Suppose That F(x, Y) = 3x4 + 3y4 2xy. = Then The Minimum Value Of F Is Round Your Answer To Four Decimal

Suppose That F(x, Y) = 3x^4 + 3y^4 - 2xy. = Then The Minimum Value Of F Is Round Your Answer To Four Decimal

Understanding the behavior of multivariable functions and finding their extrema is a fundamental aspect of calculus and mathematical analysis. In this article, we will investigate the function \(F(x, y) = 3x^4 + 3y^4 - 2xy \) to determine its minimum value. Through a systematic approach involving partial derivatives, critical point analysis, and second derivative tests, we aim to find the global minimum of the function and present the answer rounded to four decimal places.

Introduction to Multivariable Optimization

Before delving into the specifics of our function, it's crucial to understand the general framework for optimizing functions of multiple variables.

Key Concepts

1. **Critical Points:** Points where the partial derivatives of the function are zero or undefined.

2. **Second Derivative Test:** Used to classify critical points as local minima, maxima, or saddle points based on the second derivatives.

3. **Global vs. Local Extrema:** Global extrema are the absolute highest or lowest points on the entire domain, while local extrema are the highest or lowest points within a neighborhood.

In our case, because the function involves quartic terms and a mixed product, analyzing the critical points is essential to identify potential minima.

Analyzing the Function $F(x, y) = 3x^4 + 3y^4 - 2xy$

The first step involves computing the first-order partial derivatives to locate critical points.

Computing Partial Derivatives

1. Partial derivative with respect to x:
   $$
   \frac{\partial F}{\partial x} = 12x^3 - 2y
   $$

2. Partial derivative with respect to y:
   $$
   \frac{\partial F}{\partial y} = 12y^3 - 2x
   $$

Critical points occur where both derivatives simultaneously equal zero:
$$
\begin{cases}
12x^3 - 2y = 0 \\
12y^3 - 2x = 0
\end{cases}
$$

Finding Critical Points

To solve the system:
$$
\begin{cases}
12x^3 = 2y \\
12y^3 = 2x
\end{cases}
$$

we can express y in terms of x and vice versa.

Expressing \(y \) in terms of \(x \)

\[
12x^3 = 2y \Rightarrow y = 6x^3
\]

Substituting into the second equation:
\[
12(6x^3)^3 = 2x
\]

Calculate \((6x^3)^3 \):
\[
(6x^3)^3 = 6^3 x^{9} = 216 x^{9}
\]

Now, the equation becomes:
\[
12 \times 216 x^{9} = 2x \Rightarrow 2592 x^{9} = 2x
\]

Divide both sides by \(2 \):
\[
1296 x^{9} = x
\]

Rearranged:
\[
1296 x^{9} - x = 0
\]

Factor out \(x \):
\[
x (1296 x^{8} - 1) = 0
\]

This yields two possibilities:

1. \(x = 0 \)
2. \(1296 x^{8} = 1 \)

Solutions for x

Case 1: $x = 0$

- From $y = 6x^3$, we get $y = 0$.

Critical Point 1: $(0, 0)$

Case 2: $1296 x^{8} = 1$

- Solve for x:
$$
x^{8} = \frac{1}{1296}
$$
$$
x = \pm \left(\frac{1}{1296} \right)^{1/8}
$$

Note that $\left(\frac{1}{1296} \right)^{1/8} = \left(1296 \right)^{-1/8}$.

Calculate $1296^{1/8}$:

Since $1296 = 6^4$ (because $6^4 = 1296$), then:

$$
1296^{1/8} = (6^4)^{1/8} = 6^{4/8} = 6^{1/2} = \sqrt{6}
$$

Thus,

$$
x = \pm \frac{1}{\sqrt{6}}
$$

Now, compute y:

$$
y = 6x^3
$$

For $x = \frac{1}{\sqrt{6}}$:

$$
y = 6 \left(\frac{1}{\sqrt{6}} \right)^3 = 6 \times \frac{1}{(\sqrt{6})^3} = 6 \times \frac{1}{(6^{1/2})^3} = 6 \times \frac{1}{6^{3/2}} = 6 \times 6^{-3/2}
$$

Expressed as:

$$
6 \times 6^{-3/2} = 6^{1} \times 6^{-3/2} = 6^{1 - 3/2} = 6^{-1/2} = \frac{1}{\sqrt{6}}
$$

Similarly, for $x = -\frac{1}{\sqrt{6}}$:

$$
y = 6 \left(-\frac{1}{\sqrt{6}} \right)^3 = 6 \times - \frac{1}{(\sqrt{6})^3} = - \frac{6}{6^{3/2}} = -6^{1 - 3/2} = -6^{-1/2} = - \frac{1}{\sqrt{6}}
$$

Critical Points:

- $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$
- $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right)$

Summary of Critical Points

Critical Point	Coordinates	Description
1	(0, 0)	Origin
2	$\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$	Positive symmetric point
3	$\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right)$	Negative symmetric point

Next, we analyze the nature of these critical points to identify which represents the minimum value of F.

Second Derivative Test for Multivariable Functions

The second derivative test involves the Hessian matrix:

$$
H = \begin{bmatrix}
\frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} \\
\frac{\partial^2 F}{\partial y \partial x} & \frac{\partial^2 F}{\partial y^2}
\end{bmatrix}
$$

Calculate the second derivatives:

$$
\frac{\partial^2 F}{\partial x^2} = 36x^2
$$

$$
\frac{\partial^2 F}{\partial y^2} = 36y^2
$$

$$
\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} = -2
$$

The test involves computing the Hessian determinant:

$$
D = \left(\frac{\partial^2 F}{\partial x^2} \times \frac{\partial^2 F}{\partial y^2} \right) - \left(\frac{\partial^2 F}{\partial x \partial y} \right)^2
$$

Classifying Critical Points

At $(0, 0)$:

$$
\frac{\partial^2 F}{\partial x^2} = 0, \quad \frac{\partial^2 F}{\partial y^2} = 0

\]

$$
D = (0 \times 0) - (-2)^2 = -4 < 0
$$

Since $D < 0$, $(0,0)$ is a saddle point; thus,

Frequently Asked Questions

What is the function F(x, y) given in the problem?

The function is $F(x, y) = 3x^4 + 3y^4 - 2xy$.

How do you find the minimum value of the function F(x, y)?

To find the minimum, we set the partial derivatives of F with respect to x and y to zero and solve for critical points, then verify the minimum using second derivative tests.

What are the first-order partial derivatives of F(x, y)?

The partial derivatives are: $F_x = 12x^3 - 2y$ and $F_y = 12y^3 - 2x$.

How do you solve for the critical points of F(x, y)?

Set $F_x = 0$ and $F_y = 0$, leading to the system of equations: $12x^3 = 2y$ and $12y^3 = 2x$, then solve simultaneously.

What are the critical points obtained from the equations?

From the system, critical points include (0, 0), and solutions where $y = 6x^3$ and $x = 6y^3$.

How do you determine which critical point gives the minimum value of F?

Calculate F at each critical point and compare the values; the smallest value corresponds to the minimum.

What is the minimum value of F(x, y) rounded to four decimal places?

The minimum value occurs at (0, 0) with F(0, 0) = 0.0000.

Are there any other critical points besides (0,0), and what is their significance?

Yes, other solutions satisfy the equations, but evaluating F at those points shows they yield higher values, so (0,0) is the global minimum.

Additional Resources

Optimization Analysis of the Function $F(x, y) = 3x^4 + 3y^4 - 2xy$: A Deep Dive into Its Minimum Value

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Introduction: The Significance of Function Optimization

In the realm of mathematical analysis and applied sciences, understanding the behavior of multivariable functions is fundamental. Whether optimizing profit in economics, minimizing cost in engineering, or analyzing physical systems in physics, the core challenge often involves finding the functions' minimum or maximum values under certain conditions.

The function in focus, $F(x, y) = 3x^4 + 3y^4 - 2xy$, epitomizes such a challenge. Its intricate interplay of polynomial terms and the mixed product term introduces interesting complexities in determining its minimum value. This article provides a comprehensive exploration of this function's critical points, the application of calculus techniques, and the final determination of its minimum value, rounded to four decimal places.

Understanding the Function: Structure and Components

Anatomy of $F(x, y)$

The function comprises three main parts:

1. Quartic terms: $3x^4$ and $3y^4$
These terms dominate the behavior at large $|x|$ and $|y|$, tending toward positive infinity as either variable grows large. They ensure the function is bounded below, making the existence of a minimum plausible.

2. Bilinear term: \(-2xy\)
This introduces a coupling between \(x\) and \(y\), capable of creating saddle points or influencing the location of minima depending on the signs of \(x\) and \(y\).

Graphical Intuition

Plotting or visualizing the surface of \(F\) suggests a landscape with peaks and valleys. The quartic terms create a "bowl" shape that ensures the function tends upward at infinity, while the interaction term \(-2xy\) may cause the surface to dip or tilt, complicating the identification of the lowest point.

Mathematical Approach: Finding Critical Points

To determine the minimum value of \(F\), we rely on calculus techniques—primarily, finding critical points where the gradient (vector of first derivatives) equals zero.

Step 1: Computing Partial Derivatives

Calculate the first-order partial derivatives with respect to \(x\) and \(y\):

\[
\frac{\partial F}{\partial x} = 12x^3 - 2y
\]
\[
\frac{\partial F}{\partial y} = 12y^3 - 2x
\]

These derivatives set the conditions for stationary points: points where the slope of the function in any direction is zero.

Step 2: Setting Derivatives to Zero

Solve the system:

\[
\begin{cases}
12x^3 - 2y = 0 \\

12y^3 - 2x = 0
\end{cases}
\]

Rearranged:

\[
\begin{cases}
2y = 12x^3 \Rightarrow y = 6x^3 \\
2x = 12y^3 \Rightarrow x = 6y^3
\end{cases}
\]

Substituting the second into the first:

\[
y = 6(6y^3)^3
\]

Expressed explicitly:

\[
y = 6 \times 6^3 y^9
\]

Calculations:

\[
6^3 = 216
\]
\[
\Rightarrow y = 6 \times 216 y^9 = 1296 y^9
\]

Rearranged:

\[
y - 1296 y^9 = 0
\]

Factor out \(y\):

\[

y(1 - 1296 y^8) = 0
\]

This yields two possibilities:

1. \(y = 0 \)
2. \(1 - 1296 y^8 = 0 \Rightarrow y^8 = \frac{1}{1296} \)

Similarly, from \(y = 6x^3 \), we can derive corresponding \(x\):

- When \(y = 0 \):

\[
x = 6 \times 0^3 = 0
\]

- When \(y^8 = \frac{1}{1296} \):

\[
y = \pm \left(\frac{1}{1296}\right)^{1/8}
\]

Now, compute \(\left(\frac{1}{1296}\right)^{1/8}\):

\[
\left(\frac{1}{1296}\right)^{1/8} = \left(1296\right)^{-1/8}
\]

Since \(1296 = 6^4\):

\[
1296^{1/8} = (6^4)^{1/8} = 6^{4/8} = 6^{1/2} = \sqrt{6} \approx 2.4495
\]

Therefore:

\[
\left(\frac{1}{1296}\right)^{1/8} = \frac{1}{\sqrt{6}} \approx 0.4082
\]

Corresponding \(x\) values:

\[

x = 6 y^3
\]

Calculate \(x\) for positive \(y\):

\[
x = 6 \times (0.4082)^3 \approx 6 \times 0.068 \approx 0.408
\]

Similarly, for negative \(y\):

\[
x = 6 \times (-0.4082)^3 \approx 6 \times -0.068 \approx -0.408
\]

Critical Points Summary

The critical points are thus:

1. Origin: \((0, 0)\)
2. Symmetrical points: \((0.408, 0.408)\) and \((-0.408, -0.408)\)

These points are candidates for minima, maxima, or saddle points. To classify them, second derivative tests are applied.

Analyzing the Nature of Critical Points

Second Derivative (Hessian) Matrix

The Hessian matrix \(H\) is:

\[
H =
\begin{bmatrix}
\frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} \\

\frac{\partial^2 F}{\partial y \partial x} & \frac{\partial^2 F}{\partial y^2}
\end{bmatrix}
\]

Calculate second derivatives:

\[
\frac{\partial^2 F}{\partial x^2} = 36x^2
\]
\[
\frac{\partial^2 F}{\partial y^2} = 36y^2
\]
\[
\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} = -2
\]

Evaluating at Critical Points

- At \((0,0)\):

\[
H = \begin{bmatrix}
0 & -2 \\
-2 & 0
\end{bmatrix}
\]

Determinant:

\[
\det(H) = (0)(0) - (-2)(-2) = -4 < 0
\]

Negative determinant indicates a saddle point at \((0,0)\), not a minimum.

- At \((0.408, 0.408)\):

\[
36 \times (0.408)^2 \approx 36 \times 0.1667 \approx 6.001
\]

Similarly for \(y\):

$$
36 \times (0.408)^2 \approx 6.001
$$

Hessian:

$$
H \approx
\begin{bmatrix}
6.001 & -2 \\
-2 & 6.001
\end{bmatrix}
$$

Determinant:

$$
6.001 \times 6.001 - (-2)^2 = 36.012 - 4 = 32.012 > 0
$$

Since the leading principal minor (top-left element) is positive and the determinant is positive, the critical point is a local minimum.

- Similarly, at $(-0.408, -0.408)$, the second derivatives are identical, leading to the same conclusion: a local minimum.

Calculating the Minimum Value of F

Given the classification, the global minimum occurs at the critical points $(\pm 0.408, \pm 0.408)$.

Let's compute $F(0.408, 0.408)$:

$$
F(0.408, 0.408) = 3(0.408)^4 + 3(0.408)^4 - 2(0.408)(0.408)
$$

Calculate each component:

- $(0.408)^4 = (0.408)^2 \times (0.408)^2$

\[
(0.408)^2 \approx 0.1667
\]
\[
(0.

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