(b) A Free Electron Has A Wavefunction (x)=Acos[(10^10)xt], Where X Is In Meters. Find (i) The Electron's

(b) A Free Electron Has A Wavefunction (x)=Acos[(10^10)xt], Where x Is In Meters. Find (i) The Electron's

Understanding the wavefunction of a free electron is fundamental to quantum mechanics, as it encapsulates the probabilistic nature of electrons and their wave-like behavior. In this article, we will analyze the given wavefunction, determine key properties such as momentum, energy, and wavelength, and explore the significance of these quantities in the context of quantum physics. The wavefunction provided, $\psi(x,t) = A \cos[(10^{10}) x t]$, offers a fascinating glimpse into the wave behavior of electrons and serves as an excellent example to illustrate core concepts.

Introduction to Wavefunctions in Quantum Mechanics

Before delving into the specifics of the problem, it's essential to understand what a wavefunction represents in quantum mechanics.

Definition of a Wavefunction

- The wavefunction, typically denoted as $\psi(x, t)$, contains all the information about a quantum particle.
- The probability density of finding a particle at position x at time t is given by $|\psi(x,t)|^2$.
- For a free electron, the wavefunction propagates without external potential influences, exhibiting wave-like properties such as interference and diffraction.

Mathematical Form of the Wavefunction

- The form $\psi(x,t) = A \cos(k x - \omega t)$ is common for plane waves, where k is the wave number and ω is the angular frequency.
- The given wavefunction is $\psi(x,t) = A \cos[(10^{10}) x t]$, which suggests a wave-like behavior with specific parameters to be determined.

Analyzing the Given Wavefunction

Let's examine the wavefunction:

$$
\psi(x,t) = A \cos[(10^{10}) x t]
$$

This form indicates a spatial and temporal dependence intertwined through the argument of the cosine function. To analyze the physical properties, we need to interpret the parameters correctly.

Identifying the Wave Parameters

- The argument of the cosine, $(10^{10}) x t$, suggests a combined wave number and angular frequency.
- Typically, a plane wave is expressed as $\psi(x,t) = A \cos(k x - \omega t)$, but here, the dependence is $\cos(k x t)$.

This form is unusual because it involves a product $x t$, which is non-standard. However, for the purpose of analysis, we can interpret this as indicating a phase that depends on $x t$, implying a wave with properties that vary in a specific manner.

Deriving Physical Quantities

To find key properties such as the momentum and energy of the electron, we need to relate the wavefunction to standard quantum mechanical expressions.

Momentum of the Electron

- In quantum mechanics, the momentum operator in the position representation is:

$$
\hat{p} = -i \hbar \frac{\partial}{\partial x}
$$

- To find the expectation value of momentum, $\langle p \rangle$, we typically calculate:

$$
\langle p \rangle = \int \psi^(x,t) \hat{p} \psi(x,t) dx
$$

Given the form of ψ, it's more straightforward to analyze the wavevector k.

Note: The given wavefunction's argument involves $x t$. To interpret this in standard form, consider the phase:

$$
\phi(x,t) = (10^{10}) x t
$$

which suggests a phase velocity or a wave vector that depends on t. Alternatively, we can attempt to rewrite or approximate the wavefunction.

Approximate or Reinterpret the Wavefunction

- If we treat the wavefunction as a standing wave or a wave with a time-dependent wave number, we might need to consider a different approach.
- However, for simplicity, assuming the wavefunction represents a traveling wave in some form, the wave number k and angular frequency ω can be identified from the coefficients.

Given the standard form of a plane wave:

$$
\psi(x,t) = A \cos(k x - \omega t)
$$

and comparing it to:

$$
\psi(x,t) = A \cos[(10^{10}) x t]
$$

we see the phase depends on $x t$, which is non-standard. But if we consider the phase as:

$$
\phi = (10^{10}) x t
$$

then the phase velocity v_p is:

$$
v_p = \frac{\omega}{k}
$$

where, assuming the phase velocity relates to the coefficients, we can identify:

$$
k = (10^{10}) t
$$

$$
\omega = (10^{10}) x
$$

which implies both k and ω depend on t and x, respectively. This suggests a more complex behavior, possibly a wave packet or a non-traveling wave.

Standard Approach: Assuming a Plane Wave Form

Given the standard approach to such problems, we assume the wavefunction can be approximated as:

$$
\psi(x,t) = A \cos(k x - \omega t)
$$

with

$$
k = 10^{10} \text{ m}^{-1}
$$
$$
\omega = 10^{10} \text{ s}^{-1}
$$

This is a common assumption for plane waves with well-defined wave number and frequency.

Calculating the Electron's Momentum

- The momentum p relates to the wave number k:

$$
p = \hbar k
$$

where $\hbar$ is the reduced Planck's constant:

$$
\hbar \approx 1.055 \times 10^{-34} \text{ Js}
$$

Therefore,

$$
p = 1.055 \times 10^{-34} \times 10^{10} = 1.055 \times 10^{-24} \text{ kg·m/s}
$$

This is the magnitude of the electron's momentum associated with the wavefunction.

Calculating the Electron's Energy

- The total energy E of a free electron with momentum p is given by the relativistic or non-relativistic relation.

Non-Relativistic Approximation:

$$
E = \frac{p^2}{2m}
$$

where m is the electron mass:

$$
m \approx 9.11 \times 10^{-31} \text{ kg}
$$

Plugging in the values:

$$
E = \frac{(1.055 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} \approx \frac{1.113 \times 10^{-48}}{1.822 \times 10^{-30}} \approx 6.11 \times 10^{-19} \text{ J}
$$

Expressed in electronvolts (eV), using $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$:

$$
E \approx \frac{6.11 \times 10^{-19}}{1.602 \times 10^{-19}} \approx 3.81 \text{ eV}
$$

Thus, the electron's kinetic energy is approximately 3.81 eV.

Calculating the Wavelength

- The de Broglie wavelength λ is related to the wave number k:

$$
\lambda = \frac{2 \pi}{k}
$$

\]

Given $k = 10^{10} \text{ m}^{-1}$:

$$
\lambda = \frac{2 \pi}{10^{10}} \approx \frac{6.2832}{10^{10}} \approx 6.2832 \times 10^{-10} \text{ meters}
$$

This wavelength (~0.63 nanometers) is typical of high-energy electrons.

Summary of Results

Quantity	Value	Explanation
Electron's momentum (p)	$1.055 \times 10^{-24} \text{ kg·m/s}$	Derived from $p = \hbar k$
Electron's energy (E)	approximately 3.81 eV	Non-relativistic kinetic energy
Wavelength (λ)	approximately 0.63 nm	de Broglie wavelength associated with p

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Significance of These Quantities in Quantum Mechanics

Understanding the momentum, energy, and wavelength of an electron described by a wavefunction like \(

Frequently Asked Questions

What is the form of the wavefunction given for the free electron?

The wavefunction is given as Ψ(x) = A cos[(10^10) x t], where A is the amplitude, x is position in meters, and t is time.

What does the wavefunction Ψ(x) = A cos[(10^10) x t] represent for a free electron?

It represents a standing wave pattern of the electron's probability amplitude, indicating how the electron's likelihood of being found varies in space and time.

How can we determine the momentum of the electron from its wavefunction?

By analyzing the wavefunction's spatial and temporal dependence, particularly the argument of the cosine function, we can extract the wavevector and hence the momentum using p = ħk, where k is the wavevector.

What is the significance of the wavefunction's form involving a cosine function with a large coefficient (10^10)?

A large coefficient indicates a high frequency or wave number, implying the electron possesses a high momentum component associated with short-wavelength oscillations.

How do we find the electron's momentum from the given wavefunction?

Since the wavefunction is of the form Ψ(x, t) = A cos(kx - ωt), the wavevector k = 10^10 m^-1, and the momentum p = ħk, which calculates to approximately 1.055 x 10^-24 kg·m/s.

What is the physical interpretation of the wavefunction's cosine form for a free electron?

The cosine form indicates a plane wave component of the electron, representing a particle with a definite momentum and a delocalized position in space.

Are there any implications for the electron's energy based on this wavefunction?

Yes, for a free particle described by a plane wave, the energy is related to the momentum via the kinetic energy formula E = p^2 / 2m, implying the electron has a well-defined momentum and corresponding energy.

Additional Resources

A Free Electron Has A Wavefunction (x) = A cos[(10^{10}) x t], Where x is in meters. Find (i) The Electron's

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Investigating the Wavefunction of a Free Electron: A Deep Dive into Quantum Dynamics

Quantum mechanics, with its complex and often non-intuitive principles, continues to be a fertile ground for exploration and discovery. One foundational concept is the wavefunction, which encapsulates the probabilistic nature of particles at the microscopic level. In this article, we undertake a detailed analysis of a particular wavefunction describing a free electron, specifically:

> $(x) = A \cos[(10^{10}) x t]$

where x is in meters, and A is the amplitude. Our goal is to interpret this wavefunction, understand its physical implications, and compute relevant properties of the electron, such as its momentum, energy, and probability distributions.

Understanding the Form of the Wavefunction

The Given Wavefunction and Its Context

The wavefunction is provided as:

$$
\psi(x, t) = A \cos[(10^{10}) x t]
$$

At first glance, this form differs from the more common plane wave solutions $\psi(x, t) = Ae^{i(kx - \omega t)}$. Instead, it employs a cosine function with an argument proportional to the product xt. This suggests a real-valued, standing-wave-like structure, or perhaps a specific superposition, rather than a simple traveling wave.

Physical Interpretation

The cosine dependence on the product xt indicates a wave with a spatial and temporal interplay that isn't separable into the usual form:

$$
\psi(x, t) \neq \phi(x) \cdot \chi(t)
$$

unless further context or assumptions are made. However, the problem appears to treat the wavefunction as a fundamental description of a free electron's state at a given time.

To proceed, we analyze the implications of the form:

$$
\psi(x, t) = A \cos[\kappa x t]
$$

where:

$$
\kappa = 10^{10} \text{ m}^{-1} \text{s}^{-1}
$$

Physical Significance and Mathematical Structure

The Argument of the Cosine

The argument $\kappa x t$ involves both position and time, which is unusual compared to typical plane waves where the dependence is linear in either x or t, not their product.

This suggests that the wavefunction describes a wave with a phase velocity or group velocity that depends on the combination of space and time.

Real-Valued Wavefunctions and Superposition

Since $\psi(x, t)$ is real, it likely represents a superposition of plane waves:

$$
\cos(\theta) = \frac{1}{2} \left(e^{i\theta} + e^{-i\theta}\right)
$$

which can be interpreted as a superposition of traveling waves with phases $\pm \kappa x t$.

Deriving Physical Quantities

1. Probability Density

The probability density $\rho(x, t)$ is:

$$
\rho(x, t) = |\psi(x, t)|^2
$$

Since $\psi(x, t) = A \cos[\kappa x t]$:

$$
\rho(x, t) = A^2 \cos^2[\kappa x t]
$$

This oscillates between 0 and A^2, indicating a standing wave pattern, with nodes and antinodes determined by the cosine squared function.

---

2. Expectation Values of Position and Momentum

Expectation Value of Position $\langle x \rangle$

$$
\langle x \rangle = \int_{-\infty}^{\infty} x |\psi(x, t)|^2 dx
$$

Given the wavefunction's form, the probability density oscillates rapidly, but without explicit bounds or normalization, the calculation remains formal.

Expectation Value of Momentum $\langle p \rangle$

The momentum operator in quantum mechanics is:

$$
\hat{p} = -i \hbar \frac{\partial}{\partial x}
$$

Applying it to $\psi(x, t)$:

$$
\frac{\partial \psi}{\partial x} = -A \kappa t \sin[\kappa x t]
$$

Thus:

$$
\hat{p} \psi = -i \hbar \left(-A \kappa t \sin[\kappa x t]\right) = i \hbar A \kappa t \sin[\kappa x t]
$$

The expectation value:

$$
\langle p \rangle = \int_{-\infty}^{\infty} \psi^(x, t) \hat{p} \psi(x, t) dx
$$

Given the symmetry of sine and cosine functions, and the real nature of ψ, the expectation value of momentum for such a state generally evaluates to zero, especially if the wavefunction is symmetric.

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3. Energy Considerations

The total energy, expressed in terms of the expectation value of the Hamiltonian $\hat{H}$, involves the kinetic energy operator:

$$
\hat{H} = \frac{\hat{p}^2}{2m}
$$

Calculating $\langle H \rangle$:

$$
\langle H \rangle = \frac{1}{2m} \langle p^2 \rangle
$$

which requires computing:

$$
\langle p^2 \rangle = \int |\frac{\partial \psi}{\partial x}|^2 dx
$$

From earlier:

$$
\frac{\partial \psi}{\partial x} = -A \kappa t \sin[\kappa x t]
$$

Thus:

$$
|\frac{\partial \psi}{\partial x}|^2 = A^2 \kappa^2 t^2 \sin^2[\kappa x t]
$$

Integrating over all space (assuming proper normalization), yields an energy proportional to $A^2 \kappa^2 t^2$. This indicates that the energy expectation value grows with time, which is physically problematic for a free electron unless the wavefunction is carefully normalized and bounded.

Normalization and Physical Realism

Challenges with the Given Wavefunction

The form $\psi(x, t) = A \cos[\kappa x t]$ suggests an oscillatory pattern that extends over all space, with no explicit bounds. For a physically meaningful state, the wavefunction must be normalizable:

$$
\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx < \infty
$$

Since $\cos^2[\kappa x t]$ does not decay at infinity, normalization requires specific considerations, such as employing wave packets or considering finite regions.

Possible Interpretations

- Superposition of plane waves: The cosine form indicates a superposition, possibly representing a localized wave packet in the limit of certain parameters.
- Approximate or idealized state: The wavefunction may serve as a mathematical model to understand certain propagation characteristics rather than a physically realistic state.

Physical Implications and Further Analysis

Momentum Distribution

Given the wavefunction's structure, the momentum distribution $\phi(p)$ (via Fourier transform) would reveal the spread of momentum values, likely centered around zero with a certain width related to the inverse of the spatial extent.

Group Velocity and Phase Velocity

The argument's dependence on xt complicates the interpretation of phase and group velocities:

- Phase velocity $v_p = \frac{\omega}{k}$: Not directly obtainable from the given form without assumptions.
- Group velocity $v_g = \frac{d\omega}{dk}$: Also not straightforward here, but further Fourier analysis could clarify.

Conclusions and Key Takeaways

- The wavefunction $\psi(x, t) = A \cos[\kappa x t]$ describes a real-valued wave with coupled spatial and temporal oscillations.
- Its probability density oscillates in space and time, resembling a standing wave pattern.
- Calculations of expectation values suggest zero average momentum, but the time-dependent kinetic energy indicates non-stationary behavior.
- Normalization and physical realism require careful treatment, possibly involving wave packets or bounded regions.
- The form challenges typical interpretations of free electron states, highlighting the importance of context and boundary conditions in quantum mechanics.

Final Remarks

This analysis underscores how particular wavefunction forms can encode rich physical phenomena, but also how their interpretation hinges on normalization, boundary conditions, and the physical setup. The given wavefunction, while mathematically intriguing, exemplifies the complex relationship between mathematical forms and physical reality in quantum systems.

Further research could involve:

- Fourier transforming \(\psi(x, t)\) to analyze momentum distribution.
- Constructing localized wave packets from superpositions.
- Exploring the time evolution and stability of such states.

Understanding these aspects deepens our grasp of quantum dynamics and the nuanced behavior of free particles—a cornerstone in the edifice of quantum physics.

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