2. Let L Be The Line In R 3 Passing Through The Origin In The Direction Of The Vector U=(2,1,0). Consider

2. Let L Be The Line In R 3 Passing Through The Origin In The Direction Of The Vector U=(2,1,0). Consider

---

Introduction to Lines in R^3 and Their Significance in Geometry

Understanding lines in three-dimensional space is fundamental in fields such as vector calculus, physics, computer graphics, and engineering. A line in R^3 can be described mathematically using vectors and parametric equations, providing insights into spatial relationships, intersections, and geometric properties. In this article, we delve into the specifics of a particular line, denoted as L, passing through the origin in R^3 and directed along the vector U=(2,1,0). Exploring this line involves examining its parametric form, geometric interpretation, and applications in various mathematical contexts.

Defining the Line L in R^3

The Parametric Equation of the Line

The line L in R^3 passing through the origin with direction vector U=(2,1,0) can be described using a parametric equation:

$$ \mathbf{r}(t) = t \mathbf{U} = t(2, 1, 0) $$

where $t \in \mathbb{R}$ is the parameter that varies along the line.

Key points to note:

- The line passes through the origin $(0,0,0)$ since when $t=0$, $\mathbf{r}(0) = (0,0,0)$.
- The direction vector $\mathbf{U}$ indicates the line's orientation in space.
- For each value of t, $\mathbf{r}(t)$ gives a point on the line.

Geometric Interpretation of the Line L

The line L extends infinitely in both directions, following the vector $(2,1,0)$. Geometrically, this line:

- Lies in the xy-plane since the z-component of the direction vector is zero.
- Has a slope determined by the ratio of the components of the direction vector.
- Does not intersect the z-axis unless it's at the origin, which it passes through.

Visualizing the Line in R^3

Understanding the spatial orientation of line L requires visualization techniques:

Direction and Position

- The line's direction vector $\mathbf{U} = (2, 1, 0)$ indicates that for each unit increase in t, the x-coordinate increases by 2, and the y-coordinate increases by 1.
- Since the z-component is zero, the line exists entirely within the xy-plane.

Graphical Representation

Plotting the line involves:

- Marking the origin point $(0,0,0)$.
- Drawing the vector $\mathbf{U}$ starting from the origin.
- Extending the line in both directions along $\mathbf{U}$.

This visualization helps in understanding the line's slope and orientation.

Mathematical Properties of the Line L

Direction Cosines and Angles

The direction vector $\mathbf{U} = (2, 1, 0)$ can be normalized to find the direction cosines:

$$|\mathbf{U}| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{4 + 1} = \sqrt{5}$$

Normalized vector:

$$\hat{\mathbf{U}} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right)$$

The angles between the line and the coordinate axes can be found using these cosines.

Equation of the Line in Symmetric Form

The parametric equations:

$$
x = 2t, \quad y = t, \quad z = 0
$$

can be expressed in symmetric form:

$$
\frac{x}{2} = y = t, \quad z = 0
$$

\]

which simplifies to:

$$
\frac{x}{2} = y, \quad z=0
$$

This form is useful for solving intersection problems with other geometric entities.

Applications of the Line L in Real-World Contexts

Engineering and Physics

- Trajectory analysis: The line can represent the path of an object moving in space with a constant velocity vector $\mathbf{U}$.
- Force vectors: In statics, the line may depict the direction of a force acting through a point.

Computer Graphics and Visualization

- Rendering objects: Lines like L are used in wireframe models to represent edges.
- Collision detection: Determining whether points or other geometric objects intersect or align with the line.

Mathematics and Geometry

- Line-plane intersections: Calculating where the line intersects a given plane.
- Line-line intersections: Finding common points with other lines in R^3.
- Vector projections: Projecting points onto the line for analysis.

Calculating Intersections with Other Geometric Entities

Intersection with a Plane

Suppose we want to find where the line L intersects a plane, for example:

$$
ax + by + cz + d = 0
$$

Given the parametric form of L, substitute $x=2t$, $y=t$, $z=0$:

$$
a(2t) + b(t) + c(0) + d = 0
$$

which simplifies to:

$$
(2a + b) t + d = 0
$$

If $2a + b \neq 0$, solving for t:

$$
t = -\frac{d}{2a + b}
$$

then, plugging back into the parametric equations yields the intersection point.

Intersection with Other Lines

To check if two lines in R^3 intersect, set their parametric equations equal and solve for parameters. For line L:

$$
\mathbf{r}_L(t) = (2t, t, 0)
$$

And for another line $\mathbf{r}_M(s)$, with known parametric form, solving for t and s will reveal if they intersect.

Advanced Topics Related to the Line L

Orthogonality and Perpendicular Lines

- To find a line perpendicular to L passing through a given point, utilize dot products:

$$
\mathbf{U} \cdot \mathbf{V} = 0
$$

where $\mathbf{V}$ is the direction vector of the perpendicular line.

Distance from a Point to the Line

Given a point $P = (x_0, y_0, z_0)$, the shortest distance d to the line L is:

$$
d = \frac{|\left(\mathbf{P} - \mathbf{A} \right) \times \mathbf{U}|}{|\mathbf{U}|}
$$

where $\mathbf{A}$ is a point on L (e.g., the origin), and $\times$ denotes the cross product.

Summary and Key Takeaways

- The line L in R^3 passing through the origin with direction vector $\mathbf{U} = (2,1,0)$ is fundamental in understanding spatial relationships.
- Its parametric form $\mathbf{r}(t) = t(2,1,0)$ succinctly describes its infinite extent.
- Visualizing the line helps in grasping its orientation and relation to other geometric entities.
- Applications range from physics to computer graphics, demonstrating the importance of such lines in practical scenarios.
- Mathematical tools like symmetric equations, intersection calculations, and vector properties facilitate advanced analysis.

Final Thoughts

Exploring a line in three-dimensional space, especially one passing through the origin with a given direction vector, opens doors to numerous geometric and analytical concepts. The line L defined by $\mathbf{U}=(2,1,0)$ exemplifies the elegance of vector representation and parametric equations in describing spatial structures. Mastery of these concepts is essential for students and professionals working in fields that involve 3D modeling, spatial analysis, and mathematical problem-solving.

Keywords for SEO Optimization:

- Lines in R^3
- Parametric equations of lines
- Vector geometry in three dimensions
- Line passing through the origin
- Direction vector in R^3
- 3D line visualization
- Intersecting lines and planes
- Distance from point to line in R^3
- Applications of lines in physics and engineering
- 3D geometric analysis

Frequently Asked Questions

What is the parametric form of the line L in R^3 passing through the origin in the direction of vector U = (2, 1, 0)?

The parametric equations of line L are $x = 2t$, $y = t$, $z = 0$, where $t \in \mathbb{R}$.

How can we verify if a point P(x, y, z) lies on the line L passing through the origin in the direction of U = (2, 1, 0)?

A point P lies on L if there exists a real number t such that $x = 2t$, $y = t$, and $z = 0$. Alternatively, check if (x, y, z) satisfies the parametric equations for some t.

What is the significance of the vector U = (2, 1, 0) in defining the line L in R^3?

The vector U indicates the direction of the line L; it shows the line's orientation in space and determines the slope of the line passing through the origin.

Can the line L be expressed in vector form, and if so, what is it?

Yes, the line L can be expressed as $r(t) = t(2, 1, 0)$, where $t \in \mathbb{R}$, representing all points r(t) on the line.

How would you find the shortest distance from a point P in R^3 to the line L passing through the origin with direction vector U=(2,1,0)?

The shortest distance d is given by the magnitude of the cross product of the vector from the origin to P and the direction vector U, divided by the magnitude of U: $d = |(OP) \times U| / |U|$.

Is the line L in R^3 passing through the origin and in the direction of U = (2, 1, 0) a line in a plane or in space, and why?

The line L is a line in three-dimensional space (R^3), as it extends in the direction of U and passes through the origin, which is in R^3.

What is the geometric interpretation of the line L in R^3 passing through the origin in the direction of U = (2, 1, 0)?

Geometrically, L is a straight line passing through the origin, extending infinitely in both directions, oriented along the vector U, lying in a plane defined by the direction vector and the origin.

Additional Resources

A Comprehensive Analysis of the Line L in $\mathbb{R}^3$ Passing Through the Origin in the Direction of $\mathbf{U} = (2, 1, 0)$

Introduction

In the realm of vector calculus and analytic geometry, understanding the properties of lines in three-dimensional space is fundamental. Here, we focus on a specific line L in $\mathbb{R}^3$, which passes through the origin and is directed along a given vector $\mathbf{U} = (2, 1, 0)$. The goal of this discussion is to analyze this line comprehensively, exploring its parametric representation, geometric properties, relation to the vector $\mathbf{U}$, and implications in various mathematical contexts.

---

1. Defining the Line L in $\mathbb{R}^3$

1.1. Parametric Equation of the Line

A line in $\mathbb{R}^3$ passing through a point $\mathbf{P}_0$ with position vector $\mathbf{r}_0$ and directed along vector $\mathbf{U}$ can be expressed parametrically as:

$$
\boxed{
\mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{U}
}
$$

Since the line passes through the origin, the point $\mathbf{P}_0 = (0, 0, 0)$ with $\mathbf{r}_0 = \mathbf{0}$. The direction vector is given as $\mathbf{U} = (2, 1, 0)$. Therefore, the parametric equations become:

$$
\boxed{
\begin{cases}
x(t) = 2t \\
y(t) = t \\
z(t) = 0
\end{cases}
}
$$

for $t \in \mathbb{R}$.

1.2. Geometric Interpretation

- The line extends infinitely in both directions along the vector $\mathbf{U}$.

- Since $z(t) = 0$ for all t, the entire line lies within the xy-plane, which is the plane $z=0$.

---

2. Geometric Insights and Visualization

2.1. Visualizing the Line L

- The line passes through the origin $(0, 0, 0)$.
- It extends in the direction of $\mathbf{U}$, which can be visualized as a vector pointing from the origin toward the point $(2, 1, 0)$.
- The fact that $z=0$ for all points on the line indicates that the line is contained entirely within the xy-plane.

2.2. Direction and Magnitude of $\mathbf{U}$

- The vector $\mathbf{U} = (2, 1, 0)$ has a magnitude:

$$
|\mathbf{U}| = \sqrt{2^2 + 1^2 + 0^2} = \sqrt{4 + 1} = \sqrt{5}
$$

- The unit vector in the direction of L is:

$$
\hat{\mathbf{u}} = \frac{\mathbf{U}}{|\mathbf{U}|} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right)
$$

This unit vector is useful in many applications, such as projecting points onto the line or analyzing the line's relation to other vectors or geometric objects.

---

3. Algebraic and Geometric Properties

3.1. Direction Cosines

The direction cosines are the cosines of the angles between the line's direction vector and the coordinate axes. They are computed as:

$$
l = \frac{2}{\sqrt{5}}, \quad m = \frac{1}{\sqrt{5}}, \quad n = 0
$$

- The angle θ_x between $\mathbf{U}$ and the x-axis:

$$
\cos \theta_x = l = \frac{2}{\sqrt{5}}
$$

- The angle θ_y between $\mathbf{U}$ and the y-axis:

$$
\cos \theta_y = m = \frac{1}{\sqrt{5}}
$$

- The angle θ_z between $\mathbf{U}$ and the z-axis:

$$
\cos \theta_z = n = 0 \Rightarrow \theta_z = 90^\circ
$$

This confirms the line is perpendicular to the z-axis, lying entirely in the xy-plane.

3.2. Line Equation in Symmetric Form

From the parametric equations:

$$
x = 2t, \quad y = t, \quad z=0
$$

we can eliminate t:

$$
t = y \Rightarrow x = 2 y
$$

Thus, the symmetric form of the line is:

$$
\boxed{
\frac{x}{2} = y, \quad z=0
$$

or, more explicitly,

$$
x = 2 y, \quad z=0
$$

which describes a straight line in the xy-plane, passing through the origin, with slope 2 with respect to the y-axis.

---

4. Relation to Planes and Other Geometric Objects

4.1. The Line L as a Subspace

- The line L is a one-dimensional linear subspace of $\mathbb{R}^3$.

- It is a line through the origin, implying it is a vector subspace.

4.2. Orthogonal Complement

The orthogonal complement $L^\perp$ consists of all vectors in $\mathbb{R}^3$ orthogonal to $\mathbf{U}$:

$$
L^\perp = \{\mathbf{v} \in \mathbb{R}^3 : \mathbf{v} \cdot \mathbf{U} = 0 \}
$$

Calculating:

$$
\mathbf{v} = (x, y, z), \quad \mathbf{v} \cdot \mathbf{U} = 2x + y + 0 \cdot z = 0
$$

which simplifies to:

$$
2x + y = 0
$$

This is the equation of a plane passing through the origin, perpendicular to L. It has the normal vector $\mathbf{U}$.

---

5. Applications and Contexts

5.1. Projection of Points onto L

Given an arbitrary point $\mathbf{P} = (x_0, y_0, z_0)$, the orthogonal projection $\mathbf{P}_{L}$ onto L is obtained by:

$$
\mathbf{P}_L = (\mathbf{P} \cdot \hat{\mathbf{u}}) \hat{\mathbf{u}}
$$

This process is crucial in applications like:

- Computer graphics
- Engineering design
- Optimization problems

5.2. Intersection with Planes

Suppose we have a plane:

$$
ax + by + cz + d = 0
$$

\]

To find the intersection with \(L\), substitute the parametric equations:

\[
x = 2t, \quad y = t, \quad z=0
\]

and determine the value of \(t\) satisfying the plane equation.

---

6. Transformations and Coordinate Changes

6.1. Rotation to Simplify the Direction

Transformations such as rotations can align the line \(L\) with coordinate axes, simplifying analysis. For example:

- Rotate around the \(z\)-axis by an angle \(\phi\) to align the line with the \(x\)-axis.
- The rotation matrix around the \(z\)-axis:

\[
R_z(\phi) =
\begin{bmatrix}
\cos \phi & - \sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

- Choosing \(\phi\) such that the rotated vector aligns with the \(x\)-axis simplifies calculations.

6.2. Scaling and Shearing

- Applying scaling transformations can normalize the direction vector.
- Shearing transformations can change the orientation without altering the line's passing point at the origin.

---

7. Advanced Topics and Extensions

7.1. Line in the Context of Vector Spaces

- The line \(L\) is a one-dimensional subspace, characterized by its basis vector \(\mathbf{U}\).
- Its span is:

\[

L = \{\lambda \mathbf{U} : \lambda \in \mathbb{R}\}
\]

- Understanding subspaces in \(\mathbb{R}^3\) helps in linear algebra applications like basis selection, dimension counting, and orthogonal decompositions.

7.2. Line as a Ge

2 Let L Be The Line In R 3 Passing Through The Origin In The Direction Of The Vector U 2 1 0 Consider

2 Let L Be The Line In R 3 Passing Through The Origin In The Direction Of The Vector U 2 1 0 Consider

Related Articles

- To Reduce The Risk Of Aspiration Of Blood Or Other Fluids In A Seizure Victim:a. Place An Object Between
- Two Children Of Mass 20 Kg And 30 Kg Sit Balanced On A Seesaw With The Pivot Point Located At The Center
- *ANSWER PLS , DIFFICULT QUESTION* Africans Had Resisted European Colonial Rule Before World War II. Which

Back to Home