbf{v} \times \mathbf{a}| = \sqrt{(360 \sin 6t)^2 + (-360 \cos 6t)^2 + 864^2}
\]
\[
= \sqrt{129600 (\sin^2 6t + \cos^2 6t) + 864^2}
\]

Since \(\sin^2 6t + \cos^2 6t

Frequently Asked Questions

What is the vector function R(t) given as R(t) = (2 cos 6t)i + (2 sin 6t)j + (5t)k?

The vector function R(t) describes a space curve with components in the i, j, and k directions: x=2 cos 6t, y=2 sin 6t, and z=5t.

How do you find the derivative R'(t) of the given vector function?

Differentiate each component with respect to t: R'(t) = (-12 sin 6t)i + (12 cos 6t)j + 5k.

What is the magnitude of R'(t), and how is it calculated?

The magnitude |R'(t)| = sqrt((-12 sin 6t)^2 + (12 cos 6t)^2 + 5^2) = sqrt(144 sin^2 6t + 144 cos^2 6t + 25) = sqrt(144 (sin^2 6t + cos^2 6t) + 25) = sqrt(1441 + 25) = sqrt(169) = 13.

How do you compute the curvature □(t) of the curve?

Curvature □(t) = |R'(t) × R''(t)| / |R'(t)|^3. You need to find R''(t), compute the cross product with R'(t), and then divide its magnitude by |R'(t)|^3.

What is R''(t) for the given vector function?

Differentiate R'(t): R''(t) = (-72 cos 6t)i + (-72 sin 6t)j + 0k.

How do you find the cross product R'(t) × R''(t)?

Using the determinant method, R'(t) × R''(t) = |i j k|, with the components of R'(t) and R''(t) as rows, then compute accordingly.

What is the expression for the curvature □(t) after computing the cross product?

After calculation, □(t) = (864) / (13)^3 = 864 / 2197.

Where does the principal normal vector N point on the curve?

The normal vector N points in the direction of R''(t) after normalization, indicating the direction of the curve's principal normal at each point.

In which direction does the normal unit vector N point relative to the curve?

The normal unit vector N points towards the center of curvature, perpendicular to the tangent vector, and lies in the osculating plane of the curve.

Additional Resources

R(t) = (2 Cos 6t)i + (2 Sin 6t) j + (5t)k: An In-Depth Exploration of Curvature and Normal Vectors

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Introduction

R(t) = (2 Cos 6t)i + (2 Sin 6t) j + (5t)k represents a fascinating parametric vector function that encapsulates a complex spatial trajectory. Such functions are central to understanding the behavior of curves in three-dimensional space, especially in fields like physics, engineering, and computer graphics. This article aims to explore the curvature of this particular curve and analyze where its normal unit vector points, providing a comprehensive understanding that combines mathematical rigor with insightful interpretation.

Understanding the Parametric Curve R(t)

Breaking Down the Components

The vector function R(t) describes a moving point in 3D space, with each component depending on the parameter t:

- x(t) = 2 Cos 6t: Indicates oscillatory motion in the x-direction, with a cosine function scaled by 2.
- y(t) = 2 Sin 6t: Describes oscillatory motion in the y-direction, phase-shifted relative to x(t) by 90 degrees.
- z(t) = 5t: Represents linear motion along the z-axis, increasing at a rate of 5 units per unit increase in t.

This combination suggests a helical or spiral-like path, with sinusoidal motion in the xy-plane and linear ascent along the z-axis.

Graphical Intuition

Visualizing R(t), for each fixed t, the point lies on a circle of radius 2 in the xy-plane, with the angle 6t determining its position along the circle. As t increases, this circle is traced repeatedly due to the sinusoidal components, while simultaneously ascending linearly along z. The result is a helix with a constant radius and pitch determined by the 5t component.

Calculating the Derivatives: Velocity and Acceleration

First Derivative R'(t): The Velocity Vector

To analyze curvature, we need the velocity vector, which is the first derivative of R(t):

$$
R'(t) = \frac{d}{dt} \left[2 \cos 6t,\, 2 \sin 6t,\, 5t \right]
$$

Calculating component-wise:

- $\frac{d}{dt} [2 \cos 6t] = -2 \times 6 \sin 6t = -12 \sin 6t$
- $\frac{d}{dt} [2 \sin 6t] = 12 \cos 6t$
- $\frac{d}{dt} [5t] = 5$

Hence,

$$
R'(t) = (-12 \sin 6t) i + (12 \cos 6t) j + 5 k
$$

This vector indicates the instantaneous velocity of the point along the curve at any parameter t.

Second Derivative R''(t): The Acceleration Vector

Taking the derivative of R'(t):

$$
R''(t) = \frac{d}{dt} \left[-12 \sin 6t,\, 12 \cos 6t,\, 5 \right]
$$

- $\frac{d}{dt} [-12 \sin 6t] = -12 \times 6 \cos 6t = -72 \cos 6t$
- $\frac{d}{dt} [12 \cos 6t] = -12 \times 6 \sin 6t = -72 \sin 6t$
- $\frac{d}{dt} [5] = 0$

Thus,

$$
R''(t) = (-72 \cos 6t) i + (-72 \sin 6t) j + 0 k
$$

This acceleration vector reveals how the velocity vector changes over time, crucial for curvature computation.

Calculating the Curvature (κ)

Fundamental Formula for Curvature in Space

The curvature κ(t) of a space curve R(t) is a measure of how sharply the curve bends at a point. It's given by:

$$
\kappa(t) = \frac{\| R'(t) \times R''(t) \|}{\| R'(t) \|^3}
$$

Where:

- $\times$ indicates the cross product

- $\|\cdot\|$ denotes the vector magnitude

This formula encapsulates the geometric essence: the numerator assesses how much the velocity vector changes direction (via the cross product), while the denominator normalizes this change based on the speed.

Calculating $\| R'(t) \|$: The Speed

$$
\| R'(t) \| = \sqrt{(-12 \sin 6t)^2 + (12 \cos 6t)^2 + 5^2}
$$

Simplify:

$$
\| R'(t) \| = \sqrt{144 \sin^2 6t + 144 \cos^2 6t + 25}
$$

Since $\sin^2 \theta + \cos^2 \theta = 1$:

$$
\| R'(t) \| = \sqrt{144 \times 1 + 25} = \sqrt{169} = 13
$$

Remarkably, the speed is constant at 13 units, simplifying further analysis.

Calculating the Cross Product $R'(t) \times R''(t)$

$$

$$
R'(t) = (-12 \sin 6t,\, 12 \cos 6t,\, 5)
$$

$$
R''(t) = (-72 \cos 6t,\, -72 \sin 6t,\, 0)
$$

The cross product is:

$$
R'(t) \times R''(t) =
\begin{vmatrix}
i & j & k \\
-12 \sin 6t & 12 \cos 6t & 5 \\
-72 \cos 6t & -72 \sin 6t & 0
\end{vmatrix}
$$

Calculating component-wise:

- i-component:

$$
(12 \cos 6t)(0) - (5)(-72 \sin 6t) = 0 + 360 \sin 6t = 360 \sin 6t
$$

- j-component:

$$
- \left[(-12 \sin 6t)(0) - (5)(-72 \cos 6t) \right] = - [0 + 360 \cos 6t] = -360 \cos 6t
$$

- k-component:

$$
(-12 \sin 6t)(-72 \sin 6t) - (12 \cos 6t)(-72 \cos 6t) = 864 \sin^2 6t + 864 \cos^2 6t = 864 (\sin^2 6t + \cos^2 6t) = 864
$$

Therefore,

$$
R'(t) \times R''(t) = (360 \sin 6t) i - (360 \cos 6t) j + 864 k
$$

Now, the magnitude:

$$
\| R'(t) \times R''(t) \| = \sqrt{ (360 \sin 6t)^2 + (360 \cos 6t)^2 + 864^2 }
$$

Simplify:

$$
= \sqrt{ 129600 \sin^2 6t + 129600 \cos^2 6t + 746496 }
$$

$$
= \sqrt{ 129600 (\sin^2 6t + \cos^2 6t) + 746496 } = \sqrt{ 129600 \times 1 + 746496 } = \sqrt{ 876096 }
$$

Calculating the square root:

$$
\sqrt{876096} \approx 935.04
$$

Final Expression for Curvature

Recall:

$$
\kappa(t) = \frac{\| R'(t) \times R''(t) \|}{\| R'(t) \|^3}
$$

Given that:

- $\| R'(t) \| = 13$
- $\| R'(t) \times R''(t) \| \approx 935.04$

Therefore,

$$
\boxed{
\kappa(t) \approx \frac{935.04}{13^3} = \frac{935.04}{2197} \approx 0.4258
}
$$

This indicates a constant curvature along the curve, approximately 0.426, suggesting the curve has a uniform bending characteristic, typical of helices with consistent geometry.

Where Does the Normal Unit Vector N Point?

Understanding

[R T 2 Cos 6t I 2 Sin 6t J 5t K Find The Curvature Where Does The Normal Unit Vector N Point]

R T 2 Cos 6t I 2 Sin 6t J 5t K Find The Curvature Where Does The Normal Unit Vector N Point

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