

Suppose That is In Standard Position And The Given Point Is On The Terminal Side Of. Give The Exact Value

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In trigonometry, understanding the relationship between points on the coordinate plane and their corresponding angles is fundamental. When a point lies on the terminal side of an angle in standard position, it provides valuable information about the angle's measure and its associated sine, cosine, and tangent values. This article delves into the process of determining the exact trigonometric values given such a point, emphasizing the importance of coordinate geometry, the Pythagorean theorem, and the unit circle.

Understanding Standard Position and Terminal Sides

What Is Standard Position?

In trigonometry, an angle is said to be in **standard position** if its vertex is at the origin $(0,0)$ on the coordinate plane, and its initial side coincides with the positive x-axis. The angle then opens counterclockwise for positive angles and clockwise for negative angles.

The Terminal Side of an Angle

The **terminal side** of an angle in standard position is the side that intersects the coordinate plane after the initial side has been rotated by the angle measure. When a point lies on this terminal side, it indicates the direction in which the angle's terminal side points, providing a visual representation to determine the angle's trigonometric ratios.

Given a Point on the Terminal Side, How to Find the Exact Values?

Step 1: Identify the Coordinates

Suppose the point given is (x, y) . This point lies somewhere on the terminal side of the angle in standard position. The key is to use these coordinates to find the angle's measure and its trigonometric ratios.

Step 2: Calculate the Distance from the Origin

The distance from the origin to the point, denoted as r , is crucial. It can be found using the Pythagorean theorem:

- $r = \sqrt{x^2 + y^2}$

This distance r represents the hypotenuse of a right triangle formed with the x and y axes, which is essential for calculating sine and cosine.

Step 3: Determine the Trigonometric Ratios

Using the coordinates and the hypotenuse, the exact values of the primary trigonometric functions are:

- Sine: $\sin \theta = y / r$
- Cosine: $\cos \theta = x / r$
- Tangent: $\tan \theta = y / x$ (provided $x \neq 0$)

These ratios give the exact values of the trigonometric functions for the angle in question.

Example: Find Exact Trigonometric Values Given a Point

Example Problem

Suppose the point (3, 4) lies on the terminal side of an angle in standard position. Find the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$.

Solution Steps

1. Calculate r :

$$\circ r = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$$

2. Determine the ratios:

$$\circ \sin \theta = y / r = 4 / 5$$

$$\circ \cos \theta = x / r = 3 / 5$$

$$\circ \tan \theta = y / x = 4 / 3$$

Thus, the exact values are:

- $\sin \theta = 4/5$

- $\cos \theta = 3/5$

- $\tan \theta = 4/3$

Special Considerations

Quadrant Identification

The signs of the x and y coordinates determine the quadrant in which the terminal side lies, which affects the signs of the trigonometric ratios.

- Quadrant I: $x > 0, y > 0$ θ all ratios positive

- Quadrant II: $x < 0, y > 0$ θ sine positive, cosine negative
- Quadrant III: $x < 0, y < 0$ θ tangent positive, sine and cosine negative
- Quadrant IV: $x > 0, y < 0$ θ cosine positive, sine negative

Handling Special Cases

If the point lies on either the x-axis or y-axis, the ratios take particular forms:

- On the x-axis ($y=0$): $\sin \theta = 0$, $\cos \theta = \pm 1$, $\tan \theta = 0$
- On the y-axis ($x=0$): $\cos \theta = 0$, $\sin \theta = \pm 1$, $\tan \theta$ is undefined

Applications of Finding Exact Values

Solving Trigonometric Equations

Knowing the exact values allows for solving equations involving angles and their ratios precisely, without approximation errors.

Calculating Angles from Coordinates

Given a point, you can determine the angle θ by calculating:

- $\theta = \arctangent(y / x)$ (considering quadrant signs for correct angle)

Real-World Contexts

These calculations are vital in fields such as physics, engineering, navigation, and computer graphics, where precise angle measurements are crucial based on coordinate data.

Summary and Key Takeaways

- When a point lies on the terminal side of an angle in standard position, its coordinates directly inform the angle's trigonometric ratios.
- The hypotenuse r is calculated using the Pythagorean theorem: $r = \sqrt{x^2 + y^2}$.
- Trigonometric ratios are then found as $\sin \theta = y / r$, $\cos \theta = x / r$, and $\tan \theta = y / x$.
- Quadrant considerations are essential for determining the signs of the ratios.
- This method is fundamental to both theoretical and applied trigonometry, enabling precise calculations and problem solving.

Conclusion

Understanding how to derive the exact values of sine, cosine, and tangent from a point on the terminal side of an angle in standard position is a cornerstone of trigonometry. Whether dealing with simple

angle calculations or complex applications, this process offers clarity and precision. Remember to always consider the quadrant and the signs of the coordinates to correctly interpret the ratios and the angle itself. Mastery of these concepts enhances both your theoretical understanding and practical problem-solving skills in mathematics and related fields.

Frequently Asked Questions

Suppose θ is in standard position and the point $(-\sqrt{3}/2, 1/2)$ lies on its terminal side. What is the exact value of $\sin \theta$?

The exact value of $\sin \theta$ is $1/2$.

Given a point $(-1/2, \sqrt{3}/2)$ on the terminal side of θ in standard position, what is the value of $\tan \theta$?

The value of $\tan \theta$ is $(\sqrt{3}/2) / (-1/2) = -\sqrt{3}$.

If a point $(0, -1)$ lies on the terminal side of an angle θ in standard position, what is the exact value of $\cos \theta$?

The exact value of $\cos \theta$ is 0 .

Suppose the point $(1/\sqrt{2}, 1/\sqrt{2})$ is on the terminal side of θ in standard position. What is the exact value of $\sec \theta$?

Since $\cos \theta = 1/\sqrt{2}$, $\sec \theta = \sqrt{2}$.

Given the point $(-\sqrt{2}/2, -\sqrt{2}/2)$ on the terminal side of θ in standard

position, what is the exact value of $\cot \theta$?

$$\cot \theta = (-\sqrt{2}/2) / (-\sqrt{2}/2) = 1.$$

If the point $(0, 1)$ is on the terminal side of an angle θ in standard position, what are the exact values of $\sin \theta$ and $\cos \theta$?

$$\sin \theta = 1, \cos \theta = 0.$$

Suppose the point $(3/5, 4/5)$ lies on the terminal side of θ in standard position. What is the exact value of $\tan \theta$?

$$\tan \theta = (4/5) / (3/5) = 4/3.$$

Additional Resources

Suppose That θ Is In Standard Position And The Given Point Is On The Terminal Side Of θ . Give The Exact Value

Introduction

In the realm of trigonometry, understanding the relationship between angles and their corresponding points on the coordinate plane is fundamental. The phrase “Suppose θ Is In Standard Position And The Given Point Is On The Terminal Side Of θ ” encapsulates a common scenario encountered in trigonometric analysis: identifying the exact value of a trigonometric function for a specific angle based on a point's position on its terminal side. This scenario is not merely academic; it underpins numerous applications ranging from physics to engineering, navigation, and computer graphics.

This article aims to unpack this concept thoroughly, exploring the foundational principles, step-by-step

methods for calculating exact values, and practical examples to solidify understanding. Our goal is to serve as an authoritative guide for students, educators, and professionals seeking a comprehensive overview of this critical topic in trigonometry.

Understanding the Foundations: Standard Position and the Coordinate Plane

What Does "Standard Position" Mean?

In trigonometry, an angle is said to be in standard position when:

- Its vertex is at the origin (0,0).
- Its initial side coincides with the positive x-axis.
- Its terminal side is positioned somewhere in the coordinate plane, forming an angle with the initial side.

This setup provides a consistent framework for analyzing angles and their corresponding points.

The Relationship Between Angles and Coordinates

When an angle θ in standard position has a point $P(x, y)$ on its terminal side, the following relationships hold:

- The point P lies somewhere on the line that forms the terminal side of the angle.
- The distance from the origin to point P is given by the radius r , calculated as:

$$r = \sqrt{x^2 + y^2}$$

- The angle θ can be associated with these coordinates via trigonometric functions:

- $\sin(\theta) = y / r$
- $\cos(\theta) = x / r$
- $\tan(\theta) = y / x$ (where $x \neq 0$)

Understanding these relationships is crucial for determining the exact values of trigonometric functions given a point.

The Core Problem: Given a Point on the Terminal Side, Find the Exact Value of a Trigonometric Function

The General Approach

When provided with a point $P(x, y)$ on the terminal side of an angle θ in standard position:

1. Compute the radius $r = \sqrt{x^2 + y^2}$.
2. Use the definitions of sine, cosine, and tangent in terms of x , y , and r .
3. Simplify the expressions to find the exact value.

This process hinges on recognizing the coordinates associated with the point and applying fundamental identities.

Example Problem Statement

Suppose a point $P(x, y)$ lies on the terminal side of an angle θ in standard position. Given specific coordinates, determine the exact value of:

- $\sin(\theta)$
- $\cos(\theta)$
- $\tan(\theta)$

Step-by-Step Solution Strategy

- 1. Identify the Coordinates: Extract x and y from the problem.
- 2. Calculate r: Find the distance from the origin to P.
- 3. Determine the Trigonometric Values:
 - $\sin(\theta) = y / r$
 - $\cos(\theta) = x / r$
 - $\tan(\theta) = y / x$
- 4. Simplify the Results: Rationalize denominators if necessary and express in simplest radical form.

This systematic approach ensures accuracy and clarity.

Deep Dive: Special Cases and Sign Considerations

Quadrant-Based Sign Rules

The signs of x and y depend on the quadrant where the terminal side of the angle resides:

Quadrant	x	y	Sign of $\sin(\theta)$	Sign of $\cos(\theta)$	Sign of $\tan(\theta)$
I	+	+	+	+	+
II	-	+	+	-	-
III	-	-	-	-	+
IV	+	-	-	+	-

Understanding these signs is vital for accurately determining the exact value, especially when working with negative coordinates.

Points on Axes

- If the point lies on the x-axis ($y=0$), then:

- $\sin(\theta) = 0$

- $\cos(\theta) = \pm 1$ depending on x

- $\tan(\theta) = 0$

- If the point lies on the y-axis ($x=0$), then:

- $\cos(\theta) = 0$

- $\sin(\theta) = \pm 1$ depending on y

- $\tan(\theta)$ is undefined

Practical Examples and Their Solutions

Example 1: Point in Quadrant I

Given: $P(3, 4)$

Find: $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$

Solution:

1. Calculate r :

$$r = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$$

2. Compute the functions:

- $\sin(\theta) = y / r = 4 / 5$

- $\cos(\theta) = x / r = 3 / 5$

$$- \tan(\theta) = y / x = 4 / 3$$

Exact values:

$$- \sin(\theta) = 4/5$$

$$- \cos(\theta) = 3/5$$

$$- \tan(\theta) = 4/3$$

Example 2: Point in Quadrant III

Given: P(-6, -8)

Find: $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$

Solution:

1. Calculate r:

$$r = \sqrt{(-6)^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

2. Determine signs based on quadrant III: both x and y are negative, so:

$$- \sin(\theta) = (-8) / 10 = -4/5$$

$$- \cos(\theta) = (-6) / 10 = -3/5$$

$$- \tan(\theta) = (-8) / (-6) = 4/3$$

Exact values:

$$- \sin(\theta) = -4/5$$

$$- \cos(\theta) = -3/5$$

$$- \tan(\theta) = 4/3$$

Example 3: Point on the x-axis

Given: P(7, 0)

Find: $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$

Solution:

1. Calculate r:

$$r = \sqrt{7^2 + 0^2} = 7$$

2. Since $y=0$:

$$- \sin(\theta) = 0$$

$$- \cos(\theta) = x / r = 7 / 7 = 1$$

$$- \tan(\theta) = 0 / 7 = 0$$

Exact values:

$$- \sin(\theta) = 0$$

$$- \cos(\theta) = 1$$

$$- \tan(\theta) = 0$$

Extending to Other Trigonometric Functions

Beyond sine, cosine, and tangent, other functions like secant, cosecant, and cotangent can be derived:

- $\sec(\theta) = r / x$

- $\csc(\theta) = r / y$

- $\cot(\theta) = x / y$

These are particularly useful in advanced applications and when the original functions' values are undefined.

The Significance of Exact Values in Trigonometry

Exact values serve as benchmarks in mathematical analysis, enabling precise calculations and proofs without approximation. They are essential in:

- Simplifying complex expressions
- Solving equations analytically
- Validating numerical solutions
- Understanding unit circle properties

By mastering the process of deriving these values from given points, students and professionals can navigate a wide array of mathematical challenges confidently.

Practical Applications and Implications

In Physics

- Calculating vectors' directions and magnitudes.
- Analyzing oscillations and waves.

In Engineering

- Designing mechanical components with precise angular specifications.
- Signal processing and Fourier analysis.

In Navigation and Geography

- Determining bearings and course angles.
- Mapping and coordinate transformations.

In Computer Graphics

- Rotating objects based on angular calculations.
- Rendering scenes with accurate perspectives.

Conclusion

Understanding the relationship between points in the coordinate plane and their corresponding angles in standard position is a cornerstone of trigonometry. When a point lies on the terminal side of an angle, calculating the exact value of trigonometric functions becomes a matter of applying fundamental identities, considering the quadrant, and simplifying radical expressions.

This comprehensive analysis reinforces the importance of a structured approach: identifying the point, computing the hypotenuse, and applying definitions systematically. The ability to determine exact values not only deepens conceptual understanding but also enhances practical problem-solving skills across diverse disciplines.

By mastering these principles, students and practitioners can confidently analyze angles and points in the coordinate plane, unlocking a powerful toolkit for mathematical and real-world applications alike.

End of Article

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