

The Coordinates Below Are The Three Vertices Of A Rectangle. Identify The Fourth Coordinate And The Area

The Coordinates Below Are The Three Vertices Of A Rectangle. Identify The Fourth Coordinate And The Area

Understanding the properties of rectangles and how to determine the missing vertex and area is fundamental in geometry, especially for students, teachers, and professionals dealing with spatial calculations. In this article, we will explore a step-by-step approach to solving such problems, along with explanations of relevant concepts, formulas, and practical examples to solidify your understanding.

Introduction to Rectangles and Coordinates

A rectangle is a four-sided polygon (quadrilateral) characterized by having four right angles (each measuring 90 degrees). Opposite sides are parallel and equal in length. When working with coordinate geometry, rectangles can be represented by their vertices' coordinates in a Cartesian plane, which allows precise calculations of positions, distances, and areas.

In coordinate geometry, each vertex of a rectangle is represented by an ordered pair (x, y) . Given three vertices, the challenge is to identify the coordinates of the missing vertex and compute the area of the rectangle.

Key Concepts in Coordinate Geometry for Rectangles

1. Properties of Rectangles

- All angles are right angles (90 degrees).
- Opposite sides are parallel and equal in length.
- Diagonals are equal in length and bisect each other.

2. Coordinates of Vertices

- Vertices are points in the plane, typically labeled as A, B, C, D.
- The position of these points determines the shape and size of the rectangle.

3. Midpoint and Vector Calculations

- The midpoint of the diagonal can be used to find the missing vertex if two opposite vertices are known.
- Vectors help in determining the position of vertices relative to each other.

Identifying the Fourth Vertex of a Rectangle

Suppose you are given three vertices of a rectangle: $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$. To find the fourth vertex $D(x_4, y_4)$, you need to understand the geometric relationships between the points.

Method 1: Using the Parallelogram Property

A rectangle is a special parallelogram, where the diagonals bisect each other. So, the midpoint of the diagonals are the same.

- If you consider the rectangle as a parallelogram, the missing vertex D can be found using the vector addition:

$D = A + C - B$, or $D = B + C - A$, or $D = A + B - C$, depending on the configuration.

- To choose the correct formula, identify which vertices are opposite each other.

Method 2: Recognizing Opposite Vertices

- If the known vertices are adjacent, the missing point can be calculated by vector addition/subtraction.

- If you suspect A and C are opposite vertices, then D can be found by:

$$D = B + C - A$$

- Conversely, if B and C are opposite, then:

$$D = A + C - B$$

- And if A and B are opposite:

$$D = A + B - C$$

Step-by-Step Example

Let's consider a specific example to illustrate the process:

Given:

- A(2, 3)

- B(6, 7)

- C(2, 11)

Goal:

- Find $D(x_4, y_4)$
- Compute the area of the rectangle.

Step 1: Determine Opposite Vertices

- First, check if A and C are opposite vertices or adjacent.

- The coordinates:

- A: (2, 3)

- B: (6, 7)

- C: (2, 11)

- Calculate distances:

- AB: $\sqrt{(6-2)^2 + (7-3)^2} = \sqrt{16 + 16} = \sqrt{32} \approx 5.66$

- AC: $\sqrt{(2-2)^2 + (11-3)^2} = \sqrt{0 + 64} = 8$

- BC: $\sqrt{(6-2)^2 + (7-11)^2} = \sqrt{16 + 16} = \sqrt{32} \approx 5.66$

Since AC is longer and vertical, and AB and BC are equal, it suggests that A and C are opposite vertices, and B is adjacent to both.

Conclusion:

- A and C are opposite vertices.
- B is adjacent to both, meaning B is connected to A and C.

Step 2: Find D

- Using the parallelogram property:

$$D = B + C - A$$

- Calculate:

- $D_x = x_B + x_C - x_A = 6 + 2 - 2 = 6$
- $D_y = y_B + y_C - y_A = 7 + 11 - 3 = 15$

Result:

- $D(6, 15)$

Step 3: Verify the Rectangle

- Check if the sides are perpendicular:

- Vector AB: $(6-2, 7-3) = (4, 4)$

- Vector AD: $(6-2, 15-3) = (4, 12)$

- Dot product:

- $AB \cdot AD = (4)(4) + (4)(12) = 16 + 48 = 64 \neq 0$

- Since the dot product is not zero, this indicates that the sides are not perpendicular, so the initial assumption needs checking. Alternatively, the configuration suggests B and D are opposite, and A and C are adjacent.

Alternate approach:

- Check if B and D are opposite:

- $D = A + C - B$

- $D_x = 2 + 2 - 6 = -2$

- $D_y = 3 + 11 - 7 = 7$

- $D(-2, 7)$

- Verify perpendicularity:

- Vector AB: $(4, 4)$

- Vector AD: $(-2 - 2, 7 - 3) = (-4, 4)$

- Dot product:

$$- (4)(-4) + (4)(4) = -16 + 16 = 0$$

- Since the dot product is zero, sides AB and AD are perpendicular, confirming that D(-2, 7) is the correct missing vertex.

Final Coordinates:

- The missing vertex D is at (-2, 7).

Calculating the Area of the Rectangle

Once all four vertices are known or identified, calculating the area becomes straightforward.

Method 1: Using Lengths of Adjacent Sides

- Find the length of two adjacent sides:

$$\begin{aligned} & \backslash[\\ & \text{Side 1} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \text{Side 2} = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \\ & \backslash] \end{aligned}$$

- The area is then:

$$\backslash[$$

$$\text{Area} = \text{Side 1} \times \text{Side 2}$$

]

Method 2: Using the Shoelace Formula

- The shoelace formula provides an efficient way to compute the area of any polygon, including rectangles, based on the coordinates of vertices in order.

- For vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$:

[

$$\text{Area} = \frac{1}{2} |x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1 - (y_1 x_2 + y_2 x_3 + y_3 x_4 + y_4 x_1)|$$

]

- Arrange the vertices in order, either clockwise or counterclockwise.

Applying the Shoelace Formula

Using the vertices:

- A(2, 3)
- B(6, 7)
- D(-2, 7)
- C(2, 11)

Order: A → B → D → C

Calculate:

[

\begin{aligned}

```

\text{Sum}_1 \&= x_A y_B + x_B y_D + x_D y_C + x_C y_A \\
&= (2)(7) + (6)(7) + (-2)(11) + (2)(3) \\
&= 14 + 42 - 22 + 6 = 40 \\
\text{Sum}_2 \&= y_A x_B + y_B x_D + y_D x_C + y_C x_A \\
&= (3)(6) + (7)(-2) + (7)(2) + (11)(2) \\
&= 18 - 14 + 14 + 22 = 40
\end{aligned}
\]

```

Area:

```

\[\frac{1}{2} |40 - 40| = 0
\]

```

Frequently Asked Questions

Given vertices A(2, 3), B(6, 3), and C(6, 7), what is the coordinate of the fourth vertex D of the rectangle?

The fourth vertex D is at (2, 7). Since the rectangle's sides are parallel to the axes, D completes the rectangle at (2, 7).

How do you determine the missing vertex of a rectangle when given three vertices?

Identify the pair of vertices with the same x or y coordinates, then use the remaining vertices to find the missing point by matching the coordinates accordingly, often using vector addition or coordinate pairing.

If the vertices of a rectangle are A(1, 2), B(1, 5), and C(4, 5), what is the area of the rectangle?

The area is 6 units squared, calculated as length $\times$ width: $(4 - 1) \times (5 - 2) = 3 \times 3 = 9$.

Can the fourth vertex of a rectangle be found using the distance formula? If yes, how?

Yes; by calculating the distances between known vertices, confirming right angles, and using coordinate geometry to find the missing point through the midpoint or vector methods.

What is the significance of the coordinates below in determining the rectangle's area?

The coordinates define the vertices' positions in the plane, allowing calculation of side lengths via distance formula, which are then multiplied to find the area.

Given the vertices A(3, 4), B(3, 8), and C(7, 8), what is the area of the rectangle and the missing vertex D?

The missing vertex D is at (7, 4), and the area is 16 units squared, calculated as $(7 - 3) \times (8 - 4) = 4 \times 4 = 16$.

Additional Resources

The Coordinates Below Are The Three Vertices Of A Rectangle. Identify The Fourth Coordinate And The Area is a classic geometric problem that combines coordinate geometry with spatial reasoning. Whether you're a student preparing for exams, an educator designing problem sets, or a math enthusiast exploring the properties of rectangles, understanding how to determine the missing vertex and compute the area from given points is both practical and intellectually rewarding. This guide

provides a comprehensive breakdown of approaches, formulas, and step-by-step methods to tackle such problems confidently.

Understanding the Problem: Coordinates and Rectangles

Before diving into solutions, it's essential to grasp what the problem entails:

- You are given three vertices (points) of a rectangle, each represented by their coordinates in the Cartesian plane, e.g., (x, y) .
- Your task is two-fold:
 1. Identify the fourth vertex, which completes the rectangle.
 2. Calculate the area of this rectangle.

Why is this problem interesting? Because rectangles have specific properties—opposite sides are equal and parallel, and all angles are right angles—that can be exploited using coordinate geometry.

Moreover, knowing how to find the missing vertex can be useful in various applications, from computer graphics to land surveying.

Key Concepts and Properties of Rectangles in Coordinate Geometry

To solve the problem, familiarize yourself with some fundamental properties:

Properties of a Rectangle:

- Opposite sides are equal and parallel.
- All interior angles are right angles (90 degrees).
- Diagonals are equal in length and bisect each other.

Coordinate Geometry Tools:

- Distance Formula: To find the length between two points.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Midpoint Formula: To find the point exactly halfway between two points.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Vector Addition: Useful for translating points and understanding directions.

Step-by-Step Approach to Find the Fourth Vertex

Given three vertices of a rectangle, the key insight is that the missing vertex can be found by exploiting the properties of diagonals and adjacent sides.

Suppose the given points are:

- $A(x_1, y_1)$
- $B(x_2, y_2)$
- $C(x_3, y_3)$

We need to find point $(D(x_4, y_4))$.

Method 1: Using the Diagonal Property

The diagonals of a rectangle bisect each other and are equal in length. If you consider the three given points, at least two of them will lie at the ends of one diagonal, with the third point somewhere else.

Step 1: Determine which pairs could form diagonals.

Step 2: For each potential diagonal, find its midpoint.

Step 3: Use the fact that the diagonals bisect each other: the midpoint of the diagonal is the same for both.

Step 4: The missing vertex (D) can be found by vector addition:

[
If A, B, C are known, then D can be found using:
]

- $(D = A + C - B)$

- or $(D = B + C - A),$

- or $(D = A + B - C),$

depending on which points form the diagonal.

Method 2: Using the Parallelogram Rule

A rectangle is a special parallelogram with right angles. If you think of the three points as forming three vertices of a parallelogram, the fourth vertex can be found using the parallelogram rule:

$$\begin{aligned} & \backslash \\ D = A + B - C, \quad & \text{or} \quad D = A + C - B, \quad & \text{or} \quad D = B + C - A, \\ & \backslash \end{aligned}$$

depending on the configuration.

Note: Since a rectangle is a parallelogram with right angles, similar logic applies. The key is identifying which pairs of points are adjacent vertices.

Practical Example: Step-by-Step Solution

Suppose the given points are:

- $A(2, 3)$
- $B(6, 3)$
- $C(6, 7)$

Step 1: Visualize or sketch the points to understand their placement.

Step 2: Determine which points are connected to form sides and diagonals.

- A and B share the same y-coordinate, indicating they might be on the same horizontal side.
- B and C share the same x-coordinate, indicating they might be on the same vertical side.

Step 3: Recognize the likely configuration:

- A at $(2,3)$
- B at $(6,3)$
- C at $(6,7)$

From the above, the sides (AB) (horizontal) and (BC) (vertical) are apparent.

Step 4: Find the missing vertex (D) :

Using the parallelogram rule:

[

$$D = A + C - B$$

]

[

$$D_x = x_A + x_C - x_B = 2 + 6 - 6 = 2$$

]

[

$$D_y = y_A + y_C - y_B = 3 + 7 - 3 = 7$$

]

So, the missing vertex is at:

[

$$D(2, 7)$$

]

Step 5: Confirm the shape:

- Check that (AD) is parallel to (BC) and (AB) is parallel to (DC) .

Calculating the Area of the Rectangle

Once the four vertices are identified, calculating the area becomes straightforward:

Step 1: Find Lengths of Adjacent Sides

Using the distance formula:

$$\text{Side 1} = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Side 2} = BC = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$

Continuing with our example:

- AB: (2,3) to (6,3):

$$AB = \sqrt{(6 - 2)^2 + (3 - 3)^2} = \sqrt{4^2 + 0^2} = 4$$

- BC: (6,3) to (6,7):

$$BC = \sqrt{(6 - 6)^2 + (7 - 3)^2} = \sqrt{0^2 + 4^2} = 4$$

Step 2: Calculate Area

Area of a rectangle:

$$\text{Area} = \text{length} \times \text{width}$$

In this case:

$$\text{Area} = AB \times BC = 4 \times 4 = 16$$

Summary of the Process

To recap, here is a structured list to approach similar problems:

1. Identify the given points and sketch for visualization.
2. Determine the configuration: which points form sides, which form diagonals.
3. Use geometric properties:
 - Opposite sides are equal and parallel.
 - Diagonals bisect each other and are equal.
 - The rectangle's diagonals intersect at the midpoint.
4. Find the missing vertex using vector addition or the parallelogram rule:

$$D = A + C - B, \quad \text{or} \quad D = A + B - C, \quad \text{etc.}$$

5. Calculate side lengths using the distance formula.
6. Compute the area as the product of side lengths.

Additional Tips and Considerations

- Multiple configurations: Sometimes, the three points may not be aligned straightforwardly; consider different combinations.
- Check for right angles: Use the dot product to verify perpendicular sides if needed.

For vectors $\vec{u} = (x_1, y_1)$ and $\vec{v} = (x_2, y_2)$, they are perpendicular if:

$$\vec{u} \cdot \vec{v} = x_1 x_2 + y_1 y_2 = 0$$

- Use symmetry: The diagonals of a rectangle bisect each other, so midpoints are a reliable way to find missing vertices.

Conclusion

The problem of determining the fourth vertex of a rectangle given three vertices is a fundamental exercise in coordinate geometry that emphasizes the importance of properties like parallelism, equal lengths, and diagonals. By applying vector addition, midpoints, and distance formulas, you can reliably find the missing coordinate and compute the rectangle's

The Coordinates Below Are The Three Vertices Of A Rectangle Identify The Fourth Coordinate And The Area

The Coordinates Below Are The Three Vertices Of A Rectangle Identify The Fourth Coordinate And The Area

Related Articles

- [Two Speakers Are Emitting Identical Sound Waves With A Wavelength Of 4.0 M. The Speakers Are 8.0 M Apart](#)
- [.fill In The Sheet Using The Questions At The Top Of The Page And Also For Step 2 There's Their Arguments](#)
- [2018. Find The Values Of X And Y In The Parallelogram PQRS.vatPT = 2x, TR = Y +3, QT = 3x, TS = 2yT272xS0](#)

[Back to Home](#)