# The Density Function Of X Is Given Byf(x)=a+bx^2 If 0 X 10 OtherwiseIf The Expectation Is E(x)=0.5, Find

**The Density Function Of X Is Given Byf(x)=a+bx^2 If 0 X 10 OtherwiseIf The Expectation Is E(x)=0.5, Find**

Understanding probability density functions (pdfs) is essential for analyzing continuous random variables. In this article, we explore a specific density function described by $f(x) = a + bx^2$ within the interval $0 \leq x \leq 10$, and zero elsewhere. Given that the expected value $E(X)$ is 0.5, our goal is to determine the parameters $a$ and $b$ that satisfy the properties of a probability density function and the expectation constraint. This comprehensive discussion will guide you through the process of deriving these parameters step-by-step, ensuring clarity and depth in understanding.

## Understanding the Problem Setup

Before delving into calculations, it’s important to clarify the problem's components:

### The Probability Density Function (pdf)

- Defined as $f(x) = a + bx^2$ for $0 \leq x \leq 10$.
- Zero elsewhere, i.e., $f(x) = 0$ for $x < 0$ or $x > 10$.
- The parameters $a$ and $b$ are constants to be determined.

### Key Constraints

- The total probability must equal 1:
$$
\int_{-\infty}^{\infty} f(x) \, dx = 1
$$
- The expected value of $X$ is given as:
$$
E(X) = 0.5
$$
- Since $f(x)$ is non-zero only between 0 and 10, the integrals simplify over this interval.

## Step 1: Ensuring the pdf integrates to 1

The first step involves applying the normalization condition for probability density functions.

### Calculating the total probability

$$
\int_{0}^{10} (a + bx^2) \, dx = 1
$$

Performing the integral:

$$
a \int_{0}^{10} dx + b \int_{0}^{10} x^2 \, dx = 1
$$

Calculating each integral:

$$
a \times [x]_{0}^{10} + b \times \left[\frac{x^3}{3}\right]_{0}^{10} = 1
$$

Plugging in limits:

$$
a \times (10 - 0) + b \times \frac{10^3}{3} = 1
$$

$$
10a + \frac{1000b}{3} = 1
$$

This yields the first key equation:

$$
\boxed{
10a + \frac{1000b}{3} = 1
}
$$

## Step 2: Applying the expectation condition $E(X) = 0.5$

Next, we express the expectation in terms of $a$ and $b$:

$$
E(X) = \int_{0}^{10} x f(x) \, dx = 0.5
$$

Substitute $f(x) = a + bx^2$:

$$
E(X) = \int_{0}^{10} x(a + bx^2) \, dx
$$

Distribute $x$:

$$
E(X) = a \int_{0}^{10} x \, dx + b \int_{0}^{10} x^3 \, dx
$$

$$

Calculate each integral:

$$
a \times \left[\frac{x^2}{2}\right]_{0}^{10} + b \times \left[\frac{x^4}{4}\right]_{0}^{10}
$$

Plugging in limits:

$$
a \times \frac{10^2}{2} + b \times \frac{10^4}{4}
$$

$$
a \times \frac{100}{2} + b \times \frac{10000}{4}
$$

$$
50a + 2500b
$$

Set equal to 0.5:

$$
50a + 2500b = 0.5
$$

This is our second key equation:

$$
\boxed{
50a + 2500b = 0.5
}
$$

## Step 3: Solving the System of Equations

Now, we have two equations:

$$
\begin{cases}
10a + \frac{1000b}{3} = 1 \quad (1) \\
50a + 2500b = 0.5 \quad (2)
\end{cases}
$$

Our goal is to find $a$ and $b$. Let’s proceed step-by-step.

### Express $a$ in terms of $b$ from Equation (1):

$$
10a + \frac{1000b}{3} = 1
$$

$$

10a = 1 - \frac{1000b}{3}
\]
\[
a = \frac{1}{10} - \frac{1000b}{30}
\]
\[
a = \frac{1}{10} - \frac{100b}{3}
\]

## Substitute \( a \) into Equation (2):

\[
50a + 2500b = 0.5
\]
\[
50 \left( \frac{1}{10} - \frac{100b}{3} \right) + 2500b = 0.5
\]

Simplify:
\[
50 \times \frac{1}{10} - 50 \times \frac{100b}{3} + 2500b = 0.5
\]
\[
5 - \frac{50 \times 100b}{3} + 2500b = 0.5
\]

Calculate:
\[
5 - \frac{5000b}{3} + 2500b = 0.5
\]

Express all terms with denominator 3:
\[
5 = \frac{15}{3}
\]
\[
- \frac{5000b}{3} + 2500b = - \frac{5000b}{3} + \frac{7500b}{3}
\]

Combine:
\[
- \frac{5000b}{3} + \frac{7500b}{3} = \frac{2500b}{3}
\]

Now, the entire equation:
\[
\frac{15}{3} + \frac{2500b}{3} = 0.5
\]

Multiply through by 3 to clear denominators:

$$
15 + 2500b = 1.5
$$

Solve for $b$:

$$
2500b = 1.5 - 15 = -13.5
$$

$$
b = \frac{-13.5}{2500} = -0.0054
$$

Determine $a$:

$$
a = \frac{1}{10} - \frac{100b}{3}
$$

$$
a = 0.1 - \frac{100 \times (-0.0054)}{3}
$$

$$
a = 0.1 - \frac{-0.54}{3}
$$

$$
a = 0.1 + 0.18 = 0.28
$$

Final values:

$$
\boxed{
a = 0.28, \quad b = -0.0054
}
$$

# Verification of the Results

It’s important to verify that these parameters satisfy the initial constraints.

## Check the normalization condition:

$$
10a + \frac{1000b}{3} = 10 \times 0.28 + \frac{1000 \times (-0.0054)}{3}
$$

$$
= 2.8 - \frac{5.4}{3}
$$

$$
= 2.8 - 1.8 = 1
$$

Confirmed. The total probability integrates to 1.

### Check the expectation:

$$
50a + 2500b = 50 \times 0.28 + 2500 \times (-0.0054)
$$

$$
= 14 - 13.5 = 0.5
$$

Confirmed. The expectation is as given.

## Interpreting the Density Function

With the parameters known, the density function becomes:

$$
f(x) = 0.28 - 0.0054 x^2
$$

for $0 \leq x \leq 10$. Outside this interval, $f(x) = 0$.

### Key Properties:

- Shape: Since $b$ is negative, $f(x)$ is a parabola opening downward within the interval, with its maximum at $x=0$.
- Range: The density is positive over the interval because:

$$
f
$$

## Frequently Asked Questions

### Given the density function f(x) = a + b x^2 for 0 < x < 10, how do you determine the constants a and b?

You use the properties of probability density functions: integrate f(x) over (0,10) to equal 1, and use the given expectation E(X)=0.5 to set up equations to solve for a and b.

### What is the significance of the expectation E(X) = 0.5 in determining the density function?

The expectation provides a constraint that helps in solving for the parameters a and b, ensuring the density function correctly models the given mean of the distribution.

## How do you set up the integral for the total probability condition for the given density function?

You integrate f(x) from 0 to 10: $\int_0^{10} (a + b x^2) dx = 1$, and solve for a and b.

## What is the integral of the density function f(x) = a + b x^2 over 0 to 10?

The integral is $a x + (b/3) x^3$ evaluated from 0 to 10, which equals $10a + (1000/3) b$.

## How do you compute the expectation E(X) for the given density function?

Calculate $E(X) = \int_0^{10} x f(x) dx = \int_0^{10} x (a + b x^2) dx$, then evaluate the integral to relate a and b.

## What is the integral expression for E(X) given f(x) = a + b x^2?

$E(X) = \int_0^{10} x (a + b x^2) dx = a \int_0^{10} x dx + b \int_0^{10} x^3 dx$.

## How do you solve for the constants a and b using the conditions from the density function?

Set up two equations: one from the total probability (integral equals 1) and one from the expectation (integral equals 0.5), then solve these simultaneous equations.

## What are the steps to find the specific values of a and b for the given distribution?

Step 1: Integrate f(x) over (0,10) and set equal to 1. Step 2: Integrate x f(x) over (0,10) and set equal to 0.5. Step 3: Solve the resulting equations for a and b.

## Is the function f(x) = a + b x^2 a valid probability density function for 0 < x < 10?

Yes, provided that a and b are chosen such that $f(x) \geq 0$ for all x in (0,10) and the total integral equals 1.

## What is the final step after calculating a and b to verify the correctness of the density function?

Verify that the integral of f(x) over (0,10) equals 1 and that the expected value computed from the density matches E(X)=0.5.

# Additional Resources

The Density Function Of X Is Given By $f(x) = a + bx^2$ If $0 < x < 10$; Otherwise, It Is Zero: An Investigative Analysis

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Introduction

In the realm of probability theory and statistical analysis, understanding the characteristics of a random variable hinges on its probability density function (pdf). The pdf encapsulates the likelihood of the variable assuming particular values within its domain, serving as a foundational concept for inferential statistics, modeling, and various applied disciplines.

This investigative article delves into a specific case: a random variable $X$ with a density function defined as $f(x) = a + bx^2$ for $0 < x < 10$, and zero elsewhere. The problem further stipulates that the expectation $E(X)$ equals 0.5, which imposes constraints on the parameters $a$ and $b$. Our goal is to analyze this density function thoroughly, determine the values of $a$ and $b$, and explore the implications of this functional form.

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Conceptual Foundation

Before proceeding to calculations, it’s essential to establish the core principles involved:

- Probability Density Function (pdf): For a continuous random variable $X$, $f(x)$ must satisfy:
- Non-negativity: $f(x) \geq 0$ for all $x$.
- Normalization: $\int_{-\infty}^{\infty}f(x) dx = 1$.

- Expectation (Mean): The expected value $E(X)$ is given by:

$$
E(X) = \int_{-\infty}^{\infty} x f(x) dx
$$

with the integration limits confined to the support where $f(x) \neq 0$.

Given the specific functional form and support, these principles will guide our derivations.

---

Deriving the Parameters $a$ and $b$

Step 1: Normalization Condition

Since $f(x) = a + bx^2$ for $0 < x < 10$ and zero elsewhere, the total probability must be 1:

$$
\int_0^{10} (a + bx^2) dx = 1
$$

Calculating the integral:

$$
\int_0^{10} a dx + \int_0^{10} bx^2 dx = a \times 10 + b \times \frac{10^3}{3}
$$

Simplify:

$$
10a + \frac{1000b}{3} = 1
$$

This yields our first equation:

$$
\boxed{
10a + \frac{1000b}{3} = 1
}
$$

---

Step 2: Expectation Condition

Given $E(X) = 0.5$, we have:

$$
E(X) = \int_0^{10} x (a + bx^2) dx = 0.5
$$

Compute the integral:

$$
\int_0^{10} a x dx + \int_0^{10} b x^3 dx
$$

which evaluates to:

$$
a \times \frac{10^2}{2} + b \times \frac{10^4}{4}
$$

Simplify:

$$
a \times 50 + b \times 2500 = 0.5
$$

This gives our second equation:

$$
\boxed{
50a + 2500b = 0.5

}
\]

---

Solving the System of Equations

The two equations are:

1. \(10a + \frac{1000b}{3} = 1\)
2. \(50a + 2500b = 0.5\)

Rearranged:

\[
\text{Equation 1: } 10a + \frac{1000b}{3} = 1
\]
\[
\text{Equation 2: } 50a + 2500b = 0.5
\]

Step 3: Express \(a\) in terms of \(b\)

From Equation 1:

\[
10a = 1 - \frac{1000b}{3}
\]
\[
a = \frac{1}{10} - \frac{1000b}{30} = \frac{1}{10} - \frac{100b}{3}
\]

Step 4: Substitute into Equation 2

Plug into Equation 2:

\[
50 \left(\frac{1}{10} - \frac{100b}{3}\right) + 2500b = 0.5
\]

Simplify:

\[
5 - \frac{50 \times 100b}{3} + 2500b = 0.5
\]

\[
5 - \frac{5000b}{3} + 2500b = 0.5
\]

Express \(2500b\) with denominator 3:

$$
5 - \frac{5000b}{3} + \frac{7500b}{3} = 0.5
$$

Combine the $b$ terms:

$$
5 + \frac{(7500b - 5000b)}{3} = 0.5
$$

$$
5 + \frac{2500b}{3} = 0.5
$$

Subtract 5 from both sides:

$$
\frac{2500b}{3} = 0.5 - 5 = -4.5
$$

Solve for $b$:

$$
b = \frac{-4.5 \times 3}{2500} = \frac{-13.5}{2500} = -\frac{13.5}{2500}
$$

Simplify:

$$
b = -\frac{27}{5000}
$$

which is approximately:

$$
b \approx -0.0054
$$

Now, find $a$:

$$
a = \frac{1}{10} - \frac{100b}{3}
$$

$$
a = 0.1 - \frac{100 \times (-27/5000)}{3}
$$

Calculate numerator:

$$
100 \times -\frac{27}{5000} = -\frac{2700}{5000} = -0.54
$$

Divide by 3:

$$
-\frac{0.54}{3} = -0.18
$$

Now, $a$:

$$
a = 0.1 - (-0.18) = 0.1 + 0.18 = 0.28
$$

---

Final Parameters

$$
\boxed{
a = 0.28, \quad b = -0.0054
}
$$

Note: These values satisfy the constraints for the density function and the expectation.

---

Validity and Implications

Non-negativity of $f(x)$:

- For $f(x) = a + bx^2$ over $0 < x < 10$, check whether $f(x) \geq 0$.

At $x = 0$:

$$
f(0) = a = 0.28 > 0
$$

At $x = 10$:

$$
f(10) = 0.28 + (-0.0054) \times 100 = 0.28 - 0.54 = -0.26
$$

Since $f(10)$ is negative, the density function is not valid over the entire support unless the negative part is balanced by the function's shape. This indicates that the density function, as derived, violates non-negativity at $x=10$.

This suggests that either:

- The support must be restricted to a subset of $[0,10]$ where $f(x) \geq 0$,

- Or the parameters need adjustment to ensure non-negativity throughout the support.

---

Adjusting the Model for Validity

Because the initial derivation leads to negative densities at the boundary, further refinement is necessary:

- Identify the region where $f(x) \geq 0$:

Solve $f(x) = a + bx^2 \geq 0$:

$$
0.28 - 0.0054 x^2 \geq 0
$$
$$
0.0054 x^2 \leq 0.28
$$
$$
x^2 \leq \frac{0.28}{0.0054} \approx 51.85
$$
$$
x \leq \sqrt{51.85} \approx 7.2
$$

Thus, the density is non-negative for $x$ in approximately $[0, 7.2]$.

Conclusion: The original assumption of support over $[0, 10]$ leads to invalid density values at the upper end. To maintain a valid pdf, the support should be adjusted to $[0, 7.2]$ or the parameters recalculated with the support constrained accordingly.

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Broader Implications and Applications

This case study exemplifies the intricacies involved in defining probability density functions with polynomial forms. It underscores the importance of verifying the positivity and normalization conditions, especially when parameters are derived analytically.

Potential

## The Density Function Of X Is Given Byf X A Bx 2 If 0 X 10 Otherwiseif The Expectation Is E X 0 5 Find

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