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Solve The Given Initial Value Problem. Y'' - 5y' +6y=0; Y(0) = -- The Solution Is Y(t) = -1/2 2' Y'(0)=-

Understanding how to solve second-order linear differential equations with initial conditions is fundamental in various fields such as engineering, physics, and applied mathematics. In this article, we will explore in detail the process of solving the initial value problem (IVP):

$$Y'' - 5Y' + 6Y = 0$$

with certain initial conditions (which we will clarify and interpret), and how to derive the explicit solution $Y(t)$. We will also discuss related concepts, solution methods, and applications to deepen your understanding of differential equations.

Introduction to Second-Order Linear Differential Equations

A second-order linear differential equation is an equation involving the second derivative of a function $Y(t)$, its first derivative $Y'(t)$, and the function itself, typically expressed as:

$$aY'' + bY' + cY = 0$$

where a, b, and c are constants. These equations frequently model physical systems such as mechanical vibrations, electrical circuits, and population dynamics.

The general solution to such an equation depends on the roots of its characteristic (auxiliary) equation, which determines whether solutions are real and distinct, repeated, or complex conjugates.

Formulating the Characteristic Equation

Given the differential equation:

$$Y'' - 5Y' + 6Y = 0$$

we first find its characteristic equation:

$$r^2 - 5r + 6 = 0$$

This quadratic equation is obtained by replacing derivatives Y' with r, and Y'' with r^2, leading to:

$$r^2 - 5r + 6 = 0$$

To solve for r, we factor or apply the quadratic formula:

$$r^2 - 5r + 6 = (r - 2)(r - 3) = 0$$

which yields the roots:

$$r_1 = 2, \quad r_2 = 3$$

Because these roots are real and distinct, the general solution to the differential equation is:

$$Y(t) = C_1 e^{2t} + C_2 e^{3t}$$

where C_1 and C_2 are constants determined by initial conditions.

Applying Initial Conditions

To solve the initial value problem completely, initial conditions are necessary. They typically specify the value of the function and its derivative at a particular point, often $t=0$:

$$Y(0) = y_0$$
$$Y'(0) = y_1$$

In the problem statement, initial conditions are given as:

- $Y(0) = --$ (which appears incomplete or possibly a typo)
- $Y'(0) = -$ (also incomplete or missing value)

The provided snippet mentions: "The Solution Is Y(t) = -1/2 2' Y'(0)=-", which suggests some formatting issues or incomplete data.

Assumption for clarity:
Suppose the initial conditions are:

$$ Y(0) = y_0 $$
$$ Y'(0) = y_1 $$

and the constants are to be determined once these initial conditions are known.

Interpreting the Given Data

Given the incomplete information, let’s assume typical initial conditions used in such problems for demonstration purposes:

- $Y(0) = 0$
- $Y'(0) = -1$

This assumption allows us to proceed with solving the problem and understanding the method.

Calculating Constants Using Initial Conditions

Given the general solution:

$$ Y(t) = C_1 e^{2t} + C_2 e^{3t} $$

we compute its derivative:

$$ Y'(t) = 2 C_1 e^{2t} + 3 C_2 e^{3t} $$

Applying the initial conditions:

1. At $t=0$:

$$ Y(0) = C_1 e^{0} + C_2 e^{0} = C_1 + C_2 = y_0 $$

2. The derivative at $t=0$:

$$ Y'(0) = 2 C_1 e^{0} + 3 C_2 e^{0} = 2 C_1 + 3 C_2 = y_1 $$

Using the assumed initial conditions:

$$ y_0 = 0 $$
$$ y_1 = -1 $$

we get the system:

$$ C_1 + C_2 = 0 $$
$$ 2 C_1 + 3 C_2 = -1 $$

From the first equation:

$$ C_2 = -C_1 $$

Substitute into the second:

$$ 2 C_1 + 3(-C_1) = -1 $$
$$ 2 C_1 - 3 C_1 = -1 $$
$$ - C_1 = -1 $$
$$ C_1 = 1 $$

Then:

$$ C_2 = - C_1 = -1 $$

Explicit Solution to the IVP

With the constants determined, the explicit solution to our initial value problem is:

$$ \boxed{
Y(t) = e^{2t} - e^{3t}
} $$

This solution satisfies the differential equation and the initial conditions (assuming the guessed initial data).

Verifying the Solution

Verifying the solution involves substituting $Y(t)$ into the original differential equation to confirm correctness.

Calculate $Y'(t)$:

$$ Y'(t) = 2 e^{2t} - 3 e^{3t} $$

Calculate $Y''(t)$:

$$ Y''(t) = 4 e^{2t} - 9 e^{3t} $$

Substitute into the differential equation:

$$ Y'' - 5Y' + 6Y $$

which becomes:

$$
(4 e^{2t} - 9 e^{3t}) - 5 (2 e^{2t} - 3 e^{3t}) + 6 (e^{2t} - e^{3t})
$$

Simplify each term:

$$
4 e^{2t} - 9 e^{3t} - 10 e^{2t} + 15 e^{3t} + 6 e^{2t} - 6 e^{3t}
$$

Combine like terms:

$$
(4 e^{2t} - 10 e^{2t} + 6 e^{2t}) + (-9 e^{3t} + 15 e^{3t} - 6 e^{3t})
$$
$$
(0 e^{2t}) + (0 e^{3t}) = 0
$$

Thus, the solution satisfies the differential equation, confirming correctness.

Applications of Solving Second-Order Differential Equations

Understanding how to solve such equations is crucial in modeling real-world systems:

- **Mechanical Vibrations:** Modeling mass-spring systems, where the displacement $Y(t)$ obeys differential equations similar to the one discussed.
- **Electrical Circuits:** RLC circuits' charge and current dynamics are governed by second-order differential equations.
- **Population Dynamics:** Certain models incorporate second-order equations to account for acceleration effects in population growth or decline.
- **Control Systems:** Designing controllers involves solving differential

equations to predict system behavior.

Conclusion and Summary

In this comprehensive guide, we've outlined the step-by-step process of solving the initial value problem:

$$
Y'' - 5Y' + 6Y = 0
$$

with initial conditions, possibly like $Y(0) = 0$ and $Y'(0) = -1$. The key steps involved include:

1. **Formulating the characteristic equation:** $r^2 - 5r + 6 = 0$
2. **Finding roots:** $r_1=2$, $r_2=3$
3. **Constructing the general solution:** $Y(t) = C_1 e^{2t} + C_2 e^{3t}$
4. **Applying initial conditions:** to determine C_1 and C_2
5. **Verifying the solution:** by substitution into the original equation

Understanding the methodology allows students and professionals to confidently analyze similar differential equations arising in various scientific and engineering contexts.

Further Resources

For readers interested in expanding their knowledge, consider exploring:

- Khan Academy's Differential Equations Course