

Suppose A Triangle Has Two Sides Of Length 42 And 35, And That The Angle Between These Two Sides Is 120.

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This scenario presents a classic application of the Law of Cosines, a fundamental tool in trigonometry used to determine unknown side lengths or angles within a triangle when given partial information. Understanding how to analyze such a triangle involves exploring various properties, including side lengths, angles, area, and potential applications. In this article, we will delve deeply into the mathematics behind this configuration, deriving key measures, examining special cases, and illustrating the broader significance of these calculations.

Understanding the Triangle Configuration

Basic Data and Notation

Given:

- Two sides: $a = 42$ units, $b = 35$ units
- Included angle: $C = 120^\circ$ (the angle between sides a and b)

Our goal:

- Find the length of the third side c
- Determine the other angles A and B
- Calculate the area of the triangle
- Explore the triangle's properties and potential implications

Visual Representation

Picture the triangle with sides a and b meeting at vertex C with an angle of 120° . The side c runs opposite this angle, connecting the other two vertices. This visual helps clarify the relationships and the application of trigonometric laws.

Applying the Law of Cosines to Find the Third Side

Law of Cosines Formula

The Law of Cosines states that for any triangle with sides (a, b, c) and corresponding angles (A, B, C) :

$$c^2 = a^2 + b^2 - 2ab \cos C$$

In our case, since (a, b) , and (C) are known:

$$c^2 = 42^2 + 35^2 - 2 \times 42 \times 35 \times \cos 120^\circ$$

Calculating the Side (c)

Let's compute each term:

- $(42^2 = 1764)$
- $(35^2 = 1225)$
- $(2 \times 42 \times 35 = 2 \times 1470 = 2940)$
- $(\cos 120^\circ = -\frac{1}{2})$

Putting it all together:

$$c^2 = 1764 + 1225 - 2940 \times \left(-\frac{1}{2}\right) = 2989 + 1470 = 4459$$

Thus,

$$c = \sqrt{4459} \approx 66.8$$

Result:

The third side (c) is approximately 66.8 units.

Determining the Remaining Angles

Using the Law of Cosines to Find Angle (A)

To find (A) , the angle opposite side (a) , apply the Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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Rearranged for $(\cos A)$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Plugging in known values:

$$\cos A = \frac{1225 + 4459 - 1764}{2 \times 35 \times 66.8} = \frac{1225 + 4459 - 1764}{2 \times 35 \times 66.8}$$

Calculate numerator:

$$1225 + 4459 - 1764 = 3920$$

Calculate denominator:

$$2 \times 35 \times 66.8 = 70 \times 66.8 = 4676$$

Therefore:

$$\cos A = \frac{3920}{4676} \approx 0.837$$

So,

$$A \approx \arccos(0.837) \approx 33.2^\circ$$

Similarly, for (B) :

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Calculate numerator:

$$\begin{aligned} & \backslash[\\ & 1764 + 4459 - 1225 = 4998 \\ & \backslash] \end{aligned}$$

Calculate denominator:

$$\begin{aligned} & \backslash[\\ & 2 \times 42 \times 66.8 = 84 \times 66.8 = 5611.2 \\ & \backslash] \end{aligned}$$

Thus,

$$\begin{aligned} & \backslash[\\ & \cos B = \frac{4998}{5611.2} \approx 0.891 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & B \approx \arccos(0.891) \approx 27.2^\circ \\ & \backslash] \end{aligned}$$

Check sum of angles:

$$\begin{aligned} & \backslash[\\ & A + B + C \approx 33.2^\circ + 27.2^\circ + 120^\circ \approx 180.4^\circ \\ & \backslash] \end{aligned}$$

The slight discrepancy comes from rounding; the sum is approximately 180° , confirming the internal consistency.

Calculating the Area of the Triangle

Using the Given Data and Included Angle

The area Δ of a triangle with two sides and the included angle is:

$$\begin{aligned} & \backslash[\\ & \Delta = \frac{1}{2} a b \sin C \\ & \backslash] \end{aligned}$$

Calculate:

$$\begin{aligned} & \backslash[\\ & \sin 120^\circ = \frac{\sqrt{3}}{2} \approx 0.866 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \Delta = \frac{1}{2} \times 42 \times 35 \times 0.866 \\ & \backslash] \end{aligned}$$

\[
\Delta = 0.5 \times 1470 \times 0.866 \approx 735 \times 0.866 \approx 636
\]

Result:

The area of the triangle is approximately 636 square units.

Implications and Applications

Real-World Relevance

Understanding such a triangle is essential in various fields, including engineering, navigation, and architecture. For example:

- Engineering: Designing structures where specific angles and side lengths are critical.
- Navigation: Calculating distances based on angles and partial measurements.
- Architecture: Ensuring structural stability and aesthetic proportions based on geometric principles.

Theoretical Significance

From a mathematical perspective, this problem illustrates the power of the Law of Cosines and related trigonometric laws. It demonstrates how partial information can determine all aspects of a triangle, emphasizing the interconnectedness of side lengths and angles.

Exploring Special Cases and Variations

Different Angles Between the Same Sides

If the angle between sides 42 and 35 were different, the third side's length and the remaining angles would vary accordingly. For example:

- At (60°) , the third side would be shorter.
- At (180°) , the sides would form a straight line, degenerate triangle.

Varying Side Lengths

Changing the lengths of sides (a) and (b) affects the triangle's shape and size. The Law of Cosines can handle these variations and is versatile in solving for unknown dimensions.

Summary and Key Takeaways

- The third side (c) of the triangle with sides 42 and 35 and an included angle of 120° is approximately 66.8 units.
- The other angles (A) and (B) are approximately 33.2° and 27.2° , respectively.
- The area of the triangle is about 636 square units.
- The Law of Cosines and Law of Sines are powerful tools for solving triangles with partial information.
- Such geometric analyses have practical applications across numerous disciplines and deepen our understanding of spatial relationships.

Conclusion

Analyzing a triangle with given two sides and an included angle exemplifies the elegance and utility of trigonometric principles. By applying the Law of Cosines, we can derive missing side lengths and angles, enabling comprehensive understanding of the triangle's properties. This knowledge not only reinforces fundamental mathematical concepts but also provides a foundation for solving complex real-world problems involving geometric configurations.

Frequently Asked Questions

How do you find the length of the third side of a triangle with sides 42 and 35 and included angle 120° ?

Use the Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos(C)$. Plugging in the values: $c^2 = 42^2 + 35^2 - 2 \cdot 42 \cdot 35 \cos(120^\circ)$. Since $\cos(120^\circ) = -0.5$, calculate $c^2 = 1764 + 1225 + 2 \cdot 42 \cdot 35 \cdot 0.5$, giving $c^2 = 2989 + 1470 = 4459$. So, $c \approx \sqrt{4459} \approx 66.8$.

What is the approximate length of the third side of the triangle described?

The length of the third side is approximately 66.8 units.

How can I find the area of the triangle with sides 42 and 35 and an included angle of 120° ?

Use the formula: $\text{Area} = 0.5 a b \sin(C)$. Since $\sin(120^\circ) = \sqrt{3}/2 \approx 0.866$, the area is $0.5 \cdot 42 \cdot 35 \cdot 0.866 \approx 636.3$ square units.

What is the measure of the third side using the Law of Cosines?

The third side measures approximately 66.8 units, calculated as $c = \sqrt{42^2 + 35^2 - 2 \cdot 42 \cdot 35 \cdot \cos(120^\circ)}$.

Can the given sides and angle form a valid triangle?

Yes. The two sides are positive, and the included angle is between 0° and 180° , satisfying the triangle inequality conditions.

How does the included angle of 120° affect the shape of the triangle?

An angle of 120° between sides 42 and 35 makes the triangle obtuse, with the third side longer than either of the given sides.

If the side lengths were different, how would the calculations change?

You would still use the Law of Cosines and Law of Sines, but with the new side lengths and angles, adjusting the calculations accordingly.

Additional Resources

Suppose A Triangle Has Two Sides Of Length 42 And 35, And That The Angle Between These Two Sides Is 120° . This geometric scenario presents an intriguing case for exploring the properties of triangles, especially those defined by specific side lengths and angles. Such configurations are fundamental in trigonometry, providing a rich ground for examining concepts like the Law of Cosines, area calculation, and the classification of triangles based on their angles and sides. This article aims to delve deeply into this particular triangle, analyzing its properties, calculations, and practical implications, offering both theoretical insights and real-world applications.

Understanding the Triangle's Basic Properties

Before diving into calculations, it's essential to understand the basic properties of the triangle defined by two sides of lengths 42 and 35, with an included angle of 120° .

Triangle Classification

- Type by Sides: Since the two given sides are different lengths (42 and 35), the triangle is scalene, meaning all three sides are of different lengths.
- Type by Angles: The angle between these two sides is 120° , which is greater than 90° , making the triangle an obtuse triangle.

Significance of the Included Angle

- The fact that the angle between sides of lengths 42 and 35 is 120° significantly influences the triangle's shape and area.
- An angle larger than 90° implies that the triangle is obtuse, with one angle exceeding 90° , affecting how the remaining angles are calculated and the overall geometry.

Calculating the Third Side Using the Law of Cosines

One of the fundamental steps in analyzing this triangle is determining the length of the third side, which is not directly given.

The Law of Cosines

The Law of Cosines relates the lengths of sides in a triangle to the cosine of one of its angles:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Where:

- a and b are known sides (42 and 35),
- C is the included angle (120°),
- c is the side opposite the angle C .

Applying this:

$$c^2 = 42^2 + 35^2 - 2 \times 42 \times 35 \times \cos 120^\circ$$

Calculations:

- $42^2 = 1764$
- $35^2 = 1225$
- $2 \times 42 \times 35 = 2 \times 1470 = 2940$
- $\cos 120^\circ = -0.5$

Plugging in:

$$c^2 = 1764 + 1225 - 2940 \times (-0.5) = 2989 + 1470 = 4459$$

Thus:

$$\begin{aligned} & \backslash[\\ c &= \sqrt{4459} \approx 66.8 \\ & \backslash] \end{aligned}$$

Features & Implications:

- The third side of approximately 66.8 units indicates that the triangle is quite elongated, with the side opposite the 120° angle being significantly longer than the other two sides.
- This demonstrates how the Law of Cosines effectively determines unknown side lengths given two sides and the included angle.

Area Calculation of the Triangle

The area of this triangle can be calculated efficiently using the formula involving two sides and the included angle:

Formula for Area

$$\begin{aligned} & \backslash[\\ \text{Area} &= \frac{1}{2}ab \sin C \\ & \backslash] \end{aligned}$$

Where:

- $a = 42$,
- $b = 35$,
- $C = 120^\circ$.

Calculating:

$$\begin{aligned} & \backslash[\\ \sin 120^\circ &= \frac{\sqrt{3}}{2} \approx 0.866 \\ & \backslash] \end{aligned}$$

Therefore:

$$\begin{aligned} & \backslash[\\ \text{Area} &= \frac{1}{2} \times 42 \times 35 \times 0.866 \approx 0.5 \times 1470 \times 0.866 \approx 735 \times 0.866 \approx 636.3 \\ & \backslash] \end{aligned}$$

Implications:

- The area of approximately 636.3 square units highlights the substantial size of this triangle, especially considering the large included angle.
- It's a practical example illustrating how the Law of Sines and Cosines directly contribute to area calculations.

Angles of the Remaining Triangle

Knowing two sides and the included angle allows the calculation of the

remaining angles using the Law of Sines.

Using the Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Given:

- $a = 42$,
- $b = 35$,
- $c \approx 66.8$,
- $C = 120^\circ$.

Calculate $\sin A$:

$$\sin A = \frac{a \sin C}{c} = \frac{42 \times 0.866}{66.8} \approx \frac{36.4}{66.8} \approx 0.545$$

Thus:

$$A \approx \arcsin(0.545) \approx 33.1^\circ$$

Similarly, find $\sin B$:

$$\sin B = \frac{b \sin C}{c} = \frac{35 \times 0.866}{66.8} \approx \frac{30.3}{66.8} \approx 0.454$$

So:

$$B \approx \arcsin(0.454) \approx 27.0^\circ$$

The sum check:

$$A + B + C \approx 33.1^\circ + 27.0^\circ + 120^\circ \approx 180.1^\circ$$

Close enough, considering rounding errors, confirming the internal consistency of the calculations.

Features & Observations:

- The remaining angles are both acute, which aligns with the triangle being obtuse at C .
- The angles provide insight into the shape and proportions of the triangle, useful in design and architecture.

Practical Applications and Significance

Understanding this specific triangle's properties extends beyond theoretical interest, impacting various fields.

Engineering and Architecture

- Such triangles are fundamental in structural design, especially in trusses and frameworks where specific angles and lengths ensure stability.
- Calculating areas and side lengths helps in material estimation and structural analysis.

Navigation and Geographical Mapping

- Triangles with known sides and angles are essential in triangulation methods used for accurate positioning.
- The Law of Cosines and Sines aid in determining distances that are otherwise hard to measure directly.

Computer Graphics and Modeling

- Triangles serve as the basic units in mesh modeling, where knowing side lengths and angles determines shape and shading.
- Precise calculations ensure realistic rendering and efficient computational algorithms.

Pros and Cons of this Triangle Scenario

Pros:

- Clear calculation pathway: Using well-established formulas, the properties can be precisely determined.
- Demonstrates key concepts: Provides a concrete example of applying Law of Cosines and Sines.
- Applicable in real-world problems: Insights can be transferred to engineering, navigation, and computer graphics.

Cons:

- Rounding errors: Approximate calculations can lead to slight inaccuracies, especially when dealing with trigonometric functions.
- Limited scope: Focused on a specific configuration; different side lengths or angles require separate analysis.
- Complexity for learners: Requires understanding of multiple laws and trigonometric functions, which might be challenging for beginners.

Conclusion

Analyzing a triangle with two sides of lengths 42 and 35, and an included angle of 120° , reveals a wealth of geometric insights. From calculating the third side to evaluating the area and remaining angles, each step underscores the interconnectedness of trigonometric principles. This scenario not only exemplifies key mathematical concepts but also demonstrates their practical utility across various disciplines. Such explorations enhance our understanding of geometric relationships, enabling precise modeling and problem-solving in real-world contexts. Whether in engineering, navigation, or computer graphics, mastering these calculations empowers us to approach complex spatial problems with confidence and clarity.

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