3) Given The System Defined By Y(t)=2tx(t), Answer The Following And Justify Your Answer. A. Show Whether

3) Given The System Defined By Y(t)=2t x(t), Answer The Following And Justify Your Answer. A. Show Whether

Introduction

Understanding the behavior of linear time-invariant (LTI) systems is fundamental in signal processing and systems analysis. The system defined by the equation $Y(t) = 2t\,x(t)$ presents an interesting case because it involves a time-dependent coefficient, 2t, multiplying the input signal x(t). Analyzing such a system requires examining its properties such as linearity, time-invariance, causality, and stability. These properties determine how the system processes signals, its applicability in real-world scenarios, and how it might be manipulated or controlled.

This article aims to thoroughly analyze the system $Y(t) = 2t\,x(t)$. We will explore whether the system is linear, time-invariant, causal, and stable. By providing detailed justifications for each property, we will deepen the understanding of how this system functions and its classification within the broader context of system theory.

Understanding the System Equation

The system in question is described by:

$Y(t) = 2t\,x(t)$

where:

- Y(t) is the output signal.
- x(t) is the input signal.
- t is the continuous time variable.
- 2t is a multiplicative factor that varies linearly with time.

This system can be viewed as a modulation of the input signal by a time-dependent coefficient, which suggests that it might have properties different from standard LTI systems.

Analyzing the System Properties

To analyze the system comprehensively, we need to examine the following key properties:

1. Linearity
2. Time-Invariance
3. Causality
4. Stability

Each property will be discussed with detailed reasoning.

1. Linearity

Definition of Linearity:

A system is linear if it satisfies two main principles:

- Additivity: The response to a sum of inputs equals the sum of the responses to each input separately.
- Homogeneity (Scaling): The response to a scaled input is equal to the scaled response of the original input.

Mathematically:

If inputs $x_1(t)$ and $x_2(t)$, and scalars α and β, then:

System's output: $Y(t) = 2t\ x(t)$

We check if:

$$ \text{System}(\alpha x_1(t) + \beta x_2(t)) = \alpha \text{System}(x_1(t)) + \beta \text{System}(x_2(t)) $$

Verification:

$$
\begin{aligned}
Y(t) &= 2t [\alpha x_1(t) + \beta x_2(t)] \\
&= 2t \alpha x_1(t) + 2t \beta x_2(t) \\
&= \alpha (2t x_1(t)) + \beta (2t x_2(t))
\end{aligned}
$$

Since the output for each input is scaled and summed accordingly, the system satisfies both additivity and homogeneity.

Conclusion:

The system is linear because it satisfies the principles of linearity.

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2. Time-Invariance

Definition of Time-Invariance:

A system is time-invariant if a time shift in the input results in an equivalent time shift in the output, without altering the form of the output.

Mathematically, if for an input $x(t)$, the output is $y(t)$, then for a shift τ:

$$
x(t) \rightarrow x(t - \tau) \implies y(t) \rightarrow y(t - \tau)
$$

Verification:

Let's examine the response to a shifted input:

- Original input: $x(t)$
- Shifted input: $x(t - \tau)$

Output to original input:

$$
Y(t) = 2t \cdot x(t)
$$

Output to shifted input:

$$
Y_{shifted}(t) = 2t \cdot x(t - \tau)
$$

Now, compare this to the shifted original output:

$$
Y(t - \tau) = 2(t - \tau) \cdot x(t - \tau)
$$

Note that:

$$
Y_{shifted}(t) = 2t \cdot x(t - \tau) \neq Y(t - \tau) = 2(t - \tau) \cdot x(t - \tau)
$$

This implies:

$$
Y_{shifted}(t) \neq Y(t - \tau)
$$

Because of the difference in the multiplicative factor ($2t$ vs $2(t - \tau)$), the output does not shift uniformly with the input. The factor $2t$ depends explicitly on the current time t, not on the input shift τ.

Conclusion:

The system is not time-invariant because shifting the input in time does not produce a corresponding shift in the output; the multiplicative factor involving t breaks time invariance.

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3. Causality

Definition of Causality:

A system is causal if the output at time t depends only on the input at present or past times, i.e., on $x(\tau)$ where $\tau \leq t$.

Analysis:

In this system:

$$
Y(t) = 2t\, x(t)
$$

the output at time t depends solely on the input at the same time t, scaled by $2t$.

- Since the output depends only on $x(t)$, it does not depend on future inputs ($x(t + \Delta)$), satisfying causality.

Conclusion:

The system is causal because the output at time t depends only on the current input $x(t)$.

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4. Stability

Definition of Bounded Input–Bounded Output (BIBO) Stability:

A system is BIBO stable if every bounded input produces a bounded output.

- If $|x(t)| \leq M$ for all t, where M is finite, then $|Y(t)|$ should also be bounded.

Analysis:

Given:

$$
Y(t) = 2t\, x(t)
$$

Suppose:

$$
|x(t)| \leq M
$$

then:

$$
|Y(t)| = |2t\, x(t)| \leq 2|t|\, M
$$

As $t \to \infty$, $|Y(t)| \to \infty$ unless $x(t)$ tends to zero faster than $1/|t|$, which is not guaranteed for all bounded inputs.

Thus:

- For bounded inputs, the output grows unboundedly with t, implying the system is not BIBO stable.

Conclusion:

The system is unstable in the BIBO sense because the output can become unbounded for bounded inputs over time.

Summary of System Properties

Property	Result	Justification
Linearity	Yes	Satisfies additivity and homogeneity.
Time-Invariance	No	The time-dependent coefficient (2t) causes the output to not shift uniformly with input shifts.
Causality	Yes	The output depends only on the current input x(t).
Stability	No	For bounded inputs, the output grows unbounded with time due to the 2t factor.

Implications and Applications

Understanding these properties helps in determining how this system can be utilized in practical applications. For example, because the system is linear and causal, it can be implemented in real-time processing systems where causality is essential. However, its lack of time-invariance and stability limits its use in applications requiring consistent behavior over time or bounded outputs.

This system can be viewed as a time-varying gain system that scales the input signal proportionally to time, which might be useful in scenarios like time-dependent amplification or modulation where the gain needs to vary with time. Nonetheless, the unbounded growth of the output for bounded inputs suggests caution in such applications.

Conclusion

Analyzing the system with the equation $Y(t) = 2t\, x(t)$ reveals critical insights into its behavior:

- It is linear due to the linearity of the operation.
- It is not time-invariant because the multiplicative factor depends explicitly on the current time.
- It is causal since output depends only on present inputs.

- It is not BIBO stable, as the output can grow without bound even for bounded inputs.

Understanding these characteristics is vital when designing systems involving such equations, especially in signal processing and control applications. Recognizing the limitations and strengths of this system allows engineers and scientists to make informed decisions about its suitability for specific tasks.

Keywords: system analysis, linearity, time-invariance, causality, stability, signal processing, BIBO stability, time-dependent systems, system properties

Frequently Asked Questions

Is the system defined by $Y(t) = 2t\, x(t)$ linear? Justify your answer.

Yes, the system is linear because it satisfies both additivity and homogeneity. For inputs $x_1(t)$ and $x_2(t)$ and scalars a and b, the output for a combination $a\, x_1(t) + b\, x_2(t)$ results in $Y(t) = 2t\, [a\, x_1(t) + b\, x_2(t)] = a\, Y_1(t) + b\, Y_2(t)$, demonstrating linearity.

Is the system time-invariant? Justify your answer.

No, the system is not time-invariant because the output explicitly depends on the variable t. If the input is shifted in time, the output will not shift correspondingly due to the multiplication by t, indicating the system's time variance.

Is the system memoryless? Justify your answer.

No, the system is not memoryless because the output at time t depends explicitly on t itself, not solely on the input at time t. This dependence on t indicates the presence of memory.

Is the system causal? Justify your answer.

Yes, the system is causal because the output at time t depends only on the input at the same time t, with no dependence on future or past inputs.

Is the system stable? Justify your answer.

The system's stability depends on the boundedness of the input. Since the output is $Y(t) = 2t\, x(t)$, if $x(t)$ is bounded and t remains finite over the interval considered, then the system is bounded-input bounded-output (BIBO) stable. However, for unbounded t (e.g., over infinite time), the system can produce unbounded outputs even for bounded inputs, indicating potential instability.

Can this system be classified as time-invariant? Provide justification.

No, this system cannot be classified as time-invariant because the multiplication by t in the output explicitly depends on the current time variable. A time-shift in input does not produce an equivalent shift in output due to this t-dependent factor.

What is the effect of the time variable t on the system's output? Justify your answer.

The time variable t acts as a scaling factor that modulates the input x(t) linearly with time. As t increases, the output magnitude increases proportionally for the same input, making the system's response time-dependent and potentially unbounded as t grows.

Additional Resources

Given The System Defined By Y(t) = 2t x(t), Answer The Following And Justify Your Answer

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Introduction to the System Y(t) = 2t x(t)

The system characterized by the equation Y(t) = 2t x(t) presents an intriguing scenario in the analysis of linear and time-variant systems. At its core, this relationship indicates that the output Y(t) is obtained by multiplying the input x(t) by a factor that depends explicitly on time, namely 2t. This dependency introduces particular considerations regarding the system's properties, including linearity, time-invariance, causality, and stability. Analyzing this system gives us insight into how time-dependent coefficients influence system behavior, which is fundamental in signal processing, control systems, and communications.

Understanding the System: Basic Properties and Definitions

Before delving into specific questions about the system, it's essential to clarify key concepts that form the basis of system analysis:

- Linearity: A system is linear if it satisfies the principles of superposition, which include additivity and homogeneity.

- Time-invariance: A system is time-invariant if a time shift in the input results in an identical time shift in the output.
- Causality: A system is causal if its output at any time t depends only on the input at the present or past times.
- Stability: A system is BIBO (Bounded Input, Bounded Output) stable if every bounded input produces a bounded output.

With these definitions in mind, let's explore how the system $Y(t) = 2t\, x(t)$ behaves with respect to these properties.

Linearity of the System

Is the system linear? Justify your answer.

Answer:

The system $Y(t) = 2t\, x(t)$ is linear.

Justification:

To verify the linearity, consider two arbitrary inputs $x_1(t)$ and $x_2(t)$, with corresponding outputs $Y_1(t)$ and $Y_2(t)$:

- For input $x_1(t)$:

$Y_1(t) = 2t\, x_1(t)$

- For input $x_2(t)$:

$Y_2(t) = 2t\, x_2(t)$

Now, for any scalar constants a and b, the combined input is:

$x(t) = a\, x_1(t) + b\, x_2(t)$

The output corresponding to this combined input is:

$Y(t) = 2t\, [a\, x_1(t) + b\, x_2(t)] = a\, (2t\, x_1(t)) + b\, (2t\, x_2(t)) = a\, Y_1(t) + b\, Y_2(t)$

Since the output is a linear combination of the inputs with the same coefficients, the system satisfies superposition and scaling properties.

Features:

- Superposition: Holds true.
- Homogeneity: Holds true.

Pros:

- The linearity simplifies analysis and design.
- Superposition makes it easier to understand the system’s response to complex inputs.

Cons:

- The explicit time dependence means the system's properties may change over time, affecting stability and invariance.

Time-Invariance of the System

Is the system time-invariant? Justify your answer.

Answer:

The system $Y(t) = 2t\, x(t)$ is not time-invariant.

Justification:

A system is time-invariant if shifting the input in time results in an identical shift in output. Let's test this:

Suppose the input $x(t)$ produces an output:

$Y(t) = 2t\, x(t)$

Now, consider a shifted input:

$x_1(t) = x(t - t_0)$

The output for this shifted input is:

$Y_1(t) = 2t\ x(t - t_0)$

If the system were time-invariant, then shifting the original output $Y(t)$ by t_0 should give us the new output:

Shifted output: $Y(t - t_0) = 2(t - t_0)\ x(t - t_0)$

Compare this with $Y_1(t)$:

$Y_1(t) = 2t\ x(t - t_0)$

Since $2t \neq 2(t - t_0)$, the outputs differ unless $t_0 = 0$.

Conclusion:

Because the factor 2t depends explicitly on time t, shifting the input does not produce a simple shift in the output. Therefore, the system is not time-invariant.

Features:

- Explicit time dependence breaks time-invariance.
- The response changes with absolute time, not just relative shifts.

Pros:

- Can model systems where behavior varies over time.

Cons:

- Difficult to analyze and predict behavior over time.
- Not suitable for systems requiring time-invariance assumptions.

Causality of the System

Is the system causal? Justify your answer.

Answer:

Yes, the system $Y(t) = 2t\, x(t)$ is causal.

Justification:

Causality depends on whether the output at time t depends only on the current or past inputs. Here, the output at time t depends solely on $x(t)$ and the current time t:

- The output $Y(t) = 2t\, x(t)$ involves only the input $x(t)$ evaluated at the same time t.

Since there is no dependence on future inputs $x(t + \tau)$ with $\tau > 0$, the system is causal.

Features:

- Dependence only on current input: ensures causality.
- Suitable for real-time systems where future inputs are unknown.

Pros:

- Implementable in real-time processing.
- Aligns with physical systems where future inputs cannot influence the present.

Cons:

- None apparent in this context; the system is straightforwardly causal.

Stability of the System

Is the system BIBO stable? Justify your answer.

Answer:

The system $Y(t) = 2t\, x(t)$ is not BIBO stable.

Justification:

A system is BIBO stable if every bounded input produces a bounded output.

Suppose $|x(t)| \leq M$ for all t, where M is a finite constant.

Then,

$|Y(t)| = |2t\ x(t)| \leq 2|t|\ M$

As time t approaches infinity, $|Y(t)|$ grows without bound because of the factor $2|t|$. Therefore, even with a bounded input, the output becomes unbounded as t increases.

Conclusion:

The system is not stable because the multiplicative factor 2t amplifies any bounded input over time, leading to unbounded output for large t.

Features:

- Exhibits unbounded growth over time, making it unsuitable for applications requiring stability.

Pros:

- Models systems where output naturally grows with time.

Cons:

- Not practical for systems needing bounded responses.
- Sensitive to unbounded outputs in real-world applications.

Summary and Final Thoughts

The system $Y(t) = 2t\ x(t)$ serves as a classic example illustrating how explicit time dependence affects fundamental system properties. Its linearity makes it predictable in terms of superposition; however, its lack of time-invariance and stability limits its practical applications. Its causality remains intact, rendering it suitable for real-time processing, but its unbounded growth over time poses challenges for stability-critical systems.

In practical scenarios, understanding these properties aids engineers and scientists in designing systems that either leverage or mitigate such characteristics. For instance, the explicit time dependence could be exploited in applications requiring time-varying amplification, but caution is necessary to prevent instability or unbounded outputs.

Conclusion

Analyzing the system Y(t) = 2t x(t) underscores the importance of understanding how factors like explicit time dependence influence core properties such as linearity, causality, time-invariance, and stability. While the system is linear and causal, it is not time-invariant or stable, which constrains its applicability. Recognizing these attributes allows for better system design and helps anticipate behavior under various input conditions. Such analysis is crucial in advancing signal processing techniques and control systems, ensuring that systems perform as intended within their operational constraints.

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