The Circumference Of A Circle Is 24 In. What Is The Area, In Square Inches? Express Your Answer In Terms

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Understanding how to determine the area of a circle when given its circumference is a fundamental concept in geometry. This problem involves applying formulas related to circles, such as the circumference and area formulas, and solving for unknowns step-by-step. In this article, we will explore the problem comprehensively, providing detailed explanations, formulas, and methods to find the area of a circle with a given circumference of 24 inches. Additionally, we will discuss how to express the final answer in terms of variables, which is particularly useful for algebraic applications and understanding the relationships between different circle measurements.

Understanding the Basic Properties of a Circle

Before diving into calculations, it is essential to review some key properties and formulas related to circles:

Circle Definitions and Formulas

- **Circumference (C):** The distance around the circle.
- **Diameter (d):** The longest distance across the circle through its center.
- **Radius (r):** The distance from the center of the circle to any point on its edge.
- **Area (A):** The space contained within the circle's boundaries.

The fundamental formulas are:
- Circumference: $C = 2 \pi r$
- Area: $A = \pi r^2$
- Diameter: $d = 2 r$

Given Data: Circumference of the Circle

In our problem, the circumference is given as:

$$ C = 24 \text{ inches} $$

Our goal is to find the area A in square inches.

Step-by-Step Solution

To find the area, we need to determine the radius first. Since the circumference formula involves the radius, we can solve for r and then substitute into the area formula.

Step 1: Find the Radius from the Circumference

Starting with the circumference formula:

$$ C = 2 \pi r $$

Plugging in the known value:

$$ 24 = 2 \pi r $$

Solve for r:

$$ r = \frac{24}{2 \pi} $$

Simplify:

$$ r = \frac{12}{\pi} $$

Step 2: Calculate the Area

Recall the area formula:

$$ A = \pi r^2 $$

Substitute $r = \frac{12}{\pi}$:

\[A = \pi \left(\frac{12}{\pi} \right)^2 \]

Simplify the squared term:

\[A = \pi \times \frac{144}{\pi^2} \]

Now, simplify the expression:

\[A = \frac{144 \pi}{\pi^2} \]

\[A = \frac{144}{\pi} \]

Final Expression:

\[
\boxed{
A = \frac{144}{\pi} \text{ square inches}
}
\]

This is the area expressed in terms of \(\pi \).

Expressing the Answer in Terms of Variables

In many mathematical contexts, especially algebra, it is common to express the answer in terms of variables to understand relationships or for further calculations. The problem's initial data can be generalized:

- Let \(C \) represent the circumference.
- Let \(r \) represent the radius.

Given \(C \), the radius \(r \) is:

\[r = \frac{C}{2 \pi} \]

The area \(A \) can then be expressed as:

\[A = \pi r^2 = \pi \left(\frac{C}{2 \pi} \right)^2 \]

Simplify:

\[A = \pi \times \frac{C^2}{4 \pi^2} = \frac{C^2}{4 \pi} \]

Thus, the area in terms of the circumference \(C \) is:

\[
\boxed{

A = \frac{C^2}{4 \pi}
}
\]

Applying this formula to our specific problem where $C = 24$ inches:

$$
A = \frac{(24)^2}{4 \pi} = \frac{576}{4 \pi} = \frac{144}{\pi}
$$

which confirms our previous result.

Understanding the Significance of the Result

Expressing the area in terms of π provides an exact value rather than a decimal approximation. This is particularly useful for precise calculations in mathematics and engineering. If an approximate decimal value is desired, we can substitute $\pi \approx 3.1416$:

$$
A \approx \frac{144}{3.1416} \approx 45.84 \text{ square inches}
$$

However, maintaining the exact form $\frac{144}{\pi}$ preserves mathematical precision.

Additional Tips and Common Mistakes to Avoid

In solving circle problems, be cautious of the following:

1. **Mixing units:** Always ensure units are consistent throughout the calculation.

2. **Forgetting to square the radius:** Remember that the area formula involves r^2, not just r.

3. **Misapplying formulas:** Use the correct formula for the given quantity—circumference for radius, not area, and vice versa.

4. **Not simplifying correctly:** Carefully simplify algebraic expressions, especially when dealing with fractions and π.

Conclusion: Key Takeaways

- Given the circumference of a circle, the radius can be found using \(r = \frac{C}{2\pi} \).
- The area of the circle can then be calculated with \(A = \pi r^2 \).
- For a circle with a circumference of 24 inches, the area in exact form is \(\frac{144}{\pi} \) square inches.
- When expressing the answer in terms of variables, the general formula is \(A = \frac{C^2}{4 \pi} \).
- Using exact formulas preserves mathematical precision, while approximate calculations provide practical estimates.

Understanding these concepts enhances your problem-solving skills in geometry and prepares you for more advanced mathematical challenges involving circles and other geometric figures.

Meta description:
Learn how to find the area of a circle with a given circumference of 24 inches. Step-by-step explanation, formulas, and how to express the answer in terms of \(\pi \) and variables for precise and practical calculations.

Frequently Asked Questions

Given the circumference of a circle is 24 inches, how do you find its radius?

Use the formula $C = 2\pi r$; so, $r = C / (2\pi) = 24 / (2\pi) = 12 / \pi$ inches.

What is the area of a circle with a circumference of 24 inches in terms of π?

First find the radius as $r = 12 / \pi$ inches, then area $A = \pi r^2 = \pi (12 / \pi)^2 = \pi (144 / \pi^2) = 144 / \pi$ square inches.

How can the area of the circle be expressed in terms of π given the circumference?

The area is $(144 / \pi)$ square inches, derived from $r = 12 / \pi$ and $A = \pi r^2$.

If the circumference of a circle is 24 inches, what is the formula for the area in terms of π?

Area = 144 / π square inches.

Can the area of the circle be simplified further from the expression 144 / π?

Yes, it can be written as approximately 45.84 square inches, but in exact terms, it remains 144 / π.

What is the significance of expressing the area in terms of π in this problem?

Expressing in terms of π maintains exactness and avoids rounding errors associated with decimal approximations.

Additional Resources

The Circumference Of A Circle Is 24 In. What Is The Area, In Square Inches? Express Your Answer In Terms

Understanding the relationship between a circle’s circumference and its area is fundamental in geometry. When given the circumference, determining the area involves a series of mathematical steps rooted in the properties of circles. This article explores how to calculate the area of a circle when its circumference is known, specifically when the circumference measures 24 inches. We will delve into the formulas involved, the step-by-step process to find the radius, and how to express the area in terms of the known quantities, providing a comprehensive guide suitable for students, educators, and enthusiasts alike.

The Foundations: Key Formulas and Concepts

Before we begin with calculations, it’s essential to revisit some fundamental formulas related to circles:

- Circumference (C): The total distance around the circle, given by:

\[
C = 2\pi r
\]

- Area (A): The space enclosed within the circle, given by:

\[

$$
A = \pi r^2
$$

In these formulas:
- r is the radius of the circle.
- π (pi) is a mathematical constant approximately equal to 3.14159.

Our goal is to find the area A when $C = 24$ inches, expressing A in terms involving known quantities or constants.

Step 1: Calculating the Radius from the Circumference

Given the circumference:

$$
C = 24 \text{ inches}
$$

Using the circumference formula:

$$
C = 2 \pi r
$$

We solve for r:

$$
r = \frac{C}{2 \pi}
$$

Substituting the known value:

$$
r = \frac{24}{2 \pi} = \frac{24}{2 \times 3.14159}
$$

Simplify numerator and denominator:

$$
r = \frac{24}{6.28318}
$$

Now, perform the division:

$$
r \approx 3.8197 \text{ inches}
$$

This radius is approximately 3.82 inches, but for the purpose of expressing in terms, we keep the exact fractional form or symbolic expressions involving π.

Expressed exactly:

$$
r = \frac{24}{2 \pi} = \frac{12}{\pi}
$$

Step 2: Deriving the Area in Terms of Known Quantities

With the radius expressed as $r = \frac{12}{\pi}$, we can now substitute into the area formula:

$$
A = \pi r^2
$$

Substitute r:

$$
A = \pi \left(\frac{12}{\pi} \right)^2
$$

Simplify the squared term:

$$
A = \pi \times \frac{144}{\pi^2}
$$

Expressed as a fraction:

$$
A = \frac{144 \pi}{\pi^2}
$$

Cancel one π from numerator and denominator:

$$
A = \frac{144}{\pi}
$$

Thus, the area of the circle, in square inches, in terms of π, is:

$$
\boxed{A = \frac{144}{\pi} \text{ square inches}}
$$

This expression clearly shows the area in terms of the known constant \(\pi

$).

Interpreting the Result: Numerical Approximation

While the exact expression involves π, many practical applications require a decimal approximation. Using $\pi \approx 3.14159$:

$$
A \approx \frac{144}{3.14159} \approx 45.84 \text{ square inches}
$$

Therefore, when the circumference is 24 inches, the circle’s area is approximately 45.84 square inches.

Summary of the Calculation Process

1. Given: $C = 24$ inches.
2. Find the radius: $r = \frac{C}{2\pi} = \frac{12}{\pi}$.
3. Calculate the area: $A = \pi r^2$.
4. Substitute r: $A = \pi \left(\frac{12}{\pi} \right)^2$.
5. Simplify: $A = \frac{144}{\pi}$.

Expressed in terms of π, the area is $\frac{144}{\pi}$ square inches; in decimal form, approximately 45.84 square inches.

Additional Insights: Practical Applications and Variations

Understanding how to manipulate these formulas extends beyond pure mathematics, impacting fields like engineering, architecture, and design:

- Design and Manufacturing: Precise calculations for circular components.
- Navigation and Geography: Estimating areas based on circumferential measurements.
- Education: Reinforcing understanding of geometric relationships.

Furthermore, if the circumference were different, the same approach applies:

- Step 1: Solve for the radius.
- Step 2: Plug into the area formula.
- Step 3: Simplify and interpret results.

This method emphasizes the importance of algebraic manipulation in geometric contexts and highlights the interconnectedness of circle measurements.

Conclusion

Calculating a circle’s area from its circumference involves a straightforward yet insightful process, revealing the elegance of geometric relationships. Given a circumference of 24 inches, the circle’s radius is $\frac{12}{\pi}$ inches, and its area is $\frac{144}{\pi}$ square inches. Expressing the area in terms of known constants underscores the importance of symbolic understanding, while numerical approximations provide tangible values suitable for practical purposes. Whether for academic pursuits or real-world applications, mastering these calculations is fundamental to a comprehensive understanding of circle geometry.

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