(5 Points) Express 5.57575757576... As A Rational Number, In The Form Where P And Q Are Positive Integers

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Understanding how to express repeating decimals as rational numbers is a fundamental concept in mathematics, particularly in the study of rational and irrational numbers. This process not only helps in simplifying complex decimal expressions but also deepens our comprehension of the relationship between decimals and fractions. In this article, we will explore the step-by-step method to convert the repeating decimal 5.57575757576... into a rational number in the form where P and Q are positive integers, with P/Q representing the fraction.

Understanding Repeating Decimals

Before diving into the conversion process, it is essential to understand what repeating decimals are and how they relate to rational numbers.

What Are Repeating Decimals?

Repeating decimals are decimal numbers in which a certain sequence of digits repeats infinitely. For example, in the decimal 0.333..., the digit 3 repeats endlessly. Similarly, in 5.575757..., the sequence 75 repeats infinitely.

Rational Numbers and Repeating Decimals

It is a well-established mathematical fact that every repeating decimal can be expressed as a rational number — that is, as a ratio of two integers. Conversely, every rational number has a decimal expansion that either terminates or repeats periodically.

Step-by-Step Conversion of 5.57575757576... to a Fraction

Given the decimal 5.57575757576..., our goal is to express it as a fraction P/Q in the lowest terms, where P and Q are positive integers.

Step 1: Recognize the Decimal Structure

The decimal is:

5.57575757576...

Note that the repeating part is '75', and the non-repeating part is '5'. The digits after the decimal point are:

- Non-repeating part: 5
- Repeating part: 75

Since the decimal begins with 5, the integer part is 5, and the fractional part is the repeating decimal.

Step 2: Separate the Non-Repeating and Repeating Parts

Express the decimal as the sum of two parts:

- The non-repeating part: 5.5
- The repeating part: 0.07575757576...

Alternatively, to make calculations clearer, we can consider the entire decimal as:

X = 5.57575757576...

Now, focus on isolating the repeating decimal.

Step 3: Define the Variable and Create Equations

Let:

X = 5.57575757576...

Since the repeating sequence is '75', and it repeats after the decimal, we can manipulate the decimal to eliminate the repeating part.

To do this, multiply X by a power of 10 that moves the decimal point just after the repeating part.

The repeating sequence '75' is two digits, so:

- Multiply X by 100 to shift two digits to the left:

$$ 100X = 557.57575757576... $$

- Multiply X by 1 to keep the original:

$$ X = 5.57575757576... $$

Now, subtract the original from the multiplied:

$$ 100X - X = 557.57575757576... - 5.57575757576... $$

Calculations:

$$ 99X = 552 $$

Because the decimal parts cancel out due to the repeating nature:

$$ 99X = 552 $$

Step 4: Solve for X

Divide both sides by 99:

$$ X = \frac{552}{99} $$

Now, simplify the fraction:

- Find the greatest common divisor (GCD) of 552 and 99.

Prime factorization:

- 552 = 2^3 3 23
- 99 = 3^2 11

Common factors: 3

Divide numerator and denominator by 3:

$$ X = \frac{552 ÷ 3}{99 ÷ 3} = \frac{184}{33} $$

Therefore,

$$ X = \frac{184}{33} $$

Final Expression in Lowest Terms

Expressed as a fraction, the decimal 5.57575757576... equals:

$$
\boxed{\frac{184}{33}}
$$

This fraction is in simplest form since the numerator and denominator share no common divisors other than 1.

Conclusion: Rational Number Representation

The decimal 5.57575757576... can be precisely expressed as the rational number $\frac{184}{33}$. Here, both 184 and 33 are positive integers, satisfying the requirement for P and Q in the form $\frac{P}{Q}$. This conversion exemplifies the process of translating a repeating decimal into a fraction, reinforcing the foundational concept that all repeating decimals are rational numbers.

Additional Tips for Converting Repeating Decimals to Fractions

- Always identify the repeating part and the non-repeating part clearly.
- Use multiplication by powers of 10 to shift the decimal point beyond the repeating sequence.
- Subtract to eliminate the repeating component and solve for the original decimal.
- Simplify the resulting fraction to its lowest terms by dividing numerator and denominator by their GCD.

Summary of the Conversion Process

Step	Description	Mathematical Expression
1	Identify the decimal structure	5.575757...
2	Multiply to shift repeating part	100X = 557.57575...
3	Subtract equations	99X = 552
4	Solve for X	$X = \frac{552}{99} = \frac{184}{33}$
5	Simplify fraction	$\frac{184}{33}$

By following this systematic approach, converting any repeating decimal into a rational number becomes straightforward and reliable.

In conclusion, expressing 5.57575757576... as a rational number yields $\frac{184}{33}$, with P and Q being positive integers. This method underscores the importance of understanding decimal structures and algebraic manipulation in number theory and mathematics at large.

Frequently Asked Questions

How do we convert the repeating decimal 5.57575757576... into a rational number in the form P/Q?

To convert 5.575757... into a fraction, assign x = 5.575757..., then multiply by 100 to shift the repeating part, subtract to eliminate the repeating portion, and solve for x to find the ratio P/Q.

What is the first step in converting 5.575757... into a rational number?

The first step is to let x = 5.575757..., which represents the decimal as a variable for algebraic manipulation.

Why do we multiply by 100 when converting 5.575757... to a fraction?

Multiplying by 100 shifts the decimal two places to the right, aligning the

repeating parts so that subtracting the equations eliminates the repeating decimal, allowing us to solve for the rational form.

What is the value of P and Q when expressing 5.575757... as a simplified fraction?

The simplified fraction form of 5.575757... is (227/40), where P = 227 and Q = 40, both positive integers.

How do we simplify the fraction after subtracting to find P/Q for 5.575757...?

After subtracting the equations, you simplify the resulting fraction by dividing numerator and denominator by their greatest common divisor (GCD) to obtain the simplest form.

Can the decimal 5.575757... be expressed as a mixed number? If yes, what is it?

Yes, as a mixed number, 5.575757... is equivalent to 5 23/40.

Is 5.575757... a rational or irrational number, and why?

It is a rational number because it can be expressed as a fraction with positive integers P and Q, derived from its repeating decimal form.

What is the general method to convert any repeating decimal to a fraction?

Assign the decimal to a variable, multiply by a power of 10 to align repeating parts, subtract to eliminate the repeating portion, then solve for the variable and simplify the resulting fraction.

Additional Resources

Expressing 5.57575757576... as a Rational Number in the Form Where P and Q Are Positive Integers

Understanding how to convert a repeating decimal into a rational number is a fundamental skill in mathematics, especially in algebra and number theory. The decimal 5.57575757576... is a classic example of a repeating decimal, and expressing it as a fraction where the numerator (P) and denominator (Q) are positive integers is both a useful and intriguing process. This comprehensive guide delves into the conceptual framework, step-by-step methods, and nuanced considerations involved in converting such decimals into their fractional forms.

Introduction to Repeating Decimals and Rational Numbers

What Are Repeating Decimals?

Repeating decimals are decimal numbers where a sequence of digits repeats indefinitely. For example:

- 0.333... (where 3 repeats)
- 0.142857142857... (where 142857 repeats)
- 5.57575757576... (where 75 repeats)

These are distinguished from terminating decimals, which end after a finite number of digits. Repeating decimals are inherently rational because they can always be expressed as fractions of integers.

Rational Numbers and Their Connection to Repeating Decimals

A rational number is any number that can be expressed in the form $\frac{P}{Q}$, where P and Q are integers, with Q ≠ 0. The fundamental theorem of rational numbers states that:

- Every repeating decimal corresponds to a rational number.
- Every rational number can be represented as a repeating decimal.

Thus, converting a repeating decimal to a fraction is both a practical and theoretical exercise, reinforcing their interconnectedness.

Understanding the Specific Number: 5.57575757576...

Analyzing the Decimal

The decimal 5.57575757576... can be broken down into:

- A whole number part: 5
- A repeating decimal part: 0.57575757576...

The repeating sequence here is "75," which repeats indefinitely.

Significance of the Repeating Pattern

Recognizing the repeating pattern as "75" guides the conversion process. The

key steps involve isolating this repeating part and then expressing the entire number as a fraction.

Step-by-Step Method for Conversion

The process of converting a repeating decimal into a fraction generally involves the following steps:

1. Express the number as a sum of its integer and fractional parts.
2. Isolate the repeating decimal part.
3. Set the repeating decimal as a variable and manipulate algebraically to eliminate the repeating pattern.
4. Simplify the resulting fraction to lowest terms.

Let's apply this method meticulously to 5.57575757576...

Step 1: Separate the Whole Number and Decimal Part

Write the number as:

$$
N = 5 + 0.57575757576...
$$

The focus becomes converting $x = 0.57575757576...$ into a fraction.

Step 2: Assign the Repeating Decimal to a Variable

Let:

$$
x = 0.57575757576...
$$

Note: The repeating pattern is "75", which is two digits long.

Step 3: Multiply to Shift the Repeating Pattern

Since the repeating sequence length is 2, multiply x by $10^2 = 100$:

$$
100x = 57.57575757576...
$$

Now, observe:

$$
100x = 57.57575757576...
$$

and

$$
x = 0.57575757576...
$$

Subtract the original x from this to eliminate the repeating tail:

$$
100x - x = 57.57575757576... - 0.57575757576...
$$

$$
99x = 57
$$

Step 4: Solve for x and Simplify

Divide both sides by 99:

$$
x = \frac{57}{99}
$$

Simplify the fraction:

- Both numerator and denominator are divisible by 3:

$$
\frac{57 \div 3}{99 \div 3} = \frac{19}{33}
$$

Thus,

$$
x = \frac{19}{33}
$$

Recall, the original number is:

$$
N = 5 + x = 5 + \frac{19}{33}
$$

Expressed as an improper fraction:

$$
N = \frac{5 \times 33}{33} + \frac{19}{33} = \frac{165 + 19}{33} = \frac{184}{33}
$$

\]

This fraction is in its lowest terms, as 184 and 33 are coprime (their greatest common divisor is 1).

Final Expression in the Form \(\frac{P}{Q} \)

Therefore, the decimal 5.57575757576... can be expressed as:

\[
\boxed{\frac{184}{33}}
\]

where:

- \(P = 184\) (a positive integer),
- \(Q = 33\) (a positive integer).

Additional Insights and Nuances

Why Does This Method Work?

The process relies on algebraic manipulation:

- Multiplying by a power of 10 shifts the decimal to align the repeating parts.
- Subtracting cancels the repeating portion, leaving a simple linear equation.
- Solving for the decimal variable yields the fraction representation.

This technique is universally applicable for any purely repeating decimal, regardless of whether it has a whole number part or not.

Handling Different Types of Repeating Decimals

- Pure Repeating Decimals (no whole number part): For example, 0.727272..., the same process applies, but the initial step involves setting the decimal as a variable.
- Mixed Repeating Decimals (with whole number part): As demonstrated in this case, separate the whole number and fractional parts before conversion.
- Terminating Decimals: These can be viewed as repeating decimals with a repeating pattern of zeros, e.g., 0.5 = 0.5000..., which can be converted similarly.

Reducing Fractions to Lowest Terms

Always check if the resulting fraction can be simplified further:

- Use the Euclidean Algorithm to find the greatest common divisor (GCD).
- Divide numerator and denominator by the GCD to get the simplest form.

In this case, GCD(184, 33) = 1, so $\frac{184}{33}$ is already simplified.

Practical Applications and Importance

The ability to convert repeating decimals to fractions is more than an academic exercise; it has practical implications:

- Mathematical proofs and derivations: Understanding the nature of rational numbers.
- Computational mathematics: Simplifying decimal representations for algorithms.
- Financial calculations: Ensuring precision in currency and interest calculations.
- Teaching and learning: Developing number sense and algebraic manipulation skills.

Conclusion and Summary

Converting 5.57575757576... into a fraction with positive integers P and Q involves recognizing the repeating pattern, algebraically manipulating the decimal to eliminate the repeating digits, and simplifying the resulting fraction. The key steps are:

1. Separate the whole number part.
2. Assign the repeating decimal to a variable.
3. Multiply to shift the decimal point to align the repeating patterns.
4. Subtract to eliminate the repeating component.
5. Solve for the variable and simplify.

Following this method, we've demonstrated that:

$$
5.57575757576... = \frac{184}{33}
$$

This example underscores the power of algebraic techniques in understanding the properties of rational numbers and their decimal representations.

In summary, the decimal 5.57575757576... as a rational number where P and Q are positive integers is $\boxed{\frac{184}{33}}$. This process highlights the elegance of converting repeating decimals into fractions and deepens

understanding of the intrinsic connection between decimals and rational numbers.

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