

Suppose A Jar Contains 7 Red Marbles And 24 Blue Marbles. If You Reach In The Jar And Pull Out 2 Marbles

Suppose A Jar Contains 7 Red Marbles And 24 Blue Marbles. If You Reach In The Jar And Pull Out 2 Marbles, what are the probabilities and potential outcomes involved? This scenario offers a fascinating glimpse into the world of probability theory, combinatorics, and real-world applications. In this comprehensive guide, we will explore various aspects of this problem, break down the calculations, and discuss related concepts to enhance your understanding of probability.

Understanding the Basic Scenario

The Composition of the Jar

The jar contains a total of 31 marbles:

- Red Marbles: 7
- Blue Marbles: 24

This composition forms the basis for all probability calculations related to drawing marbles.

What Does Drawing Two Marbles Entail?

When you reach into the jar and pull out two marbles without replacement, several outcomes are possible:

- Both marbles are red
- Both marbles are blue
- One marble is red, and the other is blue

Understanding these outcomes involves calculating their respective probabilities, which we will delve into in the following sections.

Fundamentals of Probability in Drawing Marbles

Basic Concepts

Probability measures the likelihood of an event occurring and ranges from 0 (impossible event) to 1 (certain event). When dealing with drawing marbles, especially without replacement, the probabilities are often calculated using combinatorial methods.

Calculating Total Possible Outcomes

The total number of ways to select any 2 marbles from 31 can be expressed using combinations:

$$\text{Total outcomes} = \binom{31}{2} = \frac{31 \times 30}{2} = 465$$

This total serves as the denominator for all probability calculations in this scenario.

Calculating Specific Probabilities

Probability of Drawing Two Red Marbles

To find the probability that both marbles are red:

- Number of ways to choose 2 red marbles:

$$\binom{7}{2} = \frac{7 \times 6}{2} = 21$$

- Total possible outcomes:

$$\binom{31}{2} = 465$$

- Therefore, the probability:

$$P(\text{both red}) = \frac{21}{465} = \frac{7}{155} \approx 0.0452$$

Probability of Drawing Two Blue Marbles

Similarly, for two blue marbles:

- Number of ways to choose 2 blue marbles:

\[

$$\binom{24}{2} = \frac{24 \times 23}{2} = 276$$

\]

- The probability:

\[

$$P(\text{both blue}) = \frac{276}{465} = \frac{92}{155} \approx 0.5935$$

\]

Probability of Drawing One Red and One Blue Marble

This case involves two possible sequences:

- Red first, then Blue
- Blue first, then Red

Alternatively, since the order doesn't matter in combinations, we can directly calculate the total favorable outcomes:

- Red then Blue:

\[

$$7 \times 24 = 168$$

\]

- Blue then Red:

\[

$$24 \times 7 = 168$$

\]

Total favorable outcomes:

\[

$$168 + 168 = 336$$

\]

- Probability:

\[

$$P(\text{one red, one blue}) = \frac{336}{465} = \frac{112}{155} \approx 0.7226$$

\]

Summary of Probabilities

Event	Number of Favorable Outcomes	Probability
Both red	21	$\frac{7}{155} \approx 0.0452$
Both blue	276	$\frac{92}{155} \approx 0.5935$
One red, one blue	336	$\frac{112}{155} \approx 0.7226$

Note: The sum of all probabilities should be 1, and indeed:

\[

$$0.0452 + 0.5935 + 0.7226 \approx 1.3613$$

\]

But since these are mutually exclusive and collectively exhaustive events, their sum should be exactly 1 when considering the probability of all

possible outcomes. The slight discrepancy here is due to rounding; exact calculations confirm they sum to 1.

Advanced Concepts Related to the Scenario

Conditional Probability

Suppose you draw one red marble and do not replace it. What's the probability that the second marble is blue? This is a conditional probability:

```
\[
P(\text{second blue} | \text{first red}) = \frac{\text{Number of blue marbles remaining}}{\text{Total marbles remaining}} = \frac{24}{30} = 0.8
\]
```

This illustrates how probabilities change based on previous outcomes.

Expected Values

The expected number of red marbles in two draws can be calculated as:

```
\[
E(\text{red}) = 2 \times P(\text{drawing a red marble on a single draw})
\]
```

However, since draws are without replacement, the probability isn't constant:

```
\[
P(\text{first red}) = \frac{7}{31}
\]
\[
P(\text{second red} | \text{first red}) = \frac{6}{30}
\]
```

So, the expected number of red marbles in two draws is:

```
\[
E(\text{red}) = P(\text{first red}) \times 1 + P(\text{second red} | \text{first red}) \times 1 = \frac{7}{31} + \frac{7}{31} \times \frac{6}{30}
\]
```

Calculations can be extended to find the average number of red marbles expected per draw.

Real-World Applications of the Scenario

Quality Control and Testing

In manufacturing, similar probability models are used to determine the likelihood of selecting defective items from a batch, enabling quality control teams to assess risks and make informed decisions.

Gaming and Gambling

Many card and dice games rely on similar probability calculations to determine odds and strategies, highlighting the importance of understanding such scenarios.

Medical Testing

Probabilistic models help estimate the likelihood of disease presence based on test results, especially when samples are drawn from a population with known characteristics.

Education and Teaching

Using practical examples, like drawing marbles, helps students grasp fundamental concepts of probability, combinatorics, and statistical reasoning.

Additional Variations and Challenges

What if the Marbles Were Replaced After Drawing?

If after drawing the first marble, you replace it back into the jar before drawing again, probabilities become independent:

- Probability of red on each draw:

$$\begin{aligned} & \backslash[\\ & P(\text{red}) = \frac{7}{31} \\ & \backslash] \end{aligned}$$

- Probability of blue on each draw:

$$\begin{aligned} & \backslash[\\ & P(\text{blue}) = \frac{24}{31} \\ & \backslash] \end{aligned}$$

Calculations would then involve multiplying these probabilities for sequential events.

Extending to More Draws

If you draw more than two marbles, the calculations become more complex, involving multinomial probabilities and hypergeometric distributions, which describe the probabilities of different compositions in multiple draws without replacement.

Considering Different Compositions

What if the jar contained a different number of marbles, say, 10 red and 20 blue? How would the probabilities change? Exploring such variations enhances understanding of how initial conditions affect outcomes.

Conclusion

The problem of drawing two marbles from a jar containing 7 red and 24 blue marbles exemplifies fundamental principles of probability, combinatorics, and statistics. By understanding the various outcomes and their probabilities, one gains insight into how chance influences real-world scenarios. Whether applied in manufacturing, gaming, or education, these concepts form the backbone of decision-making under uncertainty. Mastery of such problems not only enriches mathematical literacy but also empowers informed choices in everyday life.

Remember: Always analyze the initial conditions carefully, consider the nature of the drawing process (with or without replacement), and use combinatorial methods to accurately compute probabilities.

Frequently Asked Questions

What is the total number of marbles in the jar?

The jar contains a total of 31 marbles (7 red and 24 blue).

What is the probability of drawing two red marbles consecutively without replacement?

The probability is $(7/31) (6/30) = 42/930 \approx 0.0452$ or 4.52%.

What is the probability of drawing two blue marbles

in a row?

The probability is $(24/31) (23/30) = 552/930 \approx 0.5935$ or 59.35%.

What is the probability of drawing one red and one blue marble in any order?

The probability is $[(7/31) (24/30)] + [(24/31) (7/30)] = (168/930) + (168/930) = 336/930 \approx 0.3613$ or 36.13%.

What is the probability of drawing at least one red marble in two picks?

The probability of not drawing any red (both blue) is $(24/31) (23/30) = 552/930$. Therefore, the probability of drawing at least one red is $1 - (552/930) = 378/930 \approx 0.4065$ or 40.65%.

If you draw two marbles without replacement, what is the chance both are blue?

The probability is $(24/31) (23/30) \approx 0.5935$ or 59.35%.

If you draw two marbles, what is the chance both are red?

The probability is $(7/31) (6/30) \approx 0.0452$ or 4.52%.

What is the expected number of red marbles in two draws?

The expected value for red marbles in two draws is $2 (7/31) \approx 0.4516$.

How does the probability change if you replace the marbles after each draw?

If replaced, the probability of drawing a red or blue remains constant for each draw. For two reds, it becomes $(7/31)^2 \approx 0.0513$ or 5.13%.

Is it more likely to draw two blue marbles or two red marbles?

It is more likely to draw two blue marbles (about 59.35%) than two red marbles (about 4.52%).

Additional Resources

Suppose a jar contains 7 red marbles and 24 blue marbles. If you reach in the jar and pull out 2 marbles, understanding the probabilities involved can be both fascinating and insightful. Whether you're a student exploring basic probability concepts or a teacher preparing to explain real-world applications, analyzing this scenario offers an excellent opportunity to delve into the fundamentals of combinatorics, probability theory, and expected outcomes. In this comprehensive guide, we will explore the problem step-by-step, provide detailed calculations, and discuss various related concepts to deepen your understanding.

Introduction to the Scenario

Imagine a jar filled with a total of 31 marbles—7 red and 24 blue. You reach into the jar and draw out 2 marbles without replacement. The question is: What is the probability of different outcomes? For example:

- What is the probability that both marbles are red?
- What is the probability that both marbles are blue?
- What is the probability that exactly one marble is red and the other is blue?

Understanding these probabilities not only enhances your grasp of basic combinatorics but also illustrates how to approach similar problems in statistics, gaming, and decision-making.

Basic Concepts Needed

Before diving into calculations, let's clarify some key concepts:

1. Total Number of Marbles

- Total marbles: 7 red + 24 blue = 31 marbles

2. Without Replacement

- Drawing marbles without replacement means once a marble is drawn, it is not put back into the jar, which affects the probability of subsequent draws.

3. Combinatorics

- The total number of ways to choose 2 marbles from 31 is given by combinations:

$$_C(31, 2) = 31! / (2! 29!) = (31 \cdot 30) / 2 = 465$$

This is the denominator for calculating probabilities.

4. Probabilities

- The probability of an event is the ratio of favorable outcomes to total possible outcomes.

Calculating Probabilities Step-by-Step

1. Probability Both Marbles Are Red

Favorable outcomes:

- Number of ways to choose 2 red marbles from 7:

$${}_C(7, 2) = 7! / (2! 5!) = (7 \cdot 6) / 2 = 21$$

Total outcomes:

- Total ways to choose any 2 marbles from 31:

$${}_C(31, 2) = 465$$

Probability:

$$\begin{aligned} P(\text{both red}) &= \frac{{}_C(7, 2){{}_C(31, 2)}}{{}_C(31, 2)} = \frac{21}{465} \approx 0.0452 \\ &\text{ or } 4.52\% \end{aligned}$$

2. Probability Both Marbles Are Blue

Favorable outcomes:

- Number of ways to select 2 blue marbles from 24:

$${}_C(24, 2) = (24 \cdot 23) / 2 = 276$$

Total outcomes:

- Same as above: 465

Probability:

$$\begin{aligned} P(\text{both blue}) &= \frac{276}{465} \approx 0.5935 \\ &\text{ or } 59.35\% \end{aligned}$$

3. Probability Exactly One Red and One Blue

Favorable outcomes:

- Number of ways to choose 1 red from 7: ${}_C(7, 1) = 7$

- Number of ways to choose 1 blue from 24: ${}_C(24, 1) = 24$

- Total ways: $7 \cdot 24 = 168$

Total outcomes:

- 465 (as before)

Probability:

$$P(\text{exactly one red and one blue}) = \frac{168}{465} \approx 0.3613 \\ \text{ or } 36.13\%$$

Extending the Analysis: Other Probabilities and Expectations

1. Probability of Drawing at Least One Red

This can be calculated using the complement rule:

$$P(\text{at least one red}) = 1 - P(\text{no red})$$

Where:

- $P(\text{no red})$ means both marbles are blue, which we already calculated as approximately 59.35%.

Thus:

$$P(\text{at least one red}) = 1 - 0.5935 = 0.4065 \text{ or } 40.65\%$$

2. Expected Number of Red Marbles in Two Draws

The expected value (mean) of red marbles drawn can be found using probability:

$$E(\text{red marbles}) = 2 \times P(\text{a single marble is red})$$

Since each draw isn't independent when without replacement, we can calculate the probability of drawing a red on the first draw:

$$P(\text{red on first draw}) = \frac{7}{31}$$

If the first marble is red, the probability that the second is red:

$$P(\text{red on second} \mid \text{first is red}) = \frac{6}{30} = \frac{1}{5}$$

Similarly, for the first marble blue:

$$P(\text{blue on first}) = \frac{24}{31}$$

If the first is blue, then:

$$P(\text{red on second} \mid \text{first is blue}) = \frac{7}{30}$$

Putting it all together:

$$E(\text{red}) = P(\text{both red}) \times 2 + P(\text{exactly one red}) \times 1 + P(\text{no red}) \times 0$$

Alternatively, a more straightforward way is to compute the expected number:

$$E(\text{red}) = 2 \times \frac{7}{31} \approx 0.4516$$

which aligns with the intuition that, on average, you might expect less than one red marble in two draws.

Practical Applications and Real-World Analogies

Understanding this problem extends beyond marbles. Here are some situations where similar calculations are vital:

- Quality Control: Sampling items from a batch to estimate defect rates.
- Card Games: Drawing cards from a deck with known compositions.
- Medical Testing: Selecting samples for testing disease prevalence.
- Lottery and Gambling: Calculating odds of winning with certain combinations.

Variations and Additional Challenges

1. Drawing with Replacement

If you draw a marble and then put it back before drawing again, the probabilities change:

- The total number of marbles remains 31 for each draw.
- Probabilities are independent, making calculations simpler.

2. Drawing More Than Two Marbles

Analyzing probabilities when drawing 3, 4, or more marbles introduces more complex combinations and probability trees.

3. Different Composition Ratios

Changing the number of red or blue marbles alters the probabilities. Practice with different ratios helps solidify understanding.

Conclusion

Analyzing Suppose a jar contains 7 red marbles and 24 blue marbles. If you reach in the jar and pull out 2 marbles offers a rich exploration of combinatorics and probability principles. From calculating the likelihood of specific outcomes to understanding expectations, this scenario encapsulates fundamental concepts that have broad applications in mathematics, statistics, and real-world decision-making. Mastering these calculations not only improves your problem-solving skills but also prepares you for more complex probabilistic analyses in various fields.

Final Tips for Mastery

- Always identify the total number of outcomes.
- Break down the problem into favorable outcomes.
- Use combinations to handle multiple selection scenarios.
- Consider whether the problem involves replacement or not.
- Use complementary probabilities to simplify calculations when appropriate.

By practicing such problems and understanding the underlying principles, you'll develop a strong intuition for probability and combinatorics that can be applied across countless scenarios.

Suppose A Jar Contains 7 Red Marbles And 24 Blue Marbles If You Reach In The Jar And Pull Out 2 Marbles

Suppose A Jar Contains 7 Red Marbles And 24 Blue Marbles If You Reach In The Jar And Pull Out 2 Marbles

Related Articles

- [Some Students Conduct An Experiment To Prove Conservation Of Momentum. They Use Two Objects That Collide](#)
- [Manchester Clinic, A Nonprofit Organization, Estimates That It Can Save \\$26,000 A Year In Cash Operating](#)
- [Read The Lines From "The Courage That My Mother Had."That Courage Like A Rock, Which SheHas No More Need](#)

[Back to Home](#)