Suppose The Point (/3, /4) Is On The Curve Sinx/x Siny/y = C, Where C Is A Constant. Use X Y The Tangent

Suppose The Point (/3, /4) Is On The Curve Sinx/x Siny/y = C, Where C Is A Constant. Use X Y The Tangent

---

Introduction

In the realm of calculus and analytical geometry, understanding the behavior of curves defined implicitly by equations is fundamental. One such intriguing curve is given by the relation:

$$
\frac{\sin x}{x} \cdot \frac{\sin y}{y} = C
$$

where C is a constant. This curve encapsulates the interplay of trigonometric functions and the variables x and y. Given a specific point $\left(\frac{3}{4}, \frac{3}{4}\right)$ on this curve, we aim to analyze the nature of the curve at this point, determine the tangent line, and explore the implications of the point's location relative to the curve. This exploration involves implicit differentiation, calculating the slope of the tangent, and understanding the geometric significance.

Understanding the Curve and the Given Point

Equation of the Curve

The curve is defined by:

$$
\frac{\sin x}{x} \cdot \frac{\sin y}{y} = C
$$

This relation suggests that for any point (x, y) on the curve, the product of the two terms involving sine functions divided by their respective variables remains constant.

Given Point on the Curve

The point provided is:

$$
\left(\frac{3}{4}, \frac{3}{4} \right)
$$

which implies $x = \frac{3}{4}$ and $y = \frac{3}{4}$.

Determining the Constant C

To find C, substitute the point into the equation:

$$
C = \frac{\sin (\frac{3}{4})}{\frac{3}{4}} \times \frac{\sin (\frac{3}{4})}{\frac{3}{4}}
$$

which simplifies to:

$$
C = \left(\frac{\sin (\frac{3}{4})}{\frac{3}{4}} \right)^2
$$

Calculating $\sin (\frac{3}{4})$ (with $\frac{3}{4}$ in radians):

$$
\sin \left(\frac{3}{4} \right) \approx 0.6816
$$

Thus:

$$
C \approx \left(\frac{0.6816}{0.75} \right)^2 = (0.909)^{2} \approx 0.826
$$

This confirms that the point lies on the curve with $C \approx 0.826$.

Implicit Differentiation to Find the Slope of the Tangent

Setting Up the Differentiation

Given the relation:

$$
F(x, y) = \frac{\sin x}{x} \cdot \frac{\sin y}{y} - C = 0
$$

we will differentiate implicitly with respect to x to find $\frac{dy}{dx}$, the slope of the tangent at the point.

Differentiating $\frac{\sin x}{x}$

The derivative of $\frac{\sin x}{x}$ with respect to x involves the quotient rule:

$$
\frac{d}{dx} \left(\frac{\sin x}{x} \right) = \frac{x \cos x - \sin x}{x^2}
$$

Similarly, for $\frac{\sin y}{y}$, considering y as a function of x:

$$
\frac{d}{dx} \left(\frac{\sin y}{y} \right) = \frac{d}{dy} \left(\frac{\sin y}{y} \right) \cdot \frac{dy}{dx}
$$

where:

$$
\frac{d}{dy} \left(\frac{\sin y}{y} \right) = \frac{y \cos y - \sin y}{y^2}
$$

Calculating $\frac{dy}{dx}$ at the Point $\left(\frac{3}{4}, \frac{3}{4} \right)$

Applying the Product Rule

Differentiating $F(x, y)$:

$$
\frac{d}{dx} \left(\frac{\sin x}{x} \cdot \frac{\sin y}{y} \right) = 0
$$

which yields:

$$
\left(\frac{x \cos x - \sin x}{x^2} \right) \cdot \frac{\sin y}{y} + \frac{\sin x}{x} \cdot \left(\frac{y \cos y - \sin y}{y^2} \right) \cdot \frac{dy}{dx} = 0
$$

Rearranging to solve for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = - \frac{ \left(\frac{x \cos x - \sin x}{x^2} \right) \cdot \frac{\sin y}{y} }{ \frac{\sin x}{x} \cdot \left(\frac{y \cos y - \sin y}{y^2} \right) }
$$

Substituting the Point Values

At $x = y = \frac{3}{4}$, and $C \approx 0.826$, compute:

$$
\sin \left(\frac{3}{4} \right) \approx 0.6816
$$

$$
\cos \left(\frac{3}{4} \right) \approx 0.7317
$$

Calculate numerator:

$$
x \cos x - \sin x = \frac{3}{4} \times 0.7317 - 0.6816 \approx 0.5488 - 0.6816 = -0.1328
$$

and denominator:

$$
x^2 = \left(\frac{3}{4} \right)^2 = 0.5625
$$

Thus:

$$
\frac{x \cos x - \sin x}{x^2} \approx \frac{-0.1328}{0.5625} \approx -0.2364
$$

Similarly,

$$
\frac{\sin y}{y} \approx \frac{0.6816}{0.75} \approx 0.909
$$

Now, numerator of $\frac{dy}{dx}$:

$$
-0.2364 \times 0.909 \approx -0.215
$$

Next, compute the denominator:

$$
\frac{\sin x}{x} \approx 0.909
$$

$$
y \cos y - \sin y = 0.75 \times 0.7317 - 0.6816 \approx 0.5488 - 0.6816 = -0.1328
$$

$$
y^2 = 0.5625
$$

$$
\frac{y \cos y - \sin y}{y^2} \approx \frac{-0.1328}{0.5625} \approx -0.2364
$$

Therefore, denominator:

$$
0.909 \times -0.2364 \approx -0.215
$$

Finally,

$$
\frac{dy}{dx} \approx - \frac{-0.215}{-0.215} = -1
$$

Result: The slope of the tangent line at the point $\left(\frac{3}{4}, \frac{3}{4} \right)$ is approximately -1.

Equation of the Tangent Line

Using Point-Slope Form

With the point $\left(\frac{3}{4}, \frac{3}{4} \right)$ and slope $m \approx -1$, the tangent line equation is:

$$
y - y_0 = m (x - x_0)
$$

Substituting the values:

$$
y - \frac{3}{4} = -1 \left(x - \frac{3}{4} \right)
$$

which simplifies to:

$$
y = -x + \frac{3}{4} + \frac{3}{4} = -x + \frac{3}{2}
$$

Final tangent line equation:

$$
y = -x + \frac{3}{2}
$$

Geometric and Analytical Significance

Nature of the Curve at the Point

The negative slope indicates that the curve is decreasing at that point, and the tangent line's equation provides a linear approximation to the curve near the point. The fact

Frequently Asked Questions

What is the given point in the problem and what is the curve equation?

The point is (/3, /4) and the curve is defined by the equation sin x / x sin y / y = C, where C is a constant.

How do we verify if the point (/3, /4) lies on the curve?

Substitute x = 3 and y = 4 into the equation sin x / x sin y / y = C to find the constant C. Calculate sin 3 / 3 and sin 4 / 4 and multiply them to get C.

What is the process to find the tangent line at the point (/3, /4) on the curve?

Differentiate the implicit equation with respect to x to find dy/dx, then evaluate at the point (/3, /4) to find the slope of the tangent line.

How can implicit differentiation be applied to the equation sin x / x sin y / y = C?

Differentiate both sides with respect to x, using the product rule and chain rule, treating y as a function of x to find dy/dx.

What is the derivative of sin x / x with respect to x?

Using the quotient rule, d/dx (sin x / x) = (x cos x - sin x) / x^2.

How do you find the slope of the tangent line at (/3, /4)?

Compute dy/dx at x = 3 and y = 4 using the implicit differentiation result to get the slope m, then use point-slope form to write the tangent line.

What is the final equation of the tangent line at the point (/3, /4)?

Plug the slope m and point (/3, /4) into the equation $y - y_1 = m(x - x_1)$ to obtain the tangent line equation.

Additional Resources

Suppose The Point (/3, /4) Is On The Curve Sinx/x Siny/y = C, Where C Is A Constant. Use X Y The Tangent

Understanding the intricacies of curves and their tangents is fundamental in calculus and analytical geometry. Imagine a scenario where a specific point, (/3, /4), lies on a complex curve defined by the relation Sinx/x Siny/y = C, where C remains constant. The question then arises: how do we determine the tangent to this curve at that particular point? This exploration not only deepens our grasp of derivatives and implicit differentiation but also showcases how mathematicians analyze the behavior of functions in a multi-variable context. In this article, we delve into the mathematical journey of identifying the tangent line to this curve at the given point, unraveling the underlying calculus concepts along the way.

Understanding the Curve: Sinx/x Siny/y = C

Before we proceed to find the tangent, it’s essential to understand the nature of the curve itself. The given relation:

$$ \frac{\sin x}{x} \times \frac{\sin y}{y} = C $$

describes a set of points (x, y) in the xy-plane that satisfy this equation. Here are some crucial aspects to consider:

1. The Components of the Curve

- Sinusoidal Functions: Both $\sin x / x$ and $\sin y / y$ are well-known functions in calculus, often called sinc functions, which are characterized by their oscillatory behavior and decay as their arguments tend to infinity.
- Product Equal to a Constant: The relation states that the product of these two sinc functions remains constant for all points on the curve.

2. Behavior at Specific Points

- At $x = 0$ or $y = 0$, the functions are not directly defined. However, using limits, we find:

$$
\lim_{x \to 0} \frac{\sin x}{x} = 1, \quad \lim_{y \to 0} \frac{\sin y}{y} = 1
$$

- The point $(0,0)$ then corresponds to $C = 1 \times 1 = 1$, indicating that the curve might pass through or near the origin if $C=1$.

3. The Role of the Constant C

- The value of C influences the shape and location of the curve.
- For example:
- If $C=1$, the curve includes points where $\sin x / x$ and $\sin y / y$ both are close to 1.
- For other values, the curve shifts accordingly, possibly forming disconnected branches.

Understanding these foundational elements helps in setting up the calculus tools required to find the tangent at a specific point.

Verifying the Point (/3, /4) on the Curve

Before calculating the tangent, it’s essential to confirm whether (/3, /4) indeed lies on the curve for a specific value of C.

Step 1: Calculate $\sin x / x$ at $x=3$

$$
\sin 3 \approx 0.14112
$$

$$
\frac{\sin 3}{3} \approx \frac{0.14112}{3} \approx 0.04704
$$

Step 2: Calculate $\sin y / y$ at $y=4$

$$
\sin 4 \approx -0.75680
$$

$$
\frac{\sin 4}{4} \approx \frac{-0.75680}{4} \approx -0.1892
$$

Step 3: Find C

$$
C = \left(\frac{\sin 3}{3} \right) \times \left(\frac{\sin 4}{4} \right) \approx 0.04704 \times (-0.1892) \approx -0.0089
$$

Thus, the constant C corresponding to the point (/3, /4) on the curve is approximately -0.0089.

Implicit Differentiation: Finding the Slope of the Tangent Line

Having established the point’s relevance, the next step involves finding the slope of the tangent line at this point, which requires calculus techniques—specifically, implicit differentiation.

1. Restate the equation

$$
\left(\frac{\sin x}{x} \right) \times \left(\frac{\sin y}{y} \right) = C
$$

Since C is constant, differentiating both sides with respect to x (treating y as a function of x) yields:

$$
\frac{d}{dx} \left[\frac{\sin x}{x} \times \frac{\sin y}{y} \right] = 0
$$

2. Apply the product rule

Let:

$$
U = \frac{\sin x}{x}, \quad V = \frac{\sin y}{y}
$$

Then:

$$
\frac{d}{dx} (UV) = U'V + UV' = 0
$$

3. Find derivatives U' and V'

- Derivative of U:

$$
U = \frac{\sin x}{x}
$$

Using quotient rule:

$$
U' = \frac{x \cos x - \sin x}{x^2}
$$

- Derivative of V:

$$
V = \frac{\sin y}{y}
$$

Since y is a function of x:

$$
V' = \frac{d}{dx} \left(\frac{\sin y}{y} \right)
$$

Using chain rule:

$$
V' = \frac{(y' \cos y) \times y - \sin y \times y'}{y^2} = \frac{y' y \cos y - y' \sin y}{y^2} = y' \times \frac{ y \cos y - \sin y }{ y^2 }
$$

4. Write the implicit differentiation equation

$$
U' V + U V' = 0
$$

Substitute U, V, U', V':

$$
\left(\frac{x \cos x - \sin x}{x^2} \right) \times \left(\frac{\sin y}{y} \right) + \left(\frac{\sin x}{x} \right) \times y' \times \frac{ y \cos y - \sin y }{ y^2 } = 0
$$

5. Solve for y':

$$
y' \times \frac{\sin x}{x} \times \frac{ y \cos y - \sin y }{ y^2 } = - \left(\frac{x \cos x - \sin x}{x^2} \right) \times \frac{\sin y}{ y }
$$

$$
y' = - \left[\frac{ x \cos x - \sin x }{ x^2 } \times \frac{\sin y}{ y } \right] \div \left[\frac{\sin x}{x} \times \frac{ y \cos y - \sin y }{ y^2 } \right]
$$

Simplify the division:

$$
y' = - \left(\frac{ x \cos x - \sin x }{ x^2 } \times \frac{\sin y}{ y } \right) \times \left(\frac{ y^2 }{ \sin x / x \times (y \cos y - \sin y) } \right)
$$

Calculating the Tangent Slope at (/3, /4)

Now, substitute the known values to compute the slope dy/dx at the point (/3, /4):

1. Compute necessary components:

- $x = 3$

- $y = 4$

- $\sin 3 \approx 0.14112$

- $\cos 3 \approx -0.98999$

- $\sin 4 \approx -0.75680$

- $\cos 4 \approx -0.65364$

2. Calculate numerator:

$$
x \cos x - \sin x = 3 \times (-0.98999) - 0.14112 \approx -2.97 - 0.14112 \approx -3.11112
$$

$$
\frac{ x \cos x - \sin x }{ x^2 } = \frac{ -3.11112 }{ 9 } \approx -0.34568
$$

$$
\frac{\sin y}{ y } = \frac{ -0.75680 }{ 4 } \approx -0.1892
$$

Numerator:

$$
-0.34568 \times -0.1892 \approx 0.06536
$$

3. Calculate denominator:

\

Suppose The Point 3 4 Is On The Curve Sinx X Siny Y C Where C Is A Constant Use X Y The Tangent

Suppose The Point 3 4 Is On The Curve Sinx X Siny Y C Where C Is A Constant Use X Y The Tangent

Related Articles

- Janie Walters, A 27-year-old Female, Presents Today With Burns On Her Left Hand. Over The Weekend, Janie
- Read The Excerpt From Julius Caesar, Act 2, Scene 4.PORTIA. I Prithee, Boy, Run To The Senate House;Stay
- Read The Excerpt From The Dark Game.On Several Occasions Room 40 Received An Unexpected But Welcome Gift

Back to Home