The Cadmium Isotope 109Cd Has A Half-life Of 462 Days. A Sample Begins With 1.01012109Cd Atoms. How Many

The Cadmium Isotope 109Cd Has A Half-life Of 462 Days. A Sample Begins With 1.01012109Cd Atoms. How Many atoms remain after a certain period? Understanding radioactive decay involves grasping concepts such as half-life, decay constants, and the exponential decay law. This article delves into the principles behind radioactive decay, specifically focusing on Cadmium-109 (109Cd), and provides a comprehensive guide to calculating the remaining number of atoms over time. Whether you're a student preparing for exams, a researcher analyzing radioactive samples, or just curious about nuclear physics, this exploration will clarify the process step-by-step.

Understanding Radioactive Decay and Half-life

What Is Radioactive Decay?

Radioactive decay is a spontaneous process by which unstable atomic nuclei lose energy by emitting radiation in the form of alpha particles, beta particles, or gamma rays. This process transforms one element or isotope into another, often more stable, element. The decay is random at the level of individual atoms but follows a predictable statistical pattern across large populations of atoms.

The Concept of Half-life

The half-life of a radioactive isotope is the time required for half of the initial number of radioactive atoms to decay. It is a characteristic property of each isotope and remains constant regardless of the sample size.

For Cadmium-109, the half-life is given as 462 days. This means that after 462 days, only half of the original atoms will remain undecayed, and after another 462 days, only a quarter will remain, and so on.

Mathematical Foundations of Radioactive Decay

The Exponential Decay Law

The decay of radioactive atoms follows an exponential law described by the formula:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Where:
- $N(t)$ is the number of atoms remaining after time t,
- N_0 is the initial number of atoms,
- λ is the decay constant,
- t is the elapsed time.

Relating Half-life to Decay Constant

The decay constant λ relates to the half-life $T_{1/2}$ as:

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

Using this formula, for Cadmium-109:

$$ \lambda = \frac{\ln 2}{462\, \text{days}} \approx \frac{0.6931}{462} \approx 0.001501\, \text{per day} $$

This decay constant represents the probability per unit time that a single atom will decay.

Calculating Remaining Atoms Over Time

Suppose a sample begins with $N_0 = 1.01012109 \times 10^{9}$ atoms of 109Cd. To determine how many atoms remain after a given period, say t days, we use the decay law:

$$ N(t) = N_0 \times e^{-\lambda t} $$

Alternatively, since we know the half-life, we can use a more straightforward approach:

$$ N(t) = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} $$

This formula simplifies calculations because it directly relates the remaining atoms to the number of half-lives elapsed.

Step-by-Step Calculation Example

Let's suppose we want to find the number of atoms remaining after, say, 1000 days.

1. Compute the number of half-lives elapsed:

$$ n = \frac{t}{T_{1/2}} = \frac{1000}{462} \approx 2.165 $$

2. Calculate the remaining atoms:

$$ N(t) = N_0 \times \left(\frac{1}{2}\right)^{n} $$

$$ N(t) = 1.01012109 \times 10^{9} \times \left(\frac{1}{2}\right)^{2.165} $$

3. Evaluate:

$$ \left(\frac{1}{2}\right)^{2.165} = e^{2.165 \times \ln(1/2)} = e^{-2.165 \times 0.6931} \approx e^{-1.5} \approx 0.2231 $$

4. Final calculation:

$$ N(t) \approx 1.01012109 \times 10^{9} \times 0.2231 \approx 2.254 \times 10^{8} $$

Therefore, after 1000 days, approximately 225 million atoms of 109Cd remain from the original sample.

Practical Applications of Radioactive Decay Calculations

Radioactive Dating and Age Determination

Radioactive decay calculations are fundamental in geochronology, helping scientists estimate the age of rocks and fossils. By measuring the remaining isotope and knowing its half-life, researchers can determine how long decay has been occurring.

Medical and Industrial Uses

Radioisotopes like 109Cd are used in medical imaging and industrial applications. Precise knowledge of their decay rates ensures safety and efficacy in their use.

Nuclear Waste Management

Understanding the decay patterns of radioactive waste helps in designing safe storage and disposal strategies, minimizing environmental impact.

Additional Considerations in Decay Calculations

Decay Chain Complexity

Some radioactive isotopes decay into other radioactive isotopes, creating decay chains. Calculating the quantities involved requires solving coupled differential equations, often involving the Bateman equations.

Decay Law Limitations

The exponential decay law assumes a closed system with no additional sources or sinks of the isotope. Real-world scenarios may involve leaching, contamination, or other factors affecting decay calculations.

Uncertainties and Measurement Accuracy

Experimental measurements of half-lives and initial atom counts come with uncertainties. Proper error analysis ensures reliable results.

Summary

- The half-life of 109Cd is 462 days.
- The decay follows an exponential law, with the decay constant $\lambda \approx 0.001501$ per day.
- To find the remaining number of atoms after a given period, use the formula:

$$N(t) = N_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

- This calculation is essential in various scientific fields, from dating ancient artifacts to medical applications.

Final Thoughts

Understanding the decay of radioactive isotopes like 109Cd is crucial in many scientific and practical contexts. By mastering the mathematical tools and concepts outlined above, you can accurately determine how long a sample will retain its radioactivity or how much remains after a specific period. Whether for academic purposes, research, or industry, these calculations form the backbone of nuclear science and technology.

Remember: Always consider the initial number of atoms, the isotope's half-life, and the elapsed time when performing decay calculations. With this knowledge, you can confidently analyze radioactive samples and interpret their decay behavior.

Frequently Asked Questions

What is the half-life of the cadmium isotope 109Cd?

The half-life of 109Cd is 462 days.

How many atoms of 109Cd are present in the sample initially?

The sample initially contains 1.01012109 × 10^9 atoms of 109Cd.

How do you calculate the remaining number of 109Cd atoms after a certain time?

Use the decay formula $N = N_0 \times (1/2)^{t / t_{1/2}}$, where N_0 is the initial number of atoms, t is elapsed time, and $t_{1/2}$ is the half-life.

If 100 days pass, approximately how many 109Cd atoms remain?

Approximately 6.75 × 10^8 atoms remain after 100 days, using the decay formula.

Why does the number of 109Cd atoms decrease over time?

Because 109Cd undergoes radioactive decay, transforming into other elements over its half-life period.

How long does it take for half of the initial 109Cd atoms to decay?

It takes approximately 462 days, which is the half-life of 109Cd.

What is the decay constant (λ) for 109Cd?

The decay constant λ is calculated as $\lambda = \ln(2) / t_{1/2} \approx 0.0015$ per day.

How would you determine the remaining atoms after 924 days?

Since 924 days is two half-lives, the remaining atoms would be approximately 25% of the initial, about 2.525 × 10^8 atoms.

What is the significance of knowing the half-life of a radioactive isotope like 109Cd?

It helps in predicting the decay rate, estimating the age of samples, and managing its use in applications like medical imaging or industrial radiography.

Can the initial number of atoms be used to estimate the activity of the sample?

Yes, using the decay constant and the initial number of atoms, the activity (decay rate) can be calculated as $A = \lambda N_0$.

Additional Resources

Understanding the Decay of Cadmium Isotope 109Cd and Its Radioactive Dynamics

Radioactive isotopes have fascinated scientists for centuries, providing essential insights into nuclear physics, medical applications, environmental science, and more. Among these, Cadmium isotope 109Cd is a notable example due to its specific half-life and applications. In this comprehensive review, we will explore the properties of 109Cd, its decay characteristics, the mathematical framework to understand its decay process, and practical implications of its half-life. We'll also analyze the problem statement: A sample begins with 1.01012109Cd atoms; how many remain after a given period?

Introduction to Cadmium Isotopes and 109Cd

Cadmium (Cd) is a transition metal with atomic number 48, known for its multiple stable and radioactive isotopes. Among its isotopes, 109Cd stands out because of its radioactive nature and applications in scientific research and industry.

Key facts about 109Cd:

- Atomic Number: 48
- Mass Number: 109
- Radioactive Nature: Yes, it is radioactive.
- Decay Mode: Electron capture (primarily)
- Half-life: 462 days

The isotope's decay process involves the capture of an inner orbital electron by the nucleus, leading to the transformation of a proton into a neutron, and emitting characteristic X-rays or gamma rays. This decay process makes 109Cd useful in various applications, such as calibration sources for gamma spectroscopy and radiotherapy.

Fundamentals of Radioactive Decay and Half-life

Understanding how a radioactive isotope decreases over time requires grasping the fundamental principles of nuclear decay.

Radioactive Decay Law

Radioactive decay follows an exponential decay law, which can be expressed mathematically as:

$$N(t) = N_0 e^{-\lambda t}$$

Where:
- $N(t)$ = number of radioactive atoms remaining at time t
- N_0 = initial number of radioactive atoms
- λ = decay constant (probability per unit time that an atom decays)
- t = elapsed time

The decay constant λ is related to the half-life $T_{1/2}$ by:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

where $\ln 2 \approx 0.6931$

Half-life: Definition and Significance

The half-life is the period required for half of the radioactive atoms in a sample to decay. For 109Cd, this is:

$$T_{1/2} = 462 \text{ days}$$

This means that after 462 days, exactly 50% of the initial 109Cd atoms will have decayed. The half-life is intrinsic to each isotope and remains constant regardless of the initial quantity.

Calculating Remaining Atoms Over Time

Given the initial number of atoms, N_0, and the half-life, $T_{1/2}$, one can determine the remaining quantity after a specific elapsed time t.

The Decay Formula in Terms of Half-life

Since:

$$N(t) = N_0 \times 2^{-\frac{t}{T_{1/2}}}$$

This formula is convenient because it directly relates the number of remaining atoms to the half-life and elapsed time, bypassing the decay constant.

Applying the Formula to Our Problem

Suppose:
- $N_0 = 1.01012109 \times 10^{9}$ atoms
- $T_{1/2} = 462$ days
- t = the elapsed time in days (which should be specified in the problem; if not, we can analyze the general approach)

The number of remaining atoms after time t:

$$N(t) = 1.01012109 \times 10^{9} \times 2^{-\frac{t}{462}}$$

Interpreting the Problem Statement

The problem states: "A sample begins with 1.01012109Cd atoms." The value appears to be a large number with a decimal, which suggests a formatting or typographical error, or perhaps the notation is intended as scientific notation.

Likely correction:

- It probably means: 1.01012109×10^9 atoms (i.e., approximately one billion atoms).

Assumption:

- Initial number of atoms, $N_0 = 1.01012109 \times 10^9$

Question:

- How many atoms remain after a specific period?

Since the period is not specified, typical problems in this context might ask for the remaining atoms after one or multiple half-lives, or after a certain number of days.

Calculations for Specific Time Periods

Let's explore various scenarios by applying the decay formula.

After One Half-life (462 days)

Number of atoms remaining:

$$N(462) = N_0 \times 2^{-\frac{462}{462}} = N_0 \times 2^{-1} = \frac{N_0}{2}$$

Numerically:

$$N(462) = 1.01012109 \times 10^9 \times \frac{1}{2} = 5.05060545 \times 10^8 \text{ atoms}$$

Interpretation: After 462 days, half of the original atoms decay, leaving approximately 505 million atoms.

After Two Half-lives (924 days)

$$N(924) = N_0 \times 2^{-\frac{924}{462}} = N_0 \times 2^{-2} = \frac{N_0}{4}$$

Numerically:

$$N(924) = 1.01012109 \times 10^9 \times \frac{1}{4} = 2.52530273 \times 10^8 \text{ atoms}$$

Interpretation: After 924 days, only about 252 million atoms remain, which is a quarter of the original.

After Three Half-lives (1386 days)

$$ N(1386) = N_0 \times 2^{-\frac{1386}{462}} = N_0 \times 2^{-3} = \frac{N_0}{8} $$

Numerically:

$$ N(1386) = 1.01012109 \times 10^9 \times \frac{1}{8} \approx 1.26265 \times 10^8 \text{ atoms} $$

Generalized Approach: Calculating for Any Time t

Suppose you want to find the remaining number of atoms after an arbitrary period t days.

Step-by-step:

1. Determine the ratio:

$$ \text{Fraction remaining} = 2^{-\frac{t}{T_{1/2}}} $$

2. Multiply this fraction by the initial number:

$$ N(t) = N_0 \times 2^{-\frac{t}{T_{1/2}}} $$

Example:

If $t = 300$ days,

$$ N(300) = 1.01012109 \times 10^9 \times 2^{-\frac{300}{462}} $$

Calculate:

$$ \frac{300}{462} \approx 0.649 $$

Then,

$$ 2^{-0.649} \approx e^{-\ln 2 \times 0.649} \approx e^{-0.6931 \times 0.649} \approx e^{-0.449} \approx 0.638 $$

Thus,

$$ N(300) \approx 1.01012109 \times 10^9 \times 0.638 \approx 644,200,000 \text{ atoms} $$

Decay Constants and Their Role

The decay constant λ provides an alternative way to analyze decay:

$$ \lambda = \frac{\ln 2}{T_{1/2}} $$

For 109Cd:

$$ \lambda = \frac{0.6931}{462 \text{ days}} \approx 0.001502 \text{ day}^{-1} $$

Using this:

$$ N(t) = N_0 e^{-\lambda t} $$

which can be used for more precise calculations, especially over small time intervals or when integrating decay over continuous periods.

Implications of the Half-life of 462 Days

The 462-day half-life of 109Cd makes it suitable for various practical applications:

- Calibration sources: Its predictable decay rate allows for stable calibration over periods of months.
- Medical uses: Its decay properties are exploited in radiotherapy and diagnostic imaging.
- Environmental tracing: Its decay can be used to monitor environmental processes over a period of about a year.

The relatively long half-life balances the need for a manageable decay rate with sufficient activity for practical use

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