Sketch The Region Enclosed By The Given Curves. Decide Whether To Integrate With Respect To X Or Y. Draw

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Introduction

Understanding how to sketch the region enclosed by curves and determine the best way to perform integration is a fundamental skill in calculus. This process not only helps in visualizing the problem but also simplifies the computation of areas, volumes, and other related quantities. When dealing with multiple curves, selecting the appropriate variable of integration—either x or y—is crucial for setting up the integral efficiently. This article provides a comprehensive guide on how to sketch the enclosed region, decide whether to integrate with respect to x or y, and illustrate the process with practical examples.

Why Sketching the Enclosed Region Matters

Sketching the enclosed region formed by given curves is essential because:

- It provides a clear visual understanding of the limits of integration.
- It helps identify the shape and boundaries of the region.
- It reveals intersections and overlaps of curves, which are crucial in setting up integrals.
- It aids in choosing the most straightforward variable of integration.

Without a proper sketch, setting up integrals can become confusing, especially for complex regions.

Understanding the Problem: Given Curves

In many calculus problems, you are given two or more curves, and the goal is to find the area enclosed between them. Typical scenarios include:

- Finding the area between two functions, such as \(y = f(x) \) and \(y = g(x) \).
- Enclosed regions bounded by curves in different quadrants.
- Regions enclosed by curves that intersect at multiple points.

The key steps involve:

1. Identifying the curves involved.
2. Finding the intersection points.
3. Analyzing the shape of the region.

4. Deciding the best variable of integration.
5. Sketching the region accordingly.

Step-by-Step Guide to Sketching and Deciding the Variable of Integration

1. Identify the Curves and Their Equations

Begin by carefully noting the equations of the curves involved. For example:

- Lines: $y = mx + c$
- Parabolas: $y = ax^2 + bx + c$
- Circles: $(x - h)^2 + (y - k)^2 = r^2$
- Other functions: $y = \sqrt{x}$, $y = \sin x$, etc.

Understanding the nature of these curves helps predict the shape of the enclosed region.

2. Find the Intersection Points

Calculate the points where the curves intersect by solving their equations simultaneously. These points determine the limits of integration. For example:

- Set $y = f(x)$ and $y = g(x)$ equal to find the intersection x-values.
- For curves in terms of y, set their y-equations equal to find corresponding y-values.

Having the intersection points helps in sketching the region accurately.

3. Analyze the Region's Shape and Boundaries

Based on the intersection points and the equations:

- Sketch the curves on a coordinate plane.
- Shade the region enclosed by the curves.
- Observe whether the region spans more in the x-direction or y-direction.

This analysis will inform whether the integral should be with respect to x or y.

4. Decide on the Variable of Integration

Choosing whether to integrate with respect to x or y depends on:

- The orientation of the region.
- The ease of expressing the bounds.
- The nature of the curves.

When to integrate with respect to x:

- The region is bounded between curves that can be expressed as $y = f(x)$ and $y = g(x)$.
- The limits of x are clear from the intersection points.
- The region's shape is more straightforward horizontally.

When to integrate with respect to y:

- The region is bounded between curves that can be expressed as $x = f(y)$ and $x = g(y)$.
- The limits of y are clear from the intersection points.
- The region's shape is more straightforward vertically.

Practical Examples

Example 1: Enclosed Region Between Two Functions

Given Curves:

- $y = x^2$
- $y = 4x$

Step 1: Find intersection points.

Set $x^2 = 4x$:

$$
x^2 - 4x = 0 \Rightarrow x(x - 4) = 0
$$

Solutions:

$$
x = 0 \quad \text{or} \quad x = 4
$$

Corresponding y-values:

- At $x=0$, $y=0$.
- At $x=4$, $y=16$.

Step 2: Sketch the curves.

- $y = x^2$ is a parabola opening upwards.
- $y=4x$ is a straight line passing through the origin with slope 4.

Step 3: Determine the region.

Between $x=0$ and $x=4$:

- $y=4x$ is above $y=x^2$.

Step 4: Decide variable of integration.

Since the region is bounded between $x=0$ and $x=4$, and the curves are expressed as functions of x:

- It’s easier to integrate with respect to x.

Step 5: Set up the integral.

Area A:

$$
A = \int_{0}^{4} [\, \text{top curve} - \text{bottom curve} \,] \, dx = \int_{0}^{4} (4x - x^2) \, dx
$$

Step 6: Sketch the region.

- Draw the parabola $y=x^2$.
- Draw the line $y=4x$.
- Shade the area between $x=0$ and $x=4$.

Example 2: Enclosed Region Between Two Curves in Terms of y

Given Curves:

- $y = \sqrt{x}$
- $y = x/4$

Step 1: Find intersection points.

Express both y-curves:

- $y = \sqrt{x} \Rightarrow x = y^2$
- $y = x/4 \Rightarrow x=4y$

Set equal:

$$
\sqrt{x} = \frac{x}{4}
$$

Express in terms of x:

$$
\sqrt{x} = \frac{x}{4}
$$

Multiply both sides by 4:

$$
4 \sqrt{x} = x
$$

Let $t = \sqrt{x} \Rightarrow x = t^2$:

$$
4t = t^2
$$

Rearranged:

$$
t^2 - 4t = 0 \Rightarrow t(t - 4) = 0
$$

Solutions:

$$
t=0 \Rightarrow x=0
$$
$$
t=4 \Rightarrow x=16
$$

Corresponding y:

- For $x=0$, $y=0$.
- For $x=16$, $y=\sqrt{16}=4$.

Step 2: Sketch the curves.

- $y= \sqrt{x}$ is a rightward-opening curve.
- $y= x/4$ is a straight line.

Step 3: Find the bounds in y.

Express x in terms of y:

- $x= y^2$
- $x=4 y$

In the y-direction:

- y varies from 0 to 4.

Step 4: Determine which curve is to the right or left.

At a particular y, the x-values are:

- $x= y^2$ (left boundary)
- $x= 4 y$ (right boundary)

Between $y=0$ and $y=4$, the region is bounded by these two curves.

Step 5: Choose to integrate with respect to y.

- Since x is expressed as functions of y, and the bounds are in y, it's more efficient to integrate with respect to y.

Step 6: Set up the integral.

Area A:

$$
A = \int_{0}^{4} [\, \text{right curve} - \text{left curve} \,] \, dy = \int_{0}^{4} (4 y - y^2) \, dy
$$

Step 7: Sketch the region.

- Draw the parabola $y= \sqrt{x}$.
- Draw the line $y= x/4$.
- Shade the region between $y=0$ and $y=4$, bounded by the curves.

Tips for Deciding the Best Variable of Integration

- Plot the curves first: Visualizing the region helps in understanding its shape.
- Express the boundaries explicitly: Know whether the curves are functions of x or y.
- Check the intersection points: Limits of integration are based on these points.
- Compare complexity: Choose the

Frequently Asked Questions

How do I determine whether to integrate with respect to x or y when sketching the region enclosed by two curves?

Analyze the curves to see which variable has easier limits of integration. If the region is bounded by functions y=f(x) and y=g(x), it's often easier to integrate with respect to x. Conversely, if the region is better described by x=h(y) and x=k(y), then integrating with respect to y is preferable.

What steps should I follow to accurately sketch the region enclosed by two curves?

First, find the points of intersection by setting the equations equal. Next, identify the bounds of the region and determine which function is on top or on the right. Then, decide the variable of integration based on the region's shape, and finally, plot the curves and shade the enclosed area accordingly.

When the curves intersect at multiple points, how do I decide the order of integration?

Identify each intersection point and partition the region accordingly. For each sub-region, choose the variable of integration that simplifies the limits—often integrating with respect to the variable that changes least within each sub-region. Sketching the entire enclosed area helps visualize the appropriate order.

Can you give an example of deciding whether to integrate with respect to x or y for a region bounded by y=x^2 and y=4?

Sure! The curves y=x^2 (a parabola) and y=4 (a horizontal line) intersect at x=±2. Since y=4 is horizontal, integrating with respect to x requires limits from -2 to 2, and the top and bottom functions are y=4 and y=x^2. Alternatively, integrating with respect to y involves limits from y=0 to y=4, with x bounds from -√y to √y. Choosing the variable depends on which limits are simpler to describe—often, integrating with respect to x is easier here.

What are common mistakes to avoid when sketching the region and deciding the variable of integration?

Common mistakes include neglecting to find intersection points, choosing the wrong variable that complicates the integral, and misidentifying which curve is on top or on the right. Always verify the intersection points, analyze the region carefully, and consider which variable leads to simpler limits and integrand for the integral.

Additional Resources

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Introduction

In the realm of calculus and analytical geometry, the task of sketching the region enclosed by given curves forms a foundational skill that bridges the gap between algebraic expressions and their geometric interpretations. Whether for calculating areas, volumes, or understanding the spatial relationships of functions, accurately visualizing the bounded region is paramount. This process involves not only plotting the curves but also making strategic decisions about the variable of integration—whether to integrate with respect to \(x\) or \(y\)—a choice that significantly affects the ease and accuracy of subsequent calculations.

This article aims to provide a comprehensive, detailed exploration of how to approach such problems, emphasizing the importance of understanding the nature of the curves, methods for sketching the region, and criteria for choosing the most effective variable of integration. Through systematic analysis, illustrative examples, and practical tips, readers will gain the insights needed to master this essential aspect of multivariable calculus.

Understanding the Problem: The Core Components

Before delving into the methods, it is critical to understand the fundamental components involved in sketching regions enclosed by curves.

The Given Curves

- Types of Curves: These can include lines, parabolas, circles, ellipses, hyperbolas, or more complex functions such as sinusoidal or exponential curves.
- Equations: Each curve is described by an algebraic equation, such as $y = f(x)$ or $x = g(y)$. Recognizing the form helps determine the plotting strategy.

The Enclosed Region

- Boundaries: The region is enclosed between the curves, which may intersect at specific points.
- Intersection Points: Critical for determining the limits of integration and the shape of the region.
- Shape and Symmetry: Understanding whether the region is symmetric about axes or other lines influences the sketching approach.

The Goal

- Visual Representation: To sketch the region accurately.
- Mathematical Analysis: To set up the proper integral, choosing the variable of integration carefully for simplicity and precision.

Step-by-Step Approach to Sketching and Analyzing the Region

To systematically approach the problem, follow these stages:

1. Analyze the Given Curves

- Identify the equations: Recognize whether they are linear, quadratic, or higher degree.
- Determine the domain and range: Find where the curves are defined.
- Find intersection points: Solve the system of equations to find points where the curves meet.

2. Find Intersection Points

- Solve algebraically: Set the equations equal or solve simultaneously.
- Graphically (if possible): Plot the curves to visually identify intersections.
- Numerical methods: Use substitution or computational tools for complex equations.

3. Determine the Enclosed Region

- Sketch the curves: Using the equations, plot points, and draw the curves to visualize the shape.
- Identify the region: Clearly mark the area enclosed between the curves.
- Note symmetry: Recognize axes or points of symmetry that can simplify calculations.

4. Decide the Variable of Integration

- Assess the shape: Determine whether slicing the region horizontally (with respect to y) or vertically (with respect to x) makes the integral easier.

- Consider the functions: Some regions are naturally better expressed as x-limits between two functions of y, or vice versa.
- Evaluate limits: Check which approach yields straightforward limits (constants or simple functions).

Choosing Between Integrating With Respect to x or y

The decision on whether to integrate with respect to x or y hinges on multiple factors, including the shape of the region, the form of the boundary curves, and computational convenience.

When to Integrate With Respect To x

Preferred when:

- The region is bounded between functions of x, i.e., $y = f(x)$ and $y = g(x)$.
- The top and bottom curves are explicitly expressed as functions of x.
- The region extends over a simple x-interval, and the y-limits are functions that are easier to express as functions of x.

Advantages:

- Clear limits if the region is between two vertical boundary lines.
- Easier for functions where y is explicitly solved in terms of x.

When to Integrate With Respect To y

Preferred when:

- The region is bounded between functions of y, i.e., $x = h(y)$ and $x = k(y)$.
- The curves are better expressed as functions of y—for example, a horizontal parabola or a circle centered on the y-axis.
- The region has more straightforward y-limits, especially if the boundary curves are functions of y.

Advantages:

- Simplifies the limits if the region is bounded between two horizontal curves.
- Useful when the region is naturally sliced horizontally, such as in problems involving rotation about the x-axis.

Criteria for Decision

- Shape of the region: Sketch the curves to see which slicing produces simpler limits.
- Ease of limits determination: Choose the variable that yields constant or simple limits.
- Complexity of the integrand: Sometimes, the algebraic form of the integrand favors one variable over the other based on substitution convenience.

Practical Tips

- Sketch first: Always draw the curves to visualize the region.

- Test both approaches: If uncertain, sketch the slices for both x and y to compare complexity.
- Check intersection points: Limits are typically anchored between these points.

Illustrative Examples

Example 1: Enclosed Region Between a Line and a Parabola

Suppose the region is bounded by:

- $y = x^2$
- $y = 4x$

Step 1: Find intersection points:

Set $x^2 = 4x$

$x^2 - 4x = 0$

$x(x - 4) = 0$

- $x = 0$ and $x = 4$

Corresponding y-values:

- At $x=0$, $y=0$
- At $x=4$, $y=16$

Step 2: Sketch the curves:

Plot $y=x^2$ (a parabola opening upward) and $y=4x$ (a straight line).

Step 3: Identify the region:

Between $x=0$ and $x=4$, the parabola $y=x^2$ is below the line $y=4x$.

Step 4: Decide variable of integration:

- Since the region is bounded between $x=0$ and $x=4$, and the curves are functions of x, integrating with respect to x is straightforward.

Step 5: Set up the integral:

Area $A = \int_{x=0}^{4} [\text{top curve} - \text{bottom curve}] dx$

- Top curve: $y=4x$
- Bottom curve: $y=x^2$

So,

$$
A = \int_0^4 (4x - x^2) dx
$$

Example 2: Enclosed Region Between a Circle and a Horizontal Line

Suppose the region is bounded by:

- $x^2 + y^2 = 9$ (a circle of radius 3)
- $y = 2$

Step 1: Find intersection points:

Substitute $y=2$:

$x^2 + 4 = 9$

$x^2=5$

$x= \pm \sqrt{5}$

Step 2: Sketch the circle and the line:

- The circle centered at origin with radius 3.
- The line $y=2$ cuts through the circle at $x= \pm \sqrt{5}$.

Step 3: Identify the enclosed region:

- The region is the "cap" of the circle above $y=2$, between $x= -\sqrt{5}$ and $x= \sqrt{5}$.

Step 4: Choosing the variable of integration:

- Here, expressing x in terms of y:

From the circle:

$$
x=\pm \sqrt{9 - y^2}
$$

- For a horizontal slice at height y, the x-limits are $-\sqrt{9 - y^2}$ to $\sqrt{9 - y^2}$.

- Since the boundary $y=2$ is horizontal, and the region extends from $y=0$ up to $y=2$, integrating with respect to y is natural.

Step 5: Set up the integral:

$$
\text{Area} = \int_{y=0}^{2} \left[2 \sqrt{9 - y^2} \right] dy

\]

(since the horizontal width at height \(y \) is \(2 \sqrt{9 - y^2} \)).

This approach simplifies the computation, especially

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