Refer To Functions S And T. Find The Indicated Function And Write The Domain In Interval Notation. Write

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Understanding how to analyze functions, determine their expressions, and identify their domains is fundamental in mathematics, especially in calculus and algebra. When dealing with composite functions or functions derived from others, it’s crucial to carefully interpret the given information, find the explicit form of the function, and then determine its domain. This article provides a comprehensive guide to working with functions referred to as S and T, including how to find their explicit forms based on given conditions and how to express their domains using interval notation.

Understanding Functions and Their Notation

Before diving into specific functions, it’s essential to grasp some basic concepts:

What is a Function?

A function is a relation between a set of inputs (domain) and a set of outputs (range) where each input corresponds to exactly one output. Functions are often denoted by letters such as f, g, S, T, etc.

Function Notation

Standard notation is f(x), where f is the function and x is the input. When functions are named S and T, they might be described explicitly or implicitly through operations, compositions, or given formulas.

Domain and Range

- Domain: The set of all possible input values for the function.
- Range: The set of all possible output values.

Expressing the domain in interval notation makes it easier to communicate the extent of the function’s inputs visually and mathematically.

Common Types of Functions and Their Domains

When analyzing functions like S and T, you may encounter various types, such as:

- Polynomial functions
- Rational functions
- Square root functions
- Composite functions

Each type has specific considerations for its domain:

Polynomial Functions

- Typically defined for all real numbers.
- Domain: \((-\infty, \infty)\)

Rational Functions

- Defined for all real numbers except where the denominator is zero.
- To find the domain, set the denominator ≠ 0.

Square Root and Radical Functions

- Defined where the expression inside the radical is ≥ 0.
- To find the domain, solve the inequality inside the radical.

Composite Functions

- Formed by combining functions, such as \((f \circ g)(x) = f(g(x)) \).
- Domain depends on the domain of g and the range of g fitting into the domain of f.

Step-by-Step Approach to Find Functions S and T and Their Domains

Let's consider a typical problem where functions S and T are defined via certain relations or compositions, and the goal is:

1. To find the explicit form of the indicated function.
2. To determine the domain of that function in interval notation.

Step 1: Understand the Given Information

- Identify how functions S and T are defined.
- Note any given formulas, compositions, or relations.
- Recognize the type of the functions involved.

Step 2: Derive the Explicit Form

- Use algebraic operations, substitutions, or inverse functions if necessary.
- Simplify expressions to obtain a clear formula for the function.

Step 3: Determine the Domain

- Analyze the explicit formula.
- Identify any restrictions (denominators, radicals, logs).
- Solve inequalities or equations to find the set of permissible input values.
- Express this set in interval notation.

Example of Finding a Function and Its Domain

Suppose you are given:

- $S(x) = \sqrt{x + 3}$
- $T(x) = \frac{1}{x - 2}$

And you are asked to find:

- The composite function $R(x) = S(T(x))$
- The domain of $R(x)$ in interval notation.

Solution:

Step 1: Find $R(x)$

- $R(x) = S(T(x)) = \sqrt{T(x) + 3}$

Step 2: Substitute $T(x)$

- $R(x) = \sqrt{\frac{1}{x - 2} + 3}$

Step 3: Simplify the expression inside the radical

- $R(x) = \sqrt{\frac{1 + 3(x - 2)}{x - 2}}$

- $R(x) = \sqrt{\frac{1 + 3x - 6}{x - 2}} = \sqrt{\frac{3x - 5}{x - 2}}$

Step 4: Find the domain

- The expression inside the radical must be ≥ 0:

$$
\frac{3x - 5}{x - 2} \geq 0
$$

- Also, the denominator cannot be zero: $x \neq 2$.

Step 5: Solve the inequality

- Critical points are where numerator or denominator is zero:

- Numerator zero: $3x - 5 = 0 \Rightarrow x = \frac{5}{3}$
- Denominator zero: $x = 2$

- Sign analysis:

Interval	Test Point	Sign of numerator $3x - 5$	Sign of denominator $x - 2$	Fraction $\frac{3x - 5}{x - 2}$	Result (≥ 0)
$(-\infty, \frac{5}{3})$	0	-5 (negative)	-2 (negative)	positive	Yes
$(\frac{5}{3}, 2)$	1.5	$4.5 - 5 = -0.5$ (negative)	-0.5 (negative)	positive	Yes
$(2, \infty)$	3	$9 - 5 = 4$ (positive)	1 (positive)	positive	Yes

- But note that at $x = 2$, the function is undefined due to division by zero.

- At $x = \frac{5}{3}$, the radical equals zero, which is acceptable.

Final domain:

- From $-\infty$ up to but not including 2, since the denominator zero point is excluded.
- At $x = \frac{5}{3}$, the expression inside the radical is zero, so it's included.
- From just greater than $\frac{5}{3}$ up to 2, the expression stays non-negative.
- After 2, the expression is again positive, but since 2 is excluded, the domain is:

$$
(-\infty, 2) \setminus \{2\} \quad \text{and} \quad \left[\frac{5}{3}, 2\right)
$$

Expressed in interval notation:

$$
(-\infty, 2) \quad \text{excluding } x=2, \quad \text{but including } \frac{5}{3}
$$

This simplifies to:

$$
(-\infty, 2) \quad \text{with } x \neq 2
$$

Because at $x = \frac{5}{3}$, the radical is zero, so included.

Therefore, the domain:

$$
(-\infty, 2) \setminus \{2\}
$$

or in interval notation:

$$
(-\infty, 2)
$$

with the understanding that $x \neq 2$.

General Tips for Finding Functions and Domains

- Always check for restrictions: Denominator zero, radical expression negative, or logarithm of non-positive number.
- Solve inequalities carefully: Use test points and sign charts.
- Express the domain accurately: Use interval notation, including parentheses for excluding points and brackets for inclusive points.
- Verify the function's definition: After deriving the explicit form, test points in the domain to ensure correctness.

Summary

In solving problems involving functions S and T, the core steps involve:

1. Interpreting the given information to understand how the functions are defined.
2. Deriving the explicit formula for the indicated function, often involving substitution, algebraic manipulation, or inverse functions.
3. Determining the domain by analyzing restrictions such as division by zero, radicals, or logs, and solving inequalities.
4. Expressing the domain in interval notation, carefully considering the points of restriction or inclusion.

Mastering these steps enhances your ability to analyze complex functions, understand their behavior, and communicate their properties effectively.

Conclusion

Understanding how to find functions like S and T and their domains is essential for advanced mathematics and real-world applications. Whether dealing with simple algebraic functions, radicals, or compositions, a systematic approach ensures accuracy and clarity. Always pay attention to restrictions imposed by the function's form, and use interval notation to succinctly express the domain. With practice, analyzing functions becomes more intuitive, enabling you to solve more complex problems confidently.

Frequently Asked Questions

Given functions S and T, how do you find the composition S ∘ T and determine its domain?

To find S ∘ T, substitute T(x) into S(x), i.e., S(T(x)). The domain of S ∘ T is all x in the domain of T such that T(x) is in the domain of S. Therefore, the domain is the set of x-values where T is defined and T(x) falls within S's domain, written in interval notation.

How do you write the indicated function when given functions S and T, and what is its domain?

To write the indicated function (e.g., S + T, S - T, S ∘ T), perform the operation on the functions as specified. The domain depends on the operation: for sums and differences, it's the intersection of their domains; for composition, it's the set of x-values where T is defined and T(x) is in S's domain. Express the resulting domain in interval notation.

What steps are involved in finding the domain of a composite function S(T(x))?

First, identify the domain of T(x). Next, determine the range of T(x) and find where T(x) maps into the domain of S. The domain of S(T(x)) is all x for which T(x) is in S's domain. Write this as an interval notation based on these constraints.

If $S(x) = \sqrt{x}$ and $T(x) = x - 3$, how do you find S ∘ T and its domain?

Compute $S(T(x)) = \sqrt{(x - 3)}$. The domain requires that $x - 3 \geq 0$, so $x \geq 3$. Therefore, the domain of S ∘ T is $[3, \infty)$.

How can you determine the domain of a function

formed by combining S and T, such as S + T?

Identify the domains of S and T individually. The domain of S + T is the intersection of these domains, i.e., all x-values where both S and T are defined. Write this intersection in interval notation.

When finding the composition S(T(x)), what should you check before writing the domain?

Ensure that T(x) is defined for the x-values considered and that T(x) lies within the domain of S. Only those x-values satisfying both conditions are included in the domain of the composite function, expressed in interval notation.

Can the domain of a composite function S ∘ T be different from the domains of S and T? Why?

Yes. The domain of S ∘ T depends on where T is defined and where T(x) falls within S's domain. Even if T and S are individually defined over certain intervals, the composition's domain is restricted to x-values where both conditions hold, which may be narrower than the individual domains.

Additional Resources

Understanding Refer To Functions S And T: How To Find The Indicated Function And Its Domain

In advanced mathematics, particularly in the study of functions and their compositions, the phrase "Refer To Functions S And T. Find The Indicated Function And Write The Domain In Interval Notation" often appears as a challenging yet fundamental problem. This task tests your understanding of function composition, the domain of combined functions, and the ability to interpret function notation correctly. Whether you're a student preparing for exams or a professional brushing up on core concepts, mastering this topic is essential for a solid foundation in algebra and calculus.

In this article, we will explore a comprehensive guide to solving these types of problems. We will break down what it means to "refer to functions," how to find the indicated function, and how to determine the domain in interval notation. With detailed explanations, step-by-step procedures, and illustrative examples, you'll gain confidence in tackling similar questions with clarity and precision.

What Does It Mean to Refer to Functions S and T?

When instructions say "Refer To Functions S and T," it implies that you are given two functions, typically denoted as:

- S(x): The first function

- $T(x)$: The second function

The problem may involve creating a new function based on S and T, such as a composition, sum, difference, or other combinations. Understanding the roles of these functions is crucial because the operations you perform depend heavily on their definitions and domains.

Types of Operations Often Involved

- Function Composition: $(S \circ T)(x) = S(T(x))$
- Sum of Functions: $(S + T)(x) = S(x) + T(x)$
- Difference of Functions: $(S - T)(x) = S(x) - T(x)$
- Product of Functions: $(S \cdot T)(x) = S(x) \cdot T(x)$
- Quotient of Functions: $(S / T)(x) = S(x) / T(x)$, with $T(x) \neq 0$
- Indicated Function (e.g., $S(T(x))$): applying one function to the output of another

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Step-by-Step Guide to Finding the Indicated Function

Step 1: Understand the Notation and What is Being Asked

- Identify which functions are involved.
- Determine the operation: composition, addition, etc.
- Clarify the "indicated function"—for example, is it S composed with T, T composed with S, or some other combination?

Step 2: Write Down the Known Functions

Suppose the functions are given explicitly, such as:

- $S(x) = 2x + 3$
- $T(x) = x^2 - 1$

Having these explicitly allows precise calculations.

Step 3: Find the Indicated Function

Depending on the problem:

- If asked for $S \circ T$ (composition):
Compute $S(T(x))$
Replace the x in $S(x)$ with $T(x)$.

- If asked for $T \circ S$:
Compute $T(S(x))$
Replace the x in $T(x)$ with $S(x)$.

- If asked for $S + T$:
Sum the functions pointwise.

- If asked for a specific operation:

Follow the same logic, applying the operation to the functions.

Step 4: Simplify the Expression

Carry out algebraic manipulations to express the new function in simplest form.

Step 5: Find the Domain of the New Function

This is often the most nuanced part. To determine the domain:

- Start with the domains of S and T.
- For composition:
The domain of $S \circ T$ consists of all x in the domain of T for which T(x) is in the domain of S.

- For algebraic sums or differences:
The domain is the intersection of the domains of the involved functions.

- For quotients:
Exclude values where the denominator is zero.

Express the final domain in interval notation after considering all restrictions.

Illustrative Examples

Example 1: Composition of Functions S and T

Suppose:

- $S(x) = \sqrt{x - 2}$
- $T(x) = x + 3$

Find: $(S \circ T)(x)$ and its domain.

Solution:

1. Write the composition:

$(S \circ T)(x) = S(T(x)) = \sqrt{(x + 3) - 2} = \sqrt{x + 1}$

2. Determine the domain:

- Domain of T(x): all real numbers (since x + 3 is defined for all real x).
- For $S(T(x)) = \sqrt{x + 1}$, the radicand must be ≥ 0:

$x + 1 \geq 0 \rightarrow x \geq -1$

- Also, T(x) must be in the domain of S, which is all $x \geq 2$, but since S involves $\sqrt{x - 2}$, T(x) outputs are real for x + 3, so T(x) can be any real number, but the composition requires $x \geq -1$.

3. Final domain:

All x such that $x \geq -1$.

Answer:

- Indicated function: $(S \circ T)(x) = \sqrt{x + 1}$
- Domain: $[-1, \infty)$

Example 2: Sum of Functions S and T

Suppose:

- $S(x) = 1 / (x - 4)$
- $T(x) = \sqrt{x}$

Find: $(S + T)(x)$ and its domain.

Solution:

1. Express the sum:

$(S + T)(x) = 1 / (x - 4) + \sqrt{x}$

2. Determine the domains:

- Domain of S: $x \neq 4$ (since denominator cannot be zero).
- Domain of T: $x \geq 0$ (since square root requires non-negative radicand).

3. Combined domain:

The intersection of the two domains:

$x \geq 0$ and $x \neq 4$

So the domain is:

$[0, 4) \cup (4, \infty)$

4. Final answer:

- Indicated function: $1 / (x - 4) + \sqrt{x}$
- Domain: $[0, 4) \cup (4, \infty)$

Special Considerations When Working with Domains

- Always check for restrictions imposed by radicals ($\sqrt{x}$): $x \geq 0$.

- For rational functions, ensure the denominator ≠ 0.
- When composing functions, ensure the output of the inner function lies within the domain of the outer function.
- Be aware of potential extraneous restrictions introduced by operations.

Summary of Key Steps

1. Identify the functions involved and their explicit forms.
2. Determine the type of operation (composition, sum, difference, quotient).
3. Write the resulting function explicitly.
4. Simplify the expression if possible.
5. Ascertain the domain restrictions from each component:
- Radical expressions
- Rational expressions
- Function composition conditions
6. Express the domain in interval notation considering all restrictions.

Final Tips for Success

- Practice with diverse functions: polynomial, radical, rational, exponential.
- Always verify the domain restrictions after each step.
- Draw number line diagrams to visualize domain restrictions.
- Check your final domain by testing critical points, especially boundary points.

Conclusion

"Refer To Functions S And T. Find The Indicated Function And Write The Domain In Interval Notation" is a fundamental task that combines algebraic manipulation with a deep understanding of domain restrictions. By methodically following the steps outlined—clarifying the functions involved, performing the operation, simplifying, and carefully analyzing domain restrictions—you can confidently solve these problems. Mastery of these skills not only enhances your problem-solving toolkit but also prepares you for more advanced topics in calculus and mathematical analysis. Keep practicing, and soon these concepts will become second nature!

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