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Understanding how to derive the time-domain sequence $X[n]$ from a given Z-transform $X(z)$ is a fundamental skill in signal processing and systems analysis. In this article, we will explore the step-by-step process of converting the provided Z-transform:

$$
X(z) = \frac{1 - \frac{1}{6} z^{-3}}{1 - \frac{1}{4} z^{-1}}
$$

with the Region of Convergence (ROC): $|z| > \frac{1}{4}$, into the corresponding discrete-time sequence $X[n]$.

Understanding the Given Z-transform

Before diving into the inverse Z-transform process, it's essential to interpret the given $X(z)$.

Expression Breakdown

- Numerator: $1 - \frac{1}{6} z^{-3}$
- Denominator: $1 - \frac{1}{4} z^{-1}$

This rational function resembles the ratio of polynomials in z^{-1}. To facilitate easier inverse transformation, rewriting $X(z)$ in a more manageable form is advisable.

Step 1: Rewrite $X(z)$ in a Power Series Form

To invert the Z-transform, expressing $X(z)$ as a sum of simpler terms or as a power series is helpful.

Expressing $X(z)$ as a ratio of polynomials in z^{-1}:

$$
X(z) = \frac{1 - \frac{1}{6} z^{-3}}{1 - \frac{1}{4} z^{-1}}
$$

Note that the denominator suggests a geometric series expansion if $|z| > 1/4$.

Step 2: Express $X(z)$ as a Sum of Simpler Terms

One effective approach is to perform polynomial division or to express $X(z)$ as a product involving known Z-transform pairs.

Method 1: Polynomial division

Dividing numerator and denominator to express $X(z)$ as a sum of simpler terms.

Step 3: Algebraic Manipulation

Let's manipulate $X(z)$ into a form suitable for inverse Z-transform.

Rewrite numerator:

$$
1 - \frac{1}{6} z^{-3}
$$

Rewrite denominator:

$$
1 - \frac{1}{4} z^{-1}
$$

Observe that the denominator resembles the generating function for a geometric series:

$$
\frac{1}{1 - a z^{-1}} = \sum_{k=0}^{\infty} a^{k} z^{-k}
$$
for $|z| > |a|$.

Here, $a = \frac{1}{4}$.

Step 4: Express $X(z)$ as a Sum

Using this, we can write:

$$
X(z) = \left(1 - \frac{1}{6} z^{-3}\right) \times \frac{1}{1 - \frac{1}{4} z^{-1}}
$$

which becomes:

$$
X(z) = \left(1 - \frac{1}{6} z^{-3}\right) \times \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k}
$$

valid for $|z| > 1/4$.

Step 5: Expand the Expression

Now, multiply term-by-term:

$$
X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k} - \frac{1}{6} z^{-3} \times \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k}
$$

which simplifies to:

$$
X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k} - \frac{1}{6} \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-(k+3)}
$$

Step 6: Express in Terms of z^{-n}

Reindex the second sum to align powers:

Let $m = k + 3$, so when $k=0$, $m=3$:

$$
X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k} - \frac{1}{6} \sum_{m=3}^{\infty} \left(\frac{1}{4}\right)^{m-3} z^{-m}
$$

Expressed explicitly:

$$
X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k z^{-k} - \frac{1}{6} \sum_{m=3}^{\infty} \left(\frac{1}{4}\right)^{m-3} z^{-m}
$$

Step 7: Write $X[n]$ (Inverse Z-transform)

Recall that the inverse Z-transform of z^{-n} is $\delta[n - m]$, and the sum over z^{-n} corresponds to the sequence $X[n]$.

From the previous step:

$$
X[n] = \text{coefficients of } z^{-n}
$$

Thus, the sequence $X[n]$ is:

$$
X[n] = \left(\frac{1}{4}\right)^n u[n] - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n-3]
$$

where $u[n]$ is the unit step function ensuring causality (sequence is zero for $n < 0$).

Final Expression for $X[n]$:

$$
\boxed{
X[n] = \left(\frac{1}{4}\right)^n u[n] - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n-3]
}
$$

This formula gives the explicit time-domain sequence corresponding to the given Z-transform.

Additional Details and Explanation

1. Unit Step Function $u[n]$:

- Ensures causality, meaning $X[n] = 0$ for $n < 0$.

2. Sequence Components:

- The first term $\left(\frac{1}{4}\right)^n u[n]$ corresponds to a basic geometric sequence starting at $n=0$.

- The second term $- \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n-3]$ introduces a delayed, scaled version of the geometric sequence, starting at $n=3$.

3. Region of Convergence (ROC):

- The ROC for this sequence is $|z| > 1/4$, consistent with the geometric series expansion.

Summary of the Process

To recap, here are the key steps taken to find $X[n]$:

1. Express $X(z)$ as a ratio of polynomials in z^{-1}.

2. Identify the structure suitable for series expansion.

3. Use geometric series expansion for the denominator.

4. Multiply out and reindex sums to express $X(z)$ as a sum of terms involving z^{-n}.

5. Extract the sequence coefficients corresponding to z^{-n} to determine $X[n]$.

6. Incorporate unit step functions to maintain causality.

Additional Tips for Inverse Z-Transform

- Partial Fraction Decomposition: When dealing with more complex rational functions, partial fractions can simplify the inverse process.

- Recognize Standard Z-transform Pairs: Many sequences (exponentials, delta functions, unit step sequences) have known Z-transform pairs that can be directly inverted.

- Use Power Series Expansion: For rational functions where the denominator is geometric, expanding as a geometric series is often the easiest approach.

- Check ROC and Causality: Ensure that the ROC aligns with the sequence's causality (causal sequences have ROC outside the outermost pole).

Conclusion

Deriving $X[n]$ from a Z-transform like:

$$
X(z) = \frac{1 - \frac{1}{6} z^{-3}}{1 - \frac{1}{4} z^{-1}}
$$

with ROC $|z| > 1/4$, involves strategic algebraic manipulation, series expansion, and careful reindexing. The key takeaways are to recognize geometric series patterns, properly handle delays introduced by powers of z^{-1}, and include the unit step functions to specify causality.

Understanding

Frequently Asked Questions

How do I find the time domain sequence $X[n]$ from the given Z-transform $X(z) = (1 - (1/6)z^{-3}) / (1 - (1/4)z^{-1})$?

To find $X[n]$, perform partial fraction expansion of $X(z)$, then invert each term using known Z-transform pairs, considering the ROC $|z| > 1/4$. This typically involves expressing $X(z)$ as a sum of simpler fractions whose inverse Z-transforms are standard sequences.

What is the significance of the ROC $|z| > 1/4$ in determining $X[n]$?

The ROC $|z| > 1/4$ indicates that the sequence $X[n]$ is right-sided (causal). When performing inverse Z-transform, this ROC guides us to choose the appropriate region and ensure the resulting sequence is causal and stable.

How can I apply partial fraction decomposition to $X(z)$ to simplify the inverse Z-transform process?

Express $X(z)$ as a sum of simpler fractions by factoring the denominator, then decompose into terms like $A / (1 - r z^{-1})$. Each term corresponds to a known inverse Z-transform, making it easier to find $X[n]$.

Are there standard Z-transform pairs I can use to invert the components of $X(z)$?

Yes, standard pairs include sequences like $(r)^n u[n]$, which corresponds to $1 / (1 - r z^{-1})$ with ROC $|z| > |r|$. Use these pairs to invert each partial fraction term once decomposed.

Can I use software tools like MATLAB or WolframAlpha to find $X[n]$, and how?

Yes, tools like MATLAB's 'iztrans' function or WolframAlpha's inverse Z-transform feature can compute $X[n]$ directly from $X(z)$. Input the Z-transform expression and specify the ROC to obtain the sequence efficiently.

Additional Resources

Z-transform $X(z) = 1 - (1/6)z^{-3} / [1 - (1/4)z^{-1}]$ with ROC $|z| > 1/4$: How to Find the Sequence $X[n]$

Understanding how to derive the time-domain sequence $X[n]$ from its Z-transform $X(z)$ is fundamental in signal processing, control systems, and digital filter design. The provided Z-transform expression,

$$
X(z) = \frac{1 - \frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}},
$$

along with its Region of Convergence (ROC) $|z| > \frac{1}{4}$, offers an insightful example of how rational Z-transforms translate into discrete sequences. This article aims to guide you through the process of obtaining $X[n]$, step-by-step, analyzing the underlying principles, methods, and practical considerations involved.

Understanding the Given Z-transform and Its Components

Before diving into the derivation of $X[n]$, it’s essential to interpret the structure of the given Z-transform.

Expression Breakdown

The Z-transform:

$$
X(z) = \frac{1 - \frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}
$$

can be viewed as a rational function composed of a numerator and denominator polynomial in z^{-1}. The numerator contains two terms: a constant and a cubic delay term. The denominator is a first-order polynomial in z^{-1}. The ROC specified as $|z| > 1/4$ indicates the region where the Z-transform converges.

Significance of the ROC

- $|z| > 1/4$ suggests that the sequence $X[n]$ is right-sided or causal, i.e., $X[n] = 0$ for $n < 0$.

- The ROC helps determine the nature of the sequence (causal, anti-causal, or two-sided), influencing the method used for inverse Z-transform.

Step 1: Simplify the Z-transform Expression

To find $X[n]$, it’s often easier to manipulate $X(z)$ into a form suitable for inverse Z-transform techniques like partial fraction expansion or recognizing standard Z-transform pairs.

Rewrite $X(z)$:

$$
X(z) = \frac{1 - \frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

Expressing in terms of positive powers:

$$
X(z) = \frac{z^{3} - \frac{1}{6}}{z^{3} - \frac{1}{4}z^{2}},
$$

but it’s more straightforward to work directly with the original form or manipulate it into a sum of simpler fractions.

Step 2: Express $X(z)$ as a Sum of Simpler Terms

The key idea is to perform polynomial division or partial fraction expansion to decompose $X(z)$ into terms whose inverse Z-transforms are known.

Polynomial division:

Dividing numerator by denominator:

$$
X(z) = \frac{1 - \frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

Alternatively, multiply numerator and denominator by z^{3}:

$$
X(z) = \frac{z^{3} - \frac{1}{6}}{z^{3} - \frac{1}{4}z^{2}}.
$$

But it's often more straightforward to work with the original form for inverse transforms, especially since the denominator resembles a first-order polynomial in z^{-1}.

Step 3: Recognize the Structure for Inverse Z-Transform

The original Z-transform resembles a rational function that can be broken into simpler parts:

$$
X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} - \frac{\frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

This step involves splitting the fraction:

$$
X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} - \frac{\frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

Step 4: Find the Inverse Z-transforms of Individual Terms

Inverse Z-transform of $\frac{1}{1 - a z^{-1}}$

The standard Z-transform pair:

$$
Z\{a^{n} u[n]\} = \frac{1}{1 - a z^{-1}}, \quad |z| > |a|,
$$

where $u[n]$ is the unit step function, and $|z| > |a|$.

Given ROC $|z| > 1/4$:

$$
Z^{-1}\left\{\frac{1}{1 - \frac{1}{4}z^{-1}}\right\} = \left(\frac{1}{4}\right)^n u[n].
$$

Inverse Z-transform of $\frac{z^{-k}}{1 - a z^{-1}}$

Similarly,

$$
Z\{a^{n} u[n - k]\} = a^{n} u[n - k],
$$

and multiplying by z^{-k} shifts the sequence by k samples:

$$
Z^{-1}\left\{\frac{z^{-k}}{1 - a z^{-1}}\right\} = a^{n - k} u[n - k].
$$

Applying this to the second term:

$$
\frac{\frac{1}{6} z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

Step 5: Derive $X[n]$ Using Known Inverse Z-transform Pairs

Putting it all together,

$$
X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} - \frac{\frac{1}{6}z^{-3}}{1 - \frac{1}{4}z^{-1}}.
$$

So,

$$
X[n] = \left(\frac{1}{4}\right)^n u[n] - \frac{1}{6} \times \left(\frac{1}{4}\right)^{n-3} u[n - 3].
$$

Here, the second term is shifted by 3 samples, and the sequence is scaled accordingly.

Final expression for $X[n]$:

$$
X[n] = \left(\frac{1}{4}\right)^n u[n] - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n - 3].
$$

Step 6: Simplify the Final Sequence Expression

Expressing $X[n]$ explicitly:

$$
X[n] = \left(\frac{1}{4}\right)^n u[n] - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n - 3].
$$

Note that for $n < 0$, $X[n] = 0$ due to the unit step functions and the ROC.

For $n \geq 0$:

$$
X[n] = \left(\frac{1}{4}\right)^n - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} u[n - 3].
$$

This can be written as:

$$
X[n] = \left(\frac{1}{4}\right)^n - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3} \quad \text{for } n \geq 3,
$$

and

$$
X[n] = \left(\frac{1}{4}\right)^n \quad \text{for } 0 \leq n < 3.
$$

Summary of the Sequence $X[n]$

$$
X[n] =
\begin{cases}
\left(\frac{1}{4}\right)^n, & n=0,1,2, \\
\left(\frac{1}{4}\right)^n - \frac{1}{6} \left(\frac{1}{4}\right)^{n-3}, & n \geq 3, \\
0, & n < 0.
\end{cases}
$$

Alternatively, factoring out $\left(\frac{1}{4}\right)^n$:

\[
X[n] = \left(\frac{1}{4}\right)^n \left[u[n] - \frac{1}{6} \left(\frac{1}{4}\right)^{-3} u[n - 3] \right].
\]

Since \(\left(\frac{1}{4}\right)^{-3} = 4^{3} = 64\),

\[
X[n] = \left(\frac{1}{4}\right)^n \left[u[n] - \frac{1}{6} \times 64 \times

So For This Z Transform X Z 1 1 6 Z 3 1 1 4 Z 1 Withroc Z Gt 1 4 I Need Know How To Get X N

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