The Equation Of Motion Of A Body Is Given By Dy/dt + 4 Dy/dt + 13y = E^2t Cos T, Where Y Is The Distance

The Equation Of Motion Of A Body Is Given By Dy/dt + 4 Dy/dt + 13y = E^2t Cos T, Where Y Is The Distance is a fundamental differential equation that describes the motion of a body under specific forces. This type of equation, which combines derivatives of the position function y(t) with a forcing function, is central to understanding dynamics in physics and engineering. In this article, we will explore the detailed solution process for this differential equation, its physical significance, and applications.

Understanding the Given Differential Equation

The equation Dy/dt + 4 Dy/dt + 13y = E^2t Cos T appears to contain a typographical error in the notation. Typically, the derivatives are written as y' or dy/dt. Assuming the correct form is:

dy/dt + 4 dy/dt + 13 y = E^2 t Cos T

But this seems redundant since dy/dt appears twice. Alternatively, it might be a second-order differential equation:

$d^2y/dt^2 + 4\ dy/dt + 13\ y = E^2\ t\ Cos\ T$

This form is more consistent with standard equations of motion in physics, where second derivatives represent acceleration, and first derivatives represent velocity.

Assumption:
The most probable intended form of the differential equation is:

$d^2y/dt^2 + 4\ dy/dt + 13\ y = E^2\ t\ Cos\ T$

where:
- y(t): displacement or distance as a function of time
- E, T: constants or parameters related to the system

This differential equation models a damped harmonic oscillator with a forcing function.

Components of the Differential Equation

Homogeneous Part

The homogeneous equation associated with the above is:

$$d^2y/dt^2 + 4\ dy/dt + 13\ y = 0$$

This describes the natural motion of the system without external forcing.

Particular Part

The right side, E^2 t Cos T, acts as the external forcing function, influencing the system's response.

Solution Strategy for the Differential Equation

To find the complete solution y(t), we need to:

1. Solve the homogeneous differential equation.
2. Find a particular solution to the non-homogeneous equation.
3. Combine both to get the general solution.

Step 1: Solving the Homogeneous Equation

The homogeneous differential equation:

$$d^2y/dt^2 + 4\ dy/dt + 13\ y = 0$$

has characteristic equation:

$$r^2 + 4\ r + 13 = 0$$

Solve for r:

$$r = [-4 \pm \sqrt{(16 - 52)}] / 2$$
$$r = [-4 \pm \sqrt{(-36)}] / 2$$
$$r = [-4 \pm 6i] / 2$$
$$r = -2 \pm 3i$$

Thus, the homogeneous solution is:

$$y_h(t) = e^{-2t} (A \cos 3t + B \sin 3t)$$

where A and B are arbitrary constants determined by initial conditions.

Step 2: Finding a Particular Solution

Since the forcing function is:

$f(t) = E^2 t \text{ Cos } T$

which is a product of a polynomial t and a cosine function, a suitable trial for the particular solution $y_p(t)$ is:

$y_p(t) = (C t + D) \text{ Cos } T + (E t + F) \text{ Sin } T$

But because the forcing term is more complex, and to account for the polynomial t, the method of undetermined coefficients suggests trying a particular solution of the form:

$y_p(t) = (a t + b) \text{ Cos } T + (c t + d) \text{ Sin } T$

However, because the forcing function involves t multiplied by Cos T (a constant frequency), the particular solution might be better approached using the method of variation of parameters or undetermined coefficients with the polynomial times cosine.

Alternatively, since the forcing function is $E^2 t$ Cos T, a polynomial trial multiplied by the cosine is suitable:

$y_p(t) = (\alpha t + \beta) \text{ Cos } T + (\gamma t + \delta) \text{ Sin } T$

Given the derivatives, substituting y_p into the differential equation and solving for the undetermined coefficients will yield the particular solution.

Step 3: General Solution

The total solution is:

$y(t) = y_h(t) + y_p(t)$

which combines the natural response with the forced response.

Physical Interpretation and Applications

Understanding the equation Dy/dt + 4 Dy/dt + 13 y = E^2 t Cos T aids in modeling physical systems such as:

- **Damped Mechanical Oscillators:** Systems like car suspensions or building structures subjected to periodic forces.
- **Electrical Circuits:** RLC circuits with external forcing signals.
- **Vibrational Analysis:** Analyzing how structures respond to time-dependent forces.

The damping term (represented by the coefficient 4) indicates energy loss over time, typical in real-world systems.

Key Concepts in Solving Such Differential Equations

Homogeneous vs. Non-Homogeneous Solutions

- Homogeneous solutions describe free oscillations.
- Particular solutions account for external forces.

Methods of Solution

- Characteristic equation method for homogeneous parts.
- Undetermined coefficients or variation of parameters for particular solutions.
- Use of Laplace transforms for complex forcing functions.

Example Problem and Step-by-Step Solution

Suppose the differential equation:

$d^2y/dt^2 + 4\ dy/dt + 13\ y = E^2\ t\ \mathrm{Cos}\ T$

with initial conditions $y(0) = y_0$, $y'(0) = v_0$.

Step 1: Find the homogeneous solution as shown.

Step 2: Assume a particular solution form and substitute into the equation to solve for coefficients.

Step 3: Apply initial conditions to determine the constants A and B.

Step 4: Write the complete solution y(t).

Note: For specific numerical values of E and T, the coefficients can be computed explicitly.

Conclusion

The differential equation dy/dt + 4 dy/dt + 13 y = E^2 t Cos T models a damped oscillator subjected to a time-dependent external force. Solving this equation involves understanding the homogeneous solution, particular solution, and their combination. Such equations are fundamental in physics and engineering for analyzing systems ranging from mechanical vibrations to electrical circuits. Mastering the solution techniques for these equations enables engineers and scientists to predict system behavior accurately and design more resilient structures and devices.

Additional Tips for Solving Similar Differential Equations

- Always clarify the form of the differential equation, especially the order and coefficients.
- Identify whether the forcing function is polynomial, exponential, sinusoidal, or a combination.
- Choose the appropriate method: undetermined coefficients or variation of parameters based on the forcing function.
- Check for resonance conditions where the forcing frequency matches the natural frequency, leading to large amplitude responses.
- Use initial conditions effectively to find the particular constants in the solution.

By following these steps and concepts, solving complex differential equations like the one discussed becomes a systematic process, providing insights into the dynamic behavior of physical systems.

Frequently Asked Questions

What is the differential equation governing the

motion of the body?

The differential equation is dy/dt + 4 dy/dt + 13y = e^{2t} cos t.

Is there a typo in the original equation, and if so, what is the correct form?

Yes, the original equation appears to have a typo; it should be dy/dt + 4 dy/dt + 13y = e^{2t} cos t, combining like terms properly.

What are the homogeneous solutions of this differential equation?

The homogeneous solution is found by solving the characteristic equation: r + 4r + 13 = 0, which simplifies to 5r + 13 = 0, leading to r = -13/5.

How do we find the particular solution for the nonhomogeneous differential equation?

We use methods like undetermined coefficients or variation of parameters to find a particular solution, considering the forcing term e^{2t} cos t.

What is the physical interpretation of the forcing function e^{2t} cos t in the context of the body's motion?

It represents an external force acting on the body that varies exponentially with time and oscillates sinusoidally.

How can the initial conditions be used to determine the specific solution?

Initial conditions such as initial displacement and velocity are substituted into the general solution to solve for arbitrary constants, giving the particular solution.

What is the significance of solving this differential equation in real-world applications?

It helps in predicting the body's response to external forces, crucial in engineering, physics, and vibration analysis.

Are there any particular challenges in solving this

type of differential equation?

Yes, especially due to the nonhomogeneous term e^{2t} cos t, which requires careful application of methods like undetermined coefficients or variation of parameters to find the particular solution.

Additional Resources

The Equation Of Motion Of A Body Is Given By Dy/dt + 4 Dy/dt + 13y = E^2t Cos T – this differential equation exemplifies a second-order linear differential equation with constant coefficients, a fundamental concept in the analysis of dynamic systems in physics and engineering. It models the motion of a body subjected to damping and external forcing, with the right-hand side representing an external periodic force scaled by an exponential term. Analyzing this equation provides insights into the body's response over time, including transient behavior and steady-state oscillations.

Understanding the Equation of Motion

Formulation and Components

The given differential equation appears to be:

Dy/dt + 4 Dy/dt + 13y = E^2t Cos T

At first glance, there seems to be a typographical inconsistency in the notation. Likely, the intended form is:

$a\, d^2y/dt^2 + b\, dy/dt + c\, y = f(t)$

where y is the displacement (distance), and f(t) is the external forcing function. Given the terms, an appropriate correction for clarity might be:

$d^2y/dt^2 + 4\, dy/dt + 13\, y = E^2\, t \text{ Cos } T$

This is a standard second-order linear differential equation representing a damped driven system.

- d^2y/dt^2: Represents acceleration, the second derivative of displacement.
- 4 dy/dt: Damping term, proportional to velocity.
- 13 y: Restoring force, proportional to displacement.
- E^2 t Cos T: External forcing function, a time-dependent periodic force scaled by a linear term in t.

Understanding each component helps in visualizing the physical system: a mass-spring-damper system under a periodic external force.

Physical Interpretation

This kind of equation is typical in mechanical systems where a mass attached to a spring experiences damping and is subjected to an external periodic force. The damping term causes energy dissipation, the restoring term pulls the body back toward equilibrium, and the external force drives oscillations.

The presence of E^2 t Cos T suggests that the external force not only oscillates periodically but also increases linearly with time, representing a force that grows in amplitude or intensity as time progresses.

Solution Strategy

Solving such an equation involves two main steps:

1. Find the complementary (homogeneous) solution: Solution of the associated homogeneous equation:

$d^2y/dt^2 + 4\ dy/dt + 13\ y = 0$

2. Find a particular solution: Solution that accounts for the nonhomogeneous term, E^2 t Cos T.

The total solution is then:

$y(t) = y_h(t) + y_p(t)$

Homogeneous Solution

The homogeneous equation:

$d^2y/dt^2 + 4\ dy/dt + 13\ y = 0$

has characteristic equation:

$r^2 + 4r + 13 = 0$

Solving:

$$r = [-4 \pm \sqrt{(16 - 52)}] / 2 = [-4 \pm \sqrt{(-36)}] / 2$$

$$r = -2 \pm 3i$$

The homogeneous solution:

$$y_h(t) = e^{-2t} (A \cos 3t + B \sin 3t)$$

where A and B are arbitrary constants determined by initial conditions.

Features and implications:

- The exponential decay e^{-2t} indicates damping.
- The oscillatory part ($\cos 3t$ and $\sin 3t$) signifies natural vibrations at frequency 3 rad/sec.
- The damping causes the transient oscillations to diminish over time.

Particular Solution

Given the forcing function:

$$f(t) = E^2 t \text{ Cos } T$$

Note that the forcing is a product of a polynomial term (t) and a cosine function.

Method of undetermined coefficients is suitable here, but due to the polynomial factor, the particular solution will involve terms like:

$$y_p(t) = (\alpha t + \beta) \cos T + (\gamma t + \delta) \sin T$$

or more generally, polynomial functions multiplied by cos T and sin T.

Alternatively, the method of variation of parameters could be employed for a more systematic approach, especially for complicated forcing functions.

Assuming a particular solution:

$$y_p(t) = (A t + B) \cos T + (C t + D) \sin T$$

Substituting y_p and its derivatives into the original differential equation and equating coefficients will allow solving for A, B, C, and D.

Because the forcing involves t, the particular solution involves polynomial terms multiplied by sinusoidal functions, which accounts for resonance or near-resonance behavior depending on the forcing frequency T.

Analysis of the Solution

Transient and Steady-State Behavior

The complete solution:

$y(t) = e^{-2t} (A \cos 3t + B \sin 3t)$ + particular solution

illustrates two key behaviors:

- Transient response: The homogeneous part, decaying exponentially, represents the system's initial response that diminishes over time.
- Steady-state response: The particular solution, oscillating at the forcing frequency T, dominates as $t \to \infty$.

The long-term behavior is primarily governed by the particular solution, especially if damping is sufficient to suppress the transient oscillations.

Resonance Considerations

Resonance occurs when the forcing frequency matches the system's natural frequency. Here:

- Natural frequency: $\omega_0 = 3$ rad/sec (from homogeneous solution).
- Forcing frequency: T (from the forcing term).

If $T \approx 3$, the particular solution may involve resonance, leading to large amplitude oscillations. The polynomial factor t in the forcing function further amplifies this effect, potentially resulting in unbounded growth of the response unless damping counteracts it.

Features and Applications

Features of the System Modeled by the Equation

- Damped oscillations: Due to the exponential decay term.
- External forcing with growth: The E^2 t Cos T term introduces a linearly

increasing amplitude.
- Resonance potential: When forcing frequency matches natural frequency, large oscillations can occur.
- Transient vs. steady-state: Initial conditions influence transient behavior, but long-term behavior is driven by the forcing function.

Real-world Applications

- Mechanical systems: Damped oscillations in mass-spring-damper systems under periodic loads.
- Electrical circuits: RLC circuits with external sinusoidal voltage sources.
- Structural engineering: Vibration analysis of structures subjected to periodic forces.
- Control systems: Modeling of feedback mechanisms with damping and external inputs.

Pros and Cons of Analyzing Such Equations

Pros:

- Provides a comprehensive understanding of dynamic systems' responses.
- Facilitates prediction of long-term behavior under external forcing.
- Helps in designing systems to avoid resonance and instability.
- Applicable across multiple disciplines (physics, engineering, control systems).

Cons:

- Analytical solutions can become complex, especially with polynomial forcing functions.
- Numerical methods may be required for complicated forcing terms or initial conditions.
- Resonance scenarios can lead to unbounded responses, posing practical challenges.
- Assumes linearity; real systems may involve nonlinear effects not captured here.

Conclusion

The differential equation Dy/dt + 4 Dy/dt + 13 y = E^2 t Cos T captures a rich class of dynamical behaviors characteristic of damped driven oscillatory systems. Its solution involves dissecting the transient response dictated by the homogeneous equation and the steady-state response driven by the external forcing function. Understanding such equations is crucial in analyzing real-world systems where damping, external forces, and natural frequencies interplay to produce complex motion.

Through detailed solution strategies, resonance considerations, and application contexts, this analysis underscores the importance of differential equations in modeling and controlling physical phenomena. Whether designing resilient structures or tuning electronic circuits, mastery over these equations enables engineers and scientists to predict, optimize, and innovate effectively.

The Equation Of Motion Of A Body Is Given By Dy Dt 4 Dy Dt 13y E 2t Cos T Where Y Is The Distance

The Equation Of Motion Of A Body Is Given By Dy Dt 4 Dy Dt 13y E 2t Cos T Where Y Is The Distance

Related Articles

- (MAY MARK BRAINLIST) Which Of The Following Are The Powers Of The Us Government That Come From "necessary
- The Armature Of A Small Generator Consists Of A Flat, Square Coil With 190 Turns And Sides With A Length
- To Make An Investment, A Company Has Borrowed TL 8,000,000 Annually For 10 Years With 18% Annual Capital

Back to Home