4. Find Solution Of The System Of Equations. Use D-operator Elimination Method. X' = (4 -5) X(2 -3) Write

4. Find Solution Of The System Of Equations. Use D-operator Elimination Method. X' = (4 -5) X(2 -3) Write

The process of solving systems of differential equations is fundamental in understanding the behavior of dynamic systems across physics, engineering, and applied mathematics. Among various methods available, the D-operator (or differential operator) method provides a systematic approach to transforming differential equations into algebraic equations, simplifying the process of finding solutions. In this article, we will explore the detailed steps to find the solution of the given system of equations using the D-operator elimination method, starting with the provided matrix representation and carefully working through each step to obtain the general solution.

Understanding the System of Equations

Given System in Matrix Form

The problem provides a matrix form of the system of differential equations as:

```
X' = (4 -5)
(2 -3) X
```

This indicates a system of two first-order linear differential equations, which can be written explicitly as:

$$x_1' = 4x_1 - 5x_2$$
$$x_2' = 2x_1 - 3x_2$$

where x_1 and x_2 are functions of the independent variable (say, time *t*), and the primes denote derivatives with respect to *t*.

Step 1: Write the System in Matrix Form

Matrix Representation

Expressing the system in matrix notation helps in applying the D-operator method systematically:

$$X' = A X$$

where:

- **X** = $[x_1(t), x_2(t)]^T$
- **A** = *[[4, -5], [2, -3]]*

Explicitly, the system is:

$$\frac{d}{dt} [x_1] = 4x_1 - 5x_2$$
$$\frac{d}{dt} [x_2] = 2x_1 - 3x_2$$

Step 2: Applying the D-operator

Introduction to the D-operator

The D-operator, denoted as *D*, is defined as the differentiation operator with respect to *t*. When applied to a function *f(t)*, it yields:

```
D f(t) = df/dt
```

In the context of systems, the D-operator can be used to transform matrix differential equations into algebraic equations by treating *D* as an algebraic variable.

Rewriting the system with D

```
(D I - A) X = 0
```

where *I* is the identity matrix.

In this case:

```
(D I - A) =
[[D - 4, 5],
[-2, D + 3]]
```

Note: Since the original system is *X' = A X*, the characteristic equation for the system can be obtained by solving the determinant:

```
det(D I - A) = 0
```

which leads to the characteristic polynomial used to find eigenvalues.

Step 3: Finding the Characteristic Equation

Constructing the Matrix (D I - A)

```
D I - A =
[[D - 4, 5],
[-2, D + 3]]
```

Determinant Calculation

1. Compute the determinant:

```
det(D I - A) = (D - 4)(D + 3) - (-2)(5)
```

2. Expand the terms:

```
= (D^2 - 4D + 3D - 12) + 10
= D^2 - D - 2
```

Characteristic Equation

Set the determinant to zero:

```
D^2 - D - 2 = 0
```

This quadratic equation determines the eigenvalues of the matrix A.

Step 4: Solving the Characteristic Equation

Quadratic Solution

$$D^2 - D - 2 = 0$$

Using quadratic formula:

$$
\begin{aligned}
D &= [1 \pm \sqrt{1^2 - 4 \;\; 1 \;\; (-2)}]/(2 \;\; 1) \\
&= [1 \pm \sqrt{1 + 8}]/2 \\
&= [1 \pm \sqrt{9}]/2
\end{aligned}
$$

Thus, the roots are:

$$
\begin{aligned}
D_1 &= (1 + 3)/2 = 4/2 = 2 \\
D_2 &= (1 - 3)/2 = -2/2 = -1
\end{aligned}
$$

These eigenvalues $D_1 = 2$ and $D_2 = -1$ are crucial for constructing the general solution.

Step 5: Finding Eigenvectors

Eigenvector for D = 2

Substitute D=2 into *(D I - A)* and solve *(D I - A) v = 0*:

$$
\begin{aligned}
&(2 I - A) v = 0 \\
&\rightarrow \\
&[[2 - 4, 5], \\
&[-2, 2 + 3]] v = 0 \\
&\rightarrow \\
&[[-2, 5], \\
&[-2, 5]] v = 0
\end{aligned}
$$

This simplifies to:

$$-2 v_1 + 5 v_2 = 0$$

Choose $v_2 = 1$, then $v_1 = (5/2) v_2 = 2.5$.

Eigenvector:

$v_1 = [2.5, 1]$ (or scaled as needed).

Eigenvector for D = -1

$$
(-1 I - A) v = 0
$$
→
$$
[[-1 - 4, 5],
[-2, -1 + 3]] v = 0
$$
→
$$
[[-5, 5],
[-2, 2]] v = 0
$$

The first equation:

$$
-5 v_1 + 5 v_2 = 0 \rightarrow v_2 = v_1
$$

Choose $v_1 = 1$, $v_2 = 1$.

Eigenvector:

$v_2 = [1, 1]$

Step 6: Constructing the General Solution

Form of the Solution

The general solution to the system is a linear combination of eigenvectors multiplied by exponentials of eigenvalues:

$$
X(t) = C_1 e^{D_1 t} v^{(1)} + C_2 e^{D_2 t} v^{(2)}
$$

where:

- c_1, c_2 are arbitrary constants determined by initial conditions.

- *v^{(1)}, v^{(2)}* are eigenvectors corresponding to D_1 and D_2 respectively.

Explicitly, the solution is:

$$X(t) = C_1 e^{2 t} [2.5, 1]^T + C_2 e^{-t} [1, 1]^T$$

Step 7: Final Expression and Interpretation

Component-wise Solution

Expressing explicitly:

$$x_1(t) = 2.5 C_1 e^{2 t} + C_2 e^{-t}$$
$$x_2(t) = C_1 e^{2 t} + C_2 e^{-t}$$

Summary of the Solution

The solution combines exponential terms derived from the eigenvalues, scaled by constants that depend on initial conditions. The D-operator elimination method effectively reduces the system to solving a characteristic polynomial, identifying eigenvalues and eigenvectors, and then constructing the general solution accordingly.

Additional Notes and Considerations

- **Initial Conditions:** To determine the specific constants C_1 and C_2, initial values of $x_1(0)$ and

x□(0) are required.

- **Stability:** The eigenvalues indicate the system's stability:

    - If real

Frequently Asked Questions

What is the main goal when solving a system of equations using the D-operator elimination method?

The main goal is to eliminate one variable to find the value of the other, simplifying the system to solve for the unknowns efficiently.

How do you set up the D-operator for a system of two equations?

You express each equation in operator form, typically as matrix equations involving differential operators, and then combine them to eliminate one variable by finding the determinant (D).

Given the system X' = (4 -5; 2 -3)X, how do you find the

characteristic equation using the D-operator?

You find the determinant of (D I - A), where A is the coefficient matrix, resulting in the characteristic equation D^2 + D + 10 = 0.

What is the significance of calculating the determinant D in the elimination method?

Calculating D helps determine the nature of the system's solutions and allows you to eliminate variables by solving the resulting operator equations.

Once the D-operator is applied, how do you solve for the individual variables in the system?

You solve the resulting differential equations derived from the operator equations, often by finding particular solutions and applying initial conditions if available.

How does the D-operator elimination method compare to substitution or elimination methods in solving linear systems?

The D-operator method is more systematic for differential systems, especially when dealing with higher-order derivatives, whereas substitution and elimination are more straightforward for algebraic systems.

What are the steps to solve the system X' = A X using the D-operator elimination method?

First, write the system in matrix form, derive the operator equation, compute the determinant D, find the characteristic equation, solve the resulting differential equation, and then find the particular solutions for the variables.

Can the D-operator elimination method be used for non-homogeneous systems? If so, how?

Yes, for non-homogeneous systems, after solving the homogeneous part with the D-operator method, particular solutions can be found using methods like undetermined coefficients or variation of parameters.

What is the general solution to the system X' = (4 -5; 2 -3)X using the D-operator elimination method?

The general solution involves solving the characteristic equation $D^2 + D + 10 = 0$, obtaining the eigenvalues, and then constructing the solution as a combination of exponential functions based on those eigenvalues.

Additional Resources

Finding solutions to systems of equations is a fundamental task in mathematics, particularly in fields such as engineering, physics, economics, and computer science. Among various methods available, the D-operator elimination method offers a systematic approach to solving linear systems, especially when dealing with differential equations. This article provides a detailed exploration of how to find solutions for a system of equations using the D-operator elimination method, exemplified by the given system: X' = (4 - 5)X + (2 - 3). We will delve into the theoretical foundations, step-by-step procedures, and practical applications of this technique to equip readers with a comprehensive understanding of the method.

Understanding the System of Equations

The Given System

The problem statement presents a system involving derivatives and linear combinations of variables. The notation appears to involve a differential system, where:

- X' denotes the derivative of X with respect to some independent variable, typically time (t).
- The coefficients are given as numerical matrices or constants.

However, the exact form of the system, as provided, seems to be a bit ambiguous:
X' = (4 - 5) X + (2 - 3)

Interpreted carefully, this suggests the following:

- The coefficients (4 - 5) and (2 - 3) are to be evaluated first.

- The system involves derivatives of X, with some linear combination.

Let's clarify this step-by-step:

1. Evaluate the coefficients:

- (4 - 5) = -1

- (2 - 3) = -1

2. Express the differential equation explicitly:

- X' = -1 X + (-1)

This simplifies to:

$$X' + X = -1$$

This is a first-order linear differential equation. However, the problem mentions "system of equations," which typically refers to multiple equations involving multiple variables. To generalize, we might consider an extension to a system of two differential equations involving variables X and Y, but based on the provided data, it appears to be a single equation.

Note: If the original problem intended a system involving multiple variables, additional information would be necessary. Given the current data, we proceed with solving the first-order linear differential equation using the D-operator elimination method.

Fundamentals of the D-Operator Method

What is the D-Operator?

The D-operator, also known as the differential operator, is a symbolic notation used to simplify the handling of derivatives. It is denoted by D such that:

- $D(X) = X'$
- $D^n(X)$ = the nth derivative of X

Using the D-operator, differential equations can be expressed algebraically, transforming differential equations into polynomial equations in D.

Example:

Given the differential equation:

$X' + aX = b$

Using D-operator notation:

$(D + a)X = b$

This polynomial operator can be manipulated algebraically to find the solution.

Advantages of Using D-Operator

- Simplifies the process of solving linear differential equations.
- Facilitates the elimination of variables in systems of equations.
- Enables straightforward application of algebraic techniques, such as factoring and polynomial division.

- Provides a systematic approach, especially useful for higher-order differential equations and systems.

Applying the D-Operator Elimination Method to the Given System

Step 1: Express the Differential Equation in D-Operator Form

From earlier, the simplified form of the differential equation is:

$X' + X = -1$

Expressed via the D-operator:

$(D + 1)X = -1$

This is a first-order linear differential equation, which can be approached directly or via the D-operator method.

Step 2: Identify the Homogeneous and Particular Solutions

The general solution to the differential equation is the sum of the homogeneous solution (related to the associated homogeneous equation) and a particular solution (specific to the nonhomogeneous part).

- Homogeneous equation:

$X' + X = 0$

- Particular solution:

$X_p = ?$

Step 3: Solve the Homogeneous Equation

In D-operator notation:

$(D + 1)X_h = 0$

The characteristic equation:

$D + 1 = 0$

Solution:

$X_h = C e^{-t}$
where C is an arbitrary constant.

Step 4: Find the Particular Solution

Since the RHS is a constant (-1), we look for a particular solution of the form:

$X_p = A$ (a constant)

Differentiating:

$X_p' = 0$

Substituting into the differential equation:

$0 + A = -1$

Thus:

$A = -1$

So, the particular solution:

$X_p = -1$

Step 5: Write the General Solution

Combining homogeneous and particular solutions:

$X(t) = C e^{-t} - 1$

This provides the complete solution to the differential equation.

Elimination of Variables in a System of Equations

While the problem appears to involve a single differential equation, the D-operator method shines when applied to systems with multiple variables, such as:

- $X(t)$

- $Y(t)$

Suppose we have a system:

$$
\begin{cases}
X' + a_{11}X + a_{12}Y = f_1(t) \\
Y' + a_{21}X + a_{22}Y = f_2(t)
\end{cases}
$$

Applying the D-operator, these can be expressed as:

$$
\begin{cases}
(D + a_{11})X + a_{12}Y = f_1(t) \\
a_{21}X + (D + a_{22})Y = f_2(t)
\end{cases}
$$

Eliminating one variable involves manipulating these equations algebraically, using the properties of the D-operator, to derive a single higher-order differential equation in one variable.

Steps in the elimination process:

1. Express one variable in terms of the other using one equation.
2. Substitute into the other equation to eliminate the variable.
3. Solve the resulting higher-order differential equation.

This approach streamlines the solution process, especially for complex systems.

Practical Implications and Applications

Engineering and Physics

In control systems, electrical circuits, and mechanical systems, differential equations describe dynamic behaviors. The D-operator method simplifies the process of analyzing systems, designing controllers, and predicting system responses.

Economics and Finance

Models involving differential equations, such as economic growth models or option pricing, benefit from systematic solution methods like the D-operator approach to analyze stability and long-term behavior.

Computational Tools

Modern software such as MATLAB, Mathematica, and Maple incorporate symbolic computation capabilities that implement D-operator techniques, enabling rapid solutions of complex systems.

Limitations and Considerations

While the D-operator elimination method is powerful, it has limitations:

- It primarily applies to linear differential equations and systems.
- For nonlinear systems, alternative methods are often necessary.
- The method requires familiarity with algebraic manipulation of operators and polynomial equations.
- The approach can become cumbersome with high-order systems or multiple variables without computational assistance.

Conclusion: The Significance of the D-Operator Method

The D-operator elimination method offers a structured and efficient way to solve linear differential systems, translating differential equations into algebraic forms that are more manageable. Its application to the given system demonstrates its utility in deriving explicit solutions, such as the exponential response in the simple case discussed. As systems become more complex, this method scales effectively, providing a robust framework for engineers, scientists, and mathematicians to analyze dynamic systems systematically.

Understanding and mastering the D-operator method enhances problem-solving capabilities, enabling practitioners to handle a wide array of differential systems with confidence and precision. Whether in theoretical research or practical applications, this technique remains a cornerstone of modern differential equations analysis.

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