Suppose = (3527)(32)(143) In S8. Express As A Product Of Transpositions And Determine If Is Even Or Odd.

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Introduction to Permutations and Symmetric Groups

Permutations are fundamental objects in the study of algebra, especially in the context of symmetric groups. The symmetric group S_n consists of all permutations of n elements, and it plays a central role in group theory, combinatorics, and algebraic structures. Understanding how to express permutations as products of transpositions (which are simple swaps of two elements) is crucial for analyzing their properties, such as determining whether they are even or odd permutations.

This article focuses on the permutation $\sigma = (3527)(32)(143)$ in the symmetric group S_8. We will explore how to express this permutation as a product of transpositions and determine whether it is an even or odd permutation.

Understanding the Permutation Notation

Before delving into decomposition, it's important to understand the notation used:

- Cycle Notation: Permutations are often expressed as cycles, e.g., $(a_1 a_2 \ldots a_k)$, which indicates that a_1 maps to a_2, a_2 maps to a_3, ..., and a_k maps back to a_1.

- Product of Cycles: When permutations are written as products of cycles, the composition is typically read from right to left, meaning the permutation on the right acts first.

- Disjoint Cycles: Cycles with no common elements commute, and their permutations can be combined independently.

Given the permutation:

$$
\sigma = (3527)(32)(143)
$$

we interpret it as the composition of three cycles acting on the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Step 1: Clarify the Composition Order

In permutation notation, the permutation on the right acts first. So, for the permutation:

$$
\sigma = (3527)(32)(143)
$$

the order of application is:

1. Apply (143),
2. then (32),
3. finally, (3527).

To analyze σ, we need to find the image of each element in $\{1, 2, 3, 4, 5, 6, 7, 8\}$ under this composed permutation.

Step 2: Compute σ as a Function on Elements

Let's determine $\sigma(i)$ for each element i in $\{1,\ldots,8\}$.

Note: Since the cycles are disjoint or not, and their order is from right to left, we proceed systematically.

Element 1:

- Apply (143): 1 maps to 4.
- Apply (32): 4 is unaffected (since 4 not in (32)), so remains 4.
- Apply (3527): 4 is unaffected (not in (3527)), so remains 4.

Thus, $\sigma(1) = 4$.

Element 2:

- Apply (143): 2 remains 2.
- Apply (32): 2 maps to 3.
- Apply (3527): 3 maps to 5.

Hence, $\sigma(2) = 5$.

Element 3:

- Apply (143): 3 remains 3.
- Apply (32): 3 maps to 2.
- Apply (3527): 2 maps to 3.

So, $\sigma(3) = 3$.

Element 4:

- Apply (143): 4 maps to 1.
- Apply (32): 1 not in (32), remains 1.
- Apply (3527): 1 not in (3527), remains 1.

Thus, $\sigma(4) = 1$.

Element 5:

- Apply (143): 5 unaffected, remains 5.
- Apply (32): 5 unaffected, remains 5.
- Apply (3527): 5 maps to 2.

Therefore, $\sigma(5) = 2$.

Element 6:

- Apply (143): 6 unaffected, remains 6.

- Apply (32): 6 unaffected, remains 6.
- Apply (3527): 6 unaffected, remains 6.

Hence, $\sigma(6) = 6$.

Element 7:

- Apply (143): 7 unaffected, remains 7.
- Apply (32): 7 unaffected, remains 7.
- Apply (3527): 7 maps to 2.

So, $\sigma(7) = 2$.

Element 8:

- Apply (143): 8 unaffected, remains 8.
- Apply (32): 8 unaffected, remains 8.
- Apply (3527): 8 unaffected, remains 8.

Thus, $\sigma(8) = 8$.

Summary of images:

Element	$\sigma(i)$

1	4
2	5
3	3
4	1
5	2
6	6
7	2 (from earlier, but note initial is 7, but 7 maps to 2)
8	8

Wait, for 7, earlier we found $\sigma(7) = 2$. But 7 maps to 2; in our previous, we concluded 7 maps to 2.

Actually, the previous step indicates a conflict because 7 maps to 2, and 2 maps to 5, etc. But to ensure consistency, let's verify $\sigma(7)$ again step-by-step:

- (143): 7 unaffected, remains 7.
- (32): 7 unaffected, remains 7.
- (3527): 7 maps to 2.

So, $\sigma(7) = 2$.

Similarly, for 2:

- (143): 2 unaffected.
- (32): 2 maps to 3.
- (3527): 3 maps to 5.

Yes, consistent.

Final mapping:

$$
\begin{cases}
1 \mapsto 4 \\
2 \mapsto 5 \\
3 \mapsto 3 \\
4 \mapsto 1 \\
5 \mapsto 2 \\
6 \mapsto 6 \\
7 \mapsto 2 \\
8 \mapsto 8
\end{cases}
$$

But note that 7 maps to 2, which then maps 2 to 5, meaning that the overall image of 7 is:

$$
\sigma(7) = \sigma(7) = 2 \quad (\text{from above}),
$$

but to check the consistency, we should consider the composition:

- For 7: (143): unaffected, remains 7.
- (32): unaffected, remains 7.
- (3527): 7 maps to 2.

Thus, $\sigma(7) = 2$.

Similarly, for 2:

- (143): unaffected, 2.
- (32): 2 maps to 3.
- (3527): 3 maps to 5.

So, $\sigma(2) = 5$.

Step 3: Write σ as a permutation in cycle notation

Using the above mappings:

- $1 \to 4$,
- $4 \to 1$,
- $2 \to 5$,
- $5 \to 2$,
- $3 \to 3$,
- $6 \to 6$,
- $7 \to 2$, but we've already assigned $2 \to 5$. Since 7 maps to 2, and 2 maps to 5, then:

$$
7 \to 2 \to 5,
$$

which means the cycle involving 7 is:

$$
7 \to 2 \to 5 \to 2,
$$

but 2 maps to 5, and 5 maps back to 2, so 7 maps to 2, which maps to 5, so the cycle is:

\[

(7\,2\,5).

\]

Similarly, for 3 and 3, it's fixed, and 6 is fixed.

Putting it all together, the permutation \(\sigma\) in cycle notation is:

\[

\sigma = (1\,4)(2\,5\,7)(3)(6)(8).

\]

Since fixed points

Frequently Asked Questions

How do you express the permutation (3527)(32)(143) in S8 as a product of transpositions?

First, write each cycle as transpositions: (3527) = (3 5)(5 2)(2 7), (32) = (3 2), and (143) = (1 4)(4 3). Then, combine all: (3527)(32)(143) = (3 5)(5 2)(2 7)(3 2)(1 4)(4 3).

What is the process to convert a cycle notation into a product of transpositions?

To convert a cycle (a1 a2 ... ak) into transpositions, write it as (a1 ak)(a1 a(k-1))... (a1 a2). This expresses the cycle as a product of (k-1) transpositions.

How many transpositions are in the full expression of (3527)(32)(143)?

The cycle (3527) becomes 3 transpositions, (32) is 1, and (143) is 2, totaling 6 transpositions when expressed fully.

Is the permutation (3527)(32)(143) an even or odd permutation in S8?

Since the total number of transpositions is 6 (an even number), the permutation is an even permutation.

How can you determine the parity (even or odd) of a permutation from its transposition decomposition?

Count the number of transpositions in its decomposition. If the count is even, the permutation is even; if odd, it is odd.

Does the order of the cycles in the permutation affect whether it is even or odd?

No, the parity depends on the total number of transpositions in the decomposition, which remains consistent regardless of the order of cycles.

Can the permutation (3527)(32)(143) be simplified further to a single cycle?

Yes, by composing the cycles, but the simplified form may be more complex. However, expressing it as transpositions directly helps determine its parity without full simplification.

Additional Resources

Suppose = (3527)(32)(143) In S8. Express As A Product Of Transpositions And Determine If Is Even Or Odd

Understanding permutations within symmetric groups is a fundamental aspect of group theory, a pivotal branch of abstract algebra. The permutation Suppose = (3527)(32)(143) in the symmetric group S_8 presents an interesting case for analysis. This article delves into expressing this permutation as a product of transpositions, determines whether it is even or odd, and explores the underlying concepts with clarity and depth.

Introduction to Permutations and Symmetric Groups

Before unpacking the specific permutation, it's essential to understand the context and foundational concepts.

What Is a Permutation?

A permutation is a rearrangement of elements in a set. In the context of S_8, it refers to a bijective function from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ onto itself. Each permutation can be expressed in cycle notation, which indicates how elements are permuted.

The Symmetric Group S_8

The group S_8 contains all possible permutations of 8 elements. It has $8! = 40,320$ elements, and understanding the properties of individual permutations, such as their cycle structure and parity, is crucial in many algebraic applications.

Analyzing the Given Permutation

The permutation of interest is:

$$
\text{Suppose} = (3527)(32)(143)
$$

This is a product of three cycles. To analyze and express this permutation as a product of transpositions, we need to understand the composition order and the cycle structure.

Understanding Cycle Composition

In permutation notation, the order of composition is from right to left. That is, the permutation applied to an element is computed as:

$$
(\text{Suppose}) = (3527) \circ (32) \circ (143)
$$

which reads as: first apply (143), then (32), then (3527).

Step-by-Step Conversion to Transpositions

Transpositions are permutations that swap exactly two elements. They are fundamental because any permutation can be written as a product of transpositions.

Expressing Each Cycle as Transpositions

- Cycle (143):

A cycle $(a\,b\,c)$ can be written as $(a\,c)(a\,b)$.

Thus,

$$
(143) = (1\,3)(1\,4)
$$

- Cycle (32):

$$
(3\,2) \quad \text{(a 2-cycle, already a transposition)}
$$

- Cycle (3527):

A 4-cycle $(a\,b\,c\,d)$ can be written as $(a\,d)(a\,c)(a\,b)$.

Thus,

$$
(3\,5\,2\,7) = (3\,7)(3\,2)(3\,5)
$$

Combining the Transpositions

Now, express the entire permutation as a product of these transpositions, considering the composition order:

$$
\text{Suppose} = (3527) \circ (32) \circ (143) = \left[(3\,7)(3\,2)(3\,5)\right] \circ (3\,2) \circ (1\,3)(1\,4)
$$

Remember, the composition applies from right to left:

1. First, apply $(1\,3)(1\,4)$

2. Then, apply $(3\,2)$

3. Finally, apply $(3\,7)(3\,2)(3\,5)$

Step-by-Step Calculation

Let’s compute the image of each element in $\{1,2,3,4,5,6,7,8\}$:

- Start with element 1:

1. Apply $(1\,3)(1\,4)$:

- $1 \to 3$

2. Apply $(3\,2)$:

- $3 \to 2$

3. Apply $(3\,7)(3\,2)(3\,5)$:

- First, note that $(3\,7)(3\,2)(3\,5)$ is a composition of transpositions.

- The order is from right to left, so:

- $(3\,5)$:

- $2 \to 2$ (no change)

- $(3\,2)$:

- $2 \to 3$

- $(3\,7)$:

- $3 \to 7$

Therefore,

- Starting from previous step: 1 mapped to 2, now:

- $2 \to 3$

- $3 \to 7$

So final image:

$$
1 \to 7
$$

- Element 2:

1. $(1\,3)(1\,4)$:

- $2 \to 2$

2. $(3\,2)$:

- $2 \to 3$

3. $(3\,7)(3\,2)(3\,5)$:

- $(3\,5)$:

- $3 \to 5$

- $(3\,2)$:

- $3 \to 2$, but since previous step was $3 \to 5$, and now applying $(3\,2)$, which swaps 3 and 2:

- $3 \to 2$, so the previous $3 \to 5$ becomes $5 \to 5$ (since 5 is unaffected), but in the composite, the order matters.

To clarify, perhaps it’s easier to track images step-by-step:

- Start with 2:

- $(1\,3)(1\,4)$: no change, so 2 remains 2.

- $(3\,2)$:

- $2 \to 3$

- $(3\,7)$:

- $3 \to 7$

So,

$$
2 \to 7
$$

- Element 3:

1. $(1\,3)(1\,4)$:

- $3 \to 1$

2. $(3\,2)$:

- $3 \to 2$

3. $(3\,7)(3\,2)(3\,5)$:

- Apply from right:

- $(3\,5)$:

- $3 \to 5$

- $(3\,2)$:

- $3 \to 2$, but since the previous step was $3 \to 5$, now 3 is mapped through the composite as:

- First, $(3\,5)$:

- $3 \to 5$

- $(3\,2)$:

- $3 \to 2$, but in the sequence, it acts on the previous images.

To avoid confusion, it's best to process the entire composition as a function.

Alternatively, write the composite as a function:

$$
f = (3\,7) \circ (3\,2) \circ (3\,5)
$$

Applying to 3:

- $(3\,5)$:

- \(3 \to 5\)

- \((3\,2)\):

- \(5\) unaffected; \(3 \to 5\) remains.

- \((3\,7)\):

- \(3 \to 7\), but since \(3\) was mapped to 5 by \((3\,5)\), the image is:

- The original 3 goes through the composition:

- \(3 \to 5\) (via \((3\,5)\))

- Then, \((3\,2)\) acts on 5: no change, 5 remains 5.

- \((3\,7)\):

- \(3 \to 7\), but 3 is mapped to 5 earlier, so the chain is:

- \(3 \to 5\), then \((3\,7)\) acts on the original 3, which is mapped to 7 if we consider the entire composition.

Actually, the better approach is to compose the permutations as functions and see their effect step-by-step.

Simplified Approach: Computing the Overall Permutation

Given the complexity of stepwise calculations, a more straightforward method involves composing the cycles directly as functions.

Step 1: Write each cycle as a function:

- \((143)\):

\[

1 \to 4,\quad 4 \to 3,\quad 3 \to 1

\]

- \((32)\):

\[

3 \leftrightarrow 2

\]

- \((

[Suppose 3527 32 143 In S8 Express As A Product Of Transpositions And Determine If Is Even Or Odd]()

Suppose 3527 32 143 In S8 Express As A Product Of Transpositions And Determine If Is Even Or Odd

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