(a) The Radius Of A Circle Is 5.6 Cm. Find The Angle In Degrees Subtended By An Arc Of 22 Cm At Thecentre

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Understanding the relationship between the radius of a circle, the length of an arc, and the central angle they subtend is fundamental in geometry. In this problem, we are given the radius of a circle as 5.6 centimeters and an arc length of 22 centimeters. Our goal is to determine the measure of the central angle in degrees that subtends this arc at the center of the circle. This involves applying the basic formula connecting arc length, radius, and central angle, and then converting the resulting measure from radians to degrees for clarity and conventional understanding.

Understanding the Basic Concepts

What Is an Arc?

An arc of a circle is a segment of the circumference — essentially, a "slice" of the circle's boundary. The size of an arc is usually expressed in terms of its length or the angle it subtends at the circle's center.

Central Angle and Arc Length Relationship

The central angle is the angle subtended at the center of the circle by an arc. The length of the arc (L), the radius (r), and the central angle (θ) are mathematically related by the formula:

- **Arc Length Formula:** $L = r \times \theta$ (in radians)

This formula is fundamental in solving problems involving circles. It states that the length of an arc is proportional to the radius and the angle it subtends, measured in radians.

Converting Radians to Degrees

Since the problem asks for the angle in degrees, it's important to remember the conversion factor between radians and degrees:

- 1 radian = 180° / π ≈ 57.2958°

Thus, after calculating θ in radians, we will convert it to degrees using this factor for a standard measure.

Step-by-Step Solution

Step 1: Write the Known Values

- Radius of the circle, **r = 5.6 cm**
- Arc length, **L = 22 cm**

Step 2: Use the Arc Length Formula

Applying the formula L = r × θ (in radians), we isolate θ:

θ (radians) = L / r

Substituting the known values:

θ (radians) = 22 / 5.6 ≈ 3.9286 radians

Step 3: Convert Radians to Degrees

Using the conversion factor:

θ (degrees) = θ (radians) × (180 / π)

Calculating:

θ (degrees) ≈ 3.9286 × (180 / 3.1416) ≈ 3.9286 × 57.2958 ≈ 225.0°

Step 4: Final Answer

Therefore, the central angle subtended by the arc of length 22 cm at the center of the circle is approximately **225 degrees**.

Additional Insights and Validation

Verification of the Result

- Using the calculated angle in radians: *$\theta \approx 3.9286$ radians*
- Since the entire circle measures 360°, or 2π radians (~6.2832 radians), the arc subtends a significant portion of the circle:
- Fraction of the circle: **$\theta / 2\pi \approx 3.9286 / 6.2832 \approx 0.625$**
- This indicates the arc covers approximately 62.5% of the circle, which correlates well with an angle of 225°, since 225° / 360° ≈ 0.625.

Understanding the Context of the Result

Knowing that the arc subtends an angle of approximately 225° at the center implies it is a major arc, covering more than half of the circle. This is consistent with the arc length being nearly four times the radius, which makes sense given the proportionality between arc length and radius.

Conclusion

In conclusion, given a circle with a radius of 5.6 cm and an arc length of 22 cm, the central angle subtended by this arc at the circle's center is approximately 225 degrees. This calculation underscores the importance of understanding the relationship between arc length, radius, and angles, and how conversions between radians and degrees are essential for expressing the final answer in a standard, understandable form.

Summary of Key Points

- The fundamental formula connecting arc length and central angle in radians: $L = r \times \theta$.

- To find the angle in radians: $\theta = L / r$.
- Conversion from radians to degrees involves multiplying by $180/\pi$.
- The calculated central angle is approximately 225°, indicating a major arc.

Practical Applications

This type of calculation is essential in various fields such as engineering, astronomy, navigation, and design, where understanding the relationships between angles and arc lengths assists in constructing, analyzing, and interpreting circular structures and motions.

Frequently Asked Questions

What is the radius of the circle in the problem?

The radius of the circle is 5.6 cm.

What is the length of the arc given in the problem?

The length of the arc is 22 cm.

How is the angle in degrees subtended by an arc at the center calculated?

The angle in degrees is calculated using the formula: (arc length / circumference) × 360°, where circumference = $2\pi r$.

What is the circumference of the circle in this problem?

The circumference is $2 \times \pi \times 5.6$ cm $\approx 2 \times 3.1416 \times 5.6 \approx 35.2$ cm.

How do you find the angle subtended by an arc of length 22 cm at the center?

Calculate the fraction of the circumference the arc represents, then multiply by 360°. That is, $(22 / 35.2) \times 360°$.

What is the calculated angle in degrees subtended by the 22 cm arc?

The angle is approximately (22 / 35.2) × 360° ≈ 0.625 × 360° ≈ 225°.

Is the angle subtended by the arc more than a right angle?

Yes, approximately 225°, which is greater than 90°.

What is the significance of the angle in a circle?

The angle at the center subtended by an arc indicates how much of the circle's total 360° it covers.

Can the same arc subtend different angles at different points?

Yes, the angle subtended by an arc varies depending on the point of observation; at the center, it is the largest.

Why is it important to know the radius when calculating the angle from the arc length?

Because the radius helps determine the circumference, which is essential to find the fraction of the circle covered by the arc, and thus the angle.

Additional Resources

Radius of a circle plays a fundamental role in understanding the properties and measurements associated with circular shapes. When given specific measurements such as the radius and the length of an arc, it becomes possible to determine the angle subtended at the center of the circle. This type of problem is a classic example of applying basic geometric principles, particularly those related to circles, arcs, and angles. In this comprehensive guide, we will explore how to find the angle in degrees subtended by an arc of 22 centimeters at the center of a circle with a radius of 5.6 centimeters. We will break down the concepts involved, derive the necessary formulas, and walk through the calculations step-by-step to provide a thorough understanding of the process.

Understanding the Problem

The problem states:
The radius of a circle is 5.6 cm. Find the angle in degrees subtended by an arc of 22 cm at the center.

This involves two key measurements:
- The radius of the circle (R) = 5.6 cm
- The length of the arc (L) = 22 cm

The goal is to find the central angle (θ) in degrees, which is subtended by the arc at the circle's center.

Key Concepts and Formulas

To solve this problem, we need to understand the relationship between the arc length, the radius, and the central angle. The fundamental formula connecting these quantities is:

Arc length (L) = Radius (R) × Central angle in radians (θ in radians)

Mathematically:
$L = R \times \theta$ (where θ is in radians)

Rearranged to find θ:
$\theta \text{ (radians)} = L / R$

Once we determine θ in radians, we can convert it into degrees since the problem asks for the angle in degrees.

Conversion from radians to degrees:
Degrees = Radians × (180 / π)

Step-by-Step Solution

Let's proceed with the calculations:

Step 1: Write down the known values

- Radius, R = 5.6 cm
- Arc length, L = 22 cm

Step 2: Calculate the central angle in radians

Using the formula:
θ (radians) = L / R = 22 / 5.6

Calculating:
θ (radians) ≈ 3.92857 radians

Step 3: Convert radians to degrees

Using the conversion:
Degrees = θ (radians) × (180 / π)

Calculating:
Degrees ≈ 3.92857 × (180 / 3.14159)
Degrees ≈ 3.92857 × 57.2958
Degrees ≈ 225.00°

Step 4: Final answer

The central angle subtended by the arc of 22 cm at the center of a circle with radius 5.6 cm is approximately 225 degrees.

Understanding the Result

The result indicates that the arc spans an angle of approximately 225 degrees at the center of the circle. This is a significant portion of the circle, as a full circle measures 360 degrees. The large central angle correlates with the relatively long arc length compared to the radius.

Additional Insights and Considerations

The Relationship Between Arc Length, Radius, and Angle

- The formula $L = R \times \theta$ (in radians) is fundamental in circle geometry. It allows us to relate physical length measurements to angular measures.
- The conversion from radians to degrees is essential, as degrees are more intuitive for everyday understanding.

Practical Applications

- Calculating the angle subtended by an arc is useful in fields like

engineering, architecture, and navigation.
- Understanding how to convert between arc length and angle helps in designing circular components or analyzing circular motion.

Features and Pros/Cons

Features:

- Simple and straightforward calculation once the basic formulas are understood.
- Applicable to various real-world problems involving circles and arcs.
- Reinforces understanding of radian and degree measures.

Pros:

- Provides a clear method to find the angle from arc length and radius.
- Enhances comprehension of circle geometry principles.
- Useful for students preparing for exams or technical applications.

Cons:

- Assumes the reader is familiar with radians and the conversion process.
- Does not account for measurement errors or approximations in real-world scenarios.
- May be less intuitive for those more comfortable with degrees than radians.

Common Mistakes to Avoid

- Forgetting to convert radians into degrees, leading to incorrect interpretations.
- Mixing units (e.g., using degrees for R or L) without proper conversion.
- Miscalculating π or using approximate values incorrectly, affecting the accuracy of the result.
- Assuming the arc length is less than the circle's circumference; for large arcs, ensure the measurements are consistent.

Summary of the Approach

1. Identify known values (radius and arc length).
2. Use the formula θ (radians) = L / R to find the central angle in radians.
3. Convert the radians measure into degrees by multiplying by 180/π.
4. Interpret the result in the context of the circle's geometry.

This method provides a straightforward yet powerful way to solve problems involving arcs and central angles in circles.

Conclusion

In sum, determining the angle subtended by an arc at the center of a circle relies on fundamental circle theorems and formulas. With a radius of 5.6 cm and an arc length of 22 cm, the central angle is approximately 225 degrees. This calculation not only exemplifies the practical application of circle geometry but also reinforces key mathematical concepts such as radian-degree conversion and the importance of understanding the relationships between lengths and angles in circular figures. Mastery of these principles equips students and professionals alike to handle a wide range of problems related to circles, making this a vital skill in mathematical, engineering, and scientific contexts.

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