2. A. Determine The Cartesian Equation Of The Plane With Intercepts At P(-1,0,0), (0,1,0), And R(0,0,-3).

Understanding the Problem: Determining the Cartesian Equation of a Plane with Given Intercepts

2. A. Determine The Cartesian Equation Of The Plane With Intercepts At P(-1,0,0), (0,1,0), And R(0,0,-3). This problem involves finding the equation of a plane in three-dimensional space, given specific points where the plane intersects the coordinate axes. These points are known as intercepts, and understanding how to derive the Cartesian equation from such points is fundamental in solid geometry and vector calculus.

What Are Intercepts and Why Are They Important?

Definition of Intercepts

Intercepts are the points where a plane crosses the axes in a 3D coordinate system. Specifically:

- The x-intercept is the point where the plane crosses the x-axis (y=0, z=0).
- The y-intercept is the point where the plane crosses the y-axis (x=0, z=0).
- The z-intercept is where the plane crosses the z-axis (x=0, y=0).

Significance of Intercepts in Geometry

Knowing the intercepts simplifies the process of determining the plane’s equation because the intercepts directly relate to the plane's intercept form. This form is particularly useful for visualizing and understanding the plane's position relative to the axes.

Given Data: Intercepts at P(-1,0,0), Q(0,1,0), and R(0,0,-3)

In this problem, the points P, Q, and R are the intercepts on the x, y, and z axes respectively. The points are:

- P(-1, 0, 0): x-intercept at -1
- Q(0, 1, 0): y-intercept at 1
- R(0, 0, -3): z-intercept at -3

Step-by-Step Approach to Find the Cartesian Equation

1. Understanding the Intercept Form of a Plane

The general intercept form of a plane's equation is given by:

$x/a + y/b + z/c = 1$

Where:

- a is the x-intercept (x when y=0, z=0)
- b is the y-intercept (y when x=0, z=0)
- c is the z-intercept (z when x=0, y=0)

Using the intercepts provided, the initial form of the equation becomes:

$x/(-1) + y/1 + z/(-3) = 1$

2. Simplifying the Equation

Substitute the intercept values into the intercept form:

$- x + y - (z/3) = 1$

Multiply through by 3 to clear denominators:

$-3x + 3y - z = 3$

3. Arranging the Equation in Standard Cartesian Form

Rearranged, the Cartesian equation of the plane is:

$3x - 3y + z = -3$

This is the required equation of the plane passing through the given intercepts.

Verification: Confirming the Equation Meets the Intercepts

Checking Point P(-1,0,0)

$3(-1) - 3(0) + 0 = -3 + 0 + 0 = -3$ → matches the right side of the equation

Checking Point Q(0,1,0)

$3(0) - 3(1) + 0 = 0 - 3 + 0 = -3$ → matches the right side

Checking Point R(0,0,-3)

$3(0) - 3(0) + (-3) = 0 - 0 - 3 = -3$ → matches the right side

Since all points satisfy the equation, the derived plane equation is correct.

Additional Insights: Alternative Forms and Applications

1. Standard Form of the Plane Equation

The derived equation can be written as:

$$3x - 3y + z + 3 = 0$$

This form is often used in analytical geometry, especially for calculating distances or angles between planes and points.

2. Applications of the Cartesian Equation of a Plane

- Determining the position of the plane relative to other objects in space.
- Calculating the distance from a point to the plane.
- Finding the intersection line between two planes.
- In computer graphics for rendering 3D models.
- In engineering for structural analysis.

Summary: Key Steps in Deriving the Plane Equation from Intercepts

1. Identify the intercept points on each axis.
2. Write the intercept form of the plane's equation: $x/a + y/b + z/c = 1$.
3. Substitute the intercept values into this form.
4. Clear denominators and rearrange to standard form.
5. Verify the equation with the given points on the axes.

Conclusion: Mastering the Derivation of Plane Equations from Intercepts

Understanding how to determine the Cartesian equation of a plane from given intercepts is a crucial skill in geometry and vector calculus. It enables students and professionals to analyze spatial relationships effectively, solve geometry problems, and apply these concepts in various scientific and engineering fields. By following systematic steps—using intercept form, simplifying, and verifying—you can confidently derive the equation of any plane given its intercepts on the coordinate axes.

Frequently Asked Questions

How do you find the Cartesian equation of a plane given its intercepts?

To find the Cartesian equation of a plane with intercepts at points P, Q, and R, first identify the intercepts along the axes. Then, use the intercept form of the plane equation: $x/a + y/b + z/c = 1$, where a, b, and c are the x, y, and z intercepts respectively. In this case, $a = -1$, $b = 1$, and $c = -3$.

What is the process to derive the Cartesian equation of the plane passing through points P(-1,0,0), Q(0,1,0), and R(0,0,-3)?

First, find two vectors lying on the plane, such as PQ and PR. Then, compute their cross product to get the normal vector. Using the normal vector and one point (e.g., P), write the plane equation in the form $n \cdot (r - r_0) = 0$, and expand to get the Cartesian equation.

Can you demonstrate how to compute the normal vector of the plane from the given intercept points?

Yes. The vectors are PQ = Q - P = (0 - (-1), 1 - 0, 0 - 0) = (1, 1, 0) and PR = R - P = (0 - (-1), 0 - 0, -3 - 0) = (1, 0, -3). The normal vector n is the cross product of PQ and PR: n = PQ × PR = |i j k|, which computes to ((1)(-3) - (0)(0), (0)(1) - (1)(-3), (1)(0) - (1)(1)) = (-3, 3, -1).

What is the final Cartesian equation of the plane passing through P(-1,0,0), Q(0,1,0), and R(0,0,-3)?

Using the normal vector n = (-3, 3, -1) and point P(-1,0,0), the equation is: $-3(x + 1) + 3(y - 0) - 1(z - 0) = 0$, which simplifies to $-3x - 3 + 3y - z = 0$. Therefore, the Cartesian equation is $3x - 3y + z + 3 = 0$.

How can you verify that the derived plane equation

passes through the given points?

Substitute each point into the equation 3x - 3y + z + 3 = 0. For P(-1,0,0): 3(-1) - 3(0) + 0 + 3 = -3 + 0 + 0 + 3 = 0. For Q(0,1,0): 3(0) - 3(1) + 0 + 3 = 0 - 3 + 0 + 3 = 0. For R(0,0,-3): 3(0) - 3(0) + (-3) + 3 = 0 + 0 - 3 + 3 = 0. Since all satisfy the equation, the plane passes through all three points.

Additional Resources

Plane Equation Derivation: A Comprehensive Guide to Finding the Cartesian Equation with Given Intercepts

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Introduction: Unveiling the Geometry of Planes

In the realm of three-dimensional geometry, the concept of a plane is fundamental. It represents a flat, two-dimensional surface extending infinitely in all directions within space. Determining the exact equation of a plane is a critical skill, especially when it involves specific points or intercepts. This article offers an in-depth exploration of how to derive the Cartesian equation of a plane given its intercepts with the axes—specifically, points P(-1, 0, 0), R(0, 0, -3), and the point on the y-axis, which can be inferred from the intercepts.

Understanding the process of deriving this equation not only reinforces core geometric principles but also enhances problem-solving skills in coordinate geometry. Whether you're a student preparing for exams or an educator seeking a clear explanation for classroom instruction, this comprehensive guide will illuminate each step involved in determining the plane's Cartesian equation from its intercepts.

Understanding the Fundamentals: Intercepts and Plane Equations

What Are Intercepts?

In coordinate geometry, intercepts are points where a line or a plane crosses an axis. For a plane, the intercepts with the axes are particularly significant because they provide direct information about the plane's position relative to the coordinate axes.

- x-intercept: The point where the plane crosses the x-axis, at (a, 0, 0).
- y-intercept: The point where the plane crosses the y-axis, at (0, b, 0).
- z-intercept: The point where the plane crosses the z-axis, at (0, 0, c).

Given the problem, the intercepts are:

- Point P at (-1, 0, 0), indicating the x-intercept is at -1.
- Point R at (0, 0, -3), indicating the z-intercept at -3.
- The y-intercept is at (0, 1, 0), which can be deduced from the given points or directly from the problem statement.

Note: The y-intercept point is derived from the fact that the plane crosses the y-axis at some point (0, y, 0). Since it's given that the plane intercepts at (0,1,0), we know y = 1 at the y-intercept.

The General Form of the Plane Equation

The Cartesian equation of a plane, when intercepts are known, can be expressed in the intercept form:

$$
\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1
$$

where:

- a is the x-intercept,
- b is the y-intercept,
- c is the z-intercept.

This form is particularly useful because it directly relates the intercepts to the coordinates, making it straightforward to derive the plane's equation once the intercepts are known.

Step-by-Step Derivation of the Plane Equation

Step 1: Identifying the Intercepts

From the problem statement:

- X-intercept: P(-1, 0, 0) implies $a = -1$.

- Y-intercept: (0, 1, 0) implies $b = 1$.
- Z-intercept: R(0, 0, -3) implies $c = -3$.

These three points satisfy the plane's equation in intercept form:

$$
\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1
$$

Plugging in the values:

$$
\frac{x}{-1} + \frac{y}{1} + \frac{z}{-3} = 1
$$

Simplify:

$$
- x + y - \frac{z}{3} = 1
$$

Step 2: Rearranging to the Standard Cartesian Form

Multiply through by 3 to clear denominators:

$$
3(-x + y) - z = 3
$$

which simplifies to:

$$
-3x + 3y - z = 3
$$

Rearranged into the standard form $Ax + By + Cz + D = 0$:

$$
3x - 3y + z - 3 = 0
$$

Alternatively, multiply through by -1 to make the leading coefficient positive:

$$
-3x + 3y - z + 3 = 0
$$

Either form is acceptable; the key is consistency and clarity.

Step 3: Confirming the Validity of the Equation

To verify that the derived equation represents the correct plane, check whether all three intercept points satisfy it:

- Point P(-1, 0, 0):

$$
3(-1) - 3(0) + 0 - 3 = -3 - 0 + 0 - 3 = -6 \neq 0
$$

This indicates a discrepancy, suggesting the need to revisit the initial assumptions.

Important Note: Since the intercept form is:

$$
\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1
$$

and P(-1, 0, 0) satisfies this only if:

$$
\frac{-1}{a} + 0 + 0 = 1 \Rightarrow -\frac{1}{a} = 1 \Rightarrow a = -1
$$

Similarly, for R(0, 0, -3):

$$
0 + 0 + \frac{-3}{c} = 1 \Rightarrow -\frac{3}{c} = 1 \Rightarrow c = -3
$$

And the y-intercept (0, 1, 0):

$$
0 + \frac{1}{b} + 0 = 1 \Rightarrow \frac{1}{b} = 1 \Rightarrow b = 1
$$

Thus, the intercepts are confirmed as:

$$
a = -1, \quad b=1, \quad c=-3
$$

Plugging these into the intercept form:

$$
\frac{x}{-1} + \frac{y}{1} + \frac{z}{-3} = 1
$$

which simplifies to:

$$
- x + y - \frac{z}{3} = 1
$$

Multiplying through by 3:

$$
-3x + 3 y - z = 3
$$

or equivalently:

$$
3x - 3 y + z = -3
$$

Now, check the point P(-1, 0, 0):

$$
3(-1) - 3(0) + 0 = -3 \neq -3
$$

which satisfies the equation, confirming the correctness.

Final Cartesian Equation of the Plane

Putting everything together, the Cartesian equation of the plane with intercepts at P(-1, 0, 0), R(0, 0, -3), and the y-intercept at (0, 1, 0) is:

$$
\boxed{
3x - 3 y + z = -3
}
$$

This form succinctly captures the plane's position in space relative to the axes, directly linked to the given intercepts.

Additional Insights and Applications

Understanding the Geometric Significance

- The coefficients \(3, -3,\) and \(1\) in the equation reflect the relative orientation of the plane.
- The intercepts can be visualized by plotting the points and observing the plane's tilt and position.
- The equation can be rewritten in various forms, such as standard form or symmetric form, depending on the context.

Practical Applications

- Engineering Design: Precise plane equations are essential in CAD modeling and structural analysis.
- Physics: Analyzing fields and forces where planes represent boundaries or interfaces.
- Computer Graphics: Rendering scenes with accurate geometric representations.

Further Exploration

- Derive the equation of a plane given three non-collinear points.
- Explore the normal vector of the plane and its significance in calculating angles and distances.
- Use the derived plane equation to compute the shortest distance from a given point to the plane.

Conclusion: Mastering the Art of Plane Equation Derivation

Determining the Cartesian equation of a plane from its intercepts is a fundamental skill in coordinate geometry, blending algebraic manipulation with geometric intuition. By carefully identifying the intercepts, applying the intercept form, and verifying through substitution, one can accurately represent the plane in a standard algebraic form.

This process underscores the elegance of geometric relationships in three-dimensional space and provides essential tools for advanced mathematical, engineering, and scientific applications. Whether for academic pursuits or professional design, mastering this derivation enhances spatial understanding and mathematical fluency.

In essence, the equation \(3x - 3 y + z = -3\) encapsulates the entire spatial information of the plane

2 A Determine The Cartesian Equation Of The Plane With Intercepts At P 1 0 0 0 1 0 And R 0 0 3

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