The First Term Of An Arithmetic Sequence Is A B And The Second Term Is A C. What Is The Explicit Formula

The First Term Of An Arithmetic Sequence Is A B And The Second Term Is A C. What Is The Explicit Formula

Understanding the structure of arithmetic sequences is fundamental in mathematics, especially in algebra and number theory. When given specific terms of an arithmetic sequence, such as the first and second terms, it's possible to determine an explicit formula that describes any term in the sequence. This article provides a comprehensive guide to deriving the explicit formula of an arithmetic sequence when the first term is A B and the second term is A C, offering clarity and step-by-step instructions.

What Is an Arithmetic Sequence?

Before diving into the specifics of the problem, it's essential to understand what an arithmetic sequence is and how it functions.

Definition of an Arithmetic Sequence

An arithmetic sequence is a sequence of numbers in which each term after the first is obtained by adding a fixed constant, called the common difference, to the previous term.

Mathematically, an arithmetic sequence can be represented as:
$$
a_1,\, a_2,\, a_3,\, \ldots
$$
where
$$
a_{n} = a_1 + (n - 1)d
$$
and d is the common difference between successive terms.

Key Components of an Arithmetic Sequence

- First term (a_1): The starting element of the sequence.
- Common difference (d): The constant amount added to each term to get the next.
- Explicit formula: An expression that allows you to compute the n^{th} term directly without referencing previous terms.

Understanding the Given Terms

Given:
- The first term $a_1 = A B$
- The second term $a_2 = A C$

Here, A, B, and C are variables or constants. The notation suggests that $A B$ and $A C$ are terms, possibly representing numerical values or algebraic expressions.

To clarify:
- $a_1 = A B$
- $a_2 = A C$

Our goal is to find the explicit formula for the sequence, i.e., an expression for a_n in terms of n, A, B, and C.

Note: The notation indicates that a_1 and a_2 are known or given, and the sequence's pattern is determined by these.

Deriving the Explicit Formula

To find the explicit formula for the sequence, we need to determine:
1. The first term a_1
2. The common difference d

Once these are known, the explicit formula follows directly.

The Step-by-Step Process

1. **Identify the first term:** $a_1 = A B$

2. **Identify the second term:** $a_2 = A C$

3. **Calculate the common difference:** $d = a_2 - a_1$

4. **Formulate the explicit formula:** $a_n = a_1 + (n - 1)d$

Let's carry out each step.

Calculating the Common Difference

Given:
$$
a_1 = A B
$$
$$
a_2 = A C
$$

The common difference d is:
$$
d = a_2 - a_1 = A C - A B
$$

Factoring A:
$$
d = A (C - B)
$$

This expression provides the fixed increase between successive terms.

Explicit Formula for the n^{th} Term

Using the general form:
$$
a_n = a_1 + (n - 1)d
$$

Substitute the known values:
$$
a_n = A B + (n - 1) \times A (C - B)
$$

Factor out A:
$$
a_n = A \left[B + (n - 1)(C - B) \right]
$$

This is the explicit formula for the sequence:
$$
\boxed{
a_n = A \left[B + (n - 1)(C - B) \right]
}
$$

Summary:
- The explicit formula expresses any term a_n in terms of the initial parameters A, B, C, and the term number n.
- It clearly shows how the sequence progresses based on the initial terms and the common difference.

Interpreting the Explicit Formula

The formula:

$$
a_n = A \left[B + (n - 1)(C - B) \right]
$$

has several important implications:

Understanding the Components

- A: Acts as a scaling factor applied uniformly across the sequence.
- B: Represents the initial offset or starting point of the sequence, scaled by A.
- $C - B$: The difference between the second and first terms, scaled by A, which determines the rate of change per step.

How the Sequence Evolves

- When $n = 1$, the formula simplifies to:

$$
a_1 = A \left[B + 0 \right] = A B
$$

which matches the given first term.

- When $n = 2$:

$$
a_2 = A \left[B + (2 - 1)(C - B) \right] = A (B + C - B) = A C
$$

matching the second term.

- For larger n, the sequence continues with the common difference $A (C - B)$.

Special Cases and Variations

Depending on the values of A, B, and C, the explicit formula can represent various types of sequences.

Case 1: $A = 1$

The sequence simplifies to:

$$
a_n = B + (n - 1)(C - B)
$$

which is a standard arithmetic sequence starting at B, with common difference $C - B$.

Case 2: $B = C$

The sequence's common difference becomes zero:

$$
d = A (C - B) = 0
$$

implying all terms are equal:

$$
a_n = A B = A C
$$

a constant sequence.

Case 3: Negative Differences

If $C < B$, the sequence decreases by a fixed amount each step, illustrating a decreasing arithmetic sequence.

Applications of the Explicit Formula

Knowing the explicit formula for an arithmetic sequence has numerous practical applications:

Predicting Future Terms

Given the initial terms, you can directly calculate the n^{th} term without iterating through all previous terms.

Solving Real-World Problems

Sequences often model real-world scenarios such as:
- Financial installments
- Population growth or decline
- Measurement of recurring phenomena

Analyzing Sequence Behavior

The explicit formula helps identify:
- Long-term trends
- Rate of change
- Sequence convergence or divergence

Conclusion

In summary, when the first term of an arithmetic sequence is $A B$ and the second term is $A C$, the explicit formula for the sequence can be derived systematically:

$$
a_n = A \left[B + (n - 1)(C - B) \right]
$$

This formula enables the computation of any term directly and provides insight into the sequence's behavior based on the initial parameters. Understanding how to derive and interpret this formula enhances problem-solving skills in algebra and helps in various applied mathematics contexts.

Remember: The key steps involve identifying the first two terms, calculating the common difference, and substituting these into the general explicit formula for an arithmetic sequence. Mastery of this process allows for quick analysis of similar sequences and their properties.

Frequently Asked Questions

Given the first term of an arithmetic sequence is A B and the second term is A C, how do you determine the common difference?

The common difference d is found by subtracting the first term from the second term: $d = (A C) - (A B)$.

What is the explicit formula for the nth term of an arithmetic sequence when the first term is A B and the second term is A C?

The explicit formula is $T_n = (A B) + (n - 1) [(A C) - (A B)]$.

How can I express the explicit formula if the first term is A B and the second term is A C in terms of A, B, and C?

The explicit formula is $T_n = A B + (n - 1)(A C - A B)$.

If the first term of an arithmetic sequence is A B and the second term is A C, what is the value of the third term?

The third term $T_3 = A B + 2 (A C - A B)$.

Can you derive the explicit formula for the sequence given only the first two terms A B and A C?

Yes, the explicit formula is T_n = A B + (n - 1) [(A C) - (A B)].

What assumptions are made when finding the explicit formula for this arithmetic sequence?

It is assumed that the sequence is arithmetic, meaning the difference between consecutive terms is constant, specifically (A C) - (A B).

How does knowing only the first two terms help in constructing the explicit formula for an arithmetic sequence?

Knowing the first two terms allows you to determine the common difference and then write the general explicit formula based on that difference.

If A B = 3 and A C = 7, what is the explicit formula for the sequence?

The common difference is 7 - 3 = 4, so the explicit formula is T_n = 3 + (n - 1) 4.

What is the significance of the explicit formula in an arithmetic sequence?

The explicit formula allows you to find any term in the sequence directly without calculating all previous terms.

Additional Resources

Arithmetic Sequence: Unlocking the Explicit Formula from the First Two Terms

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Introduction: The Significance of Arithmetic Sequences in Mathematics

Mathematics, often regarded as the language of the universe, is replete with fascinating patterns and sequences that reveal underlying structures in nature, science, and technology. Among these, arithmetic sequences stand out for their simplicity and widespread applications—from financial calculations to computer science algorithms. Understanding how to derive the explicit formula of an arithmetic sequence given its initial terms is fundamental for both students and professionals aiming to decode these patterns efficiently.

In this article, we delve into a specific problem: Given that the first term of an arithmetic sequence is $A B$, and the second term is $A C$, what is its explicit formula? We will explore the concepts step-by-step, providing clarity for readers at all levels, and demonstrate how to articulate the explicit formula based solely on the information about the first two terms.

Understanding the Foundations: What Is an Arithmetic Sequence?

Definition and Characteristics

An arithmetic sequence (also known as an arithmetic progression) is a sequence of numbers where each term after the first is obtained by adding a constant difference to the preceding term. This constant is called the common difference (d).

Mathematically, an arithmetic sequence can be expressed as:

$$
a_n = a_1 + (n - 1)d
$$

where:
- a_1 is the first term,
- d is the common difference,
- a_n is the n^{th} term.

Key Points:
- The sequence is linear in nature.
- Knowledge of the first term and the common difference fully determines the sequence.
- The sequence progresses predictably, making it easy to compute any term directly.

Deciphering the Given Information: First and Second Terms

Interpreting the Terms: $A B$ and $A C$

The problem states:
- First term: $A B$
- Second term: $A C$

At first glance, these terms resemble algebraic expressions or perhaps concatenated symbols. To interpret them thoroughly, consider the following:

1. Potential interpretations:
- Literal variables: A, B, C represent constants or variables, and $A B$ and $A C$ are products or concatenations.
- String concatenation: If A, B, and C are strings, then $A B$ and $A C$ are concatenations.
- Numeric notation: If A, B, C are numbers, then $A B$ and $A C$ could be two-digit numbers where A is the tens digit, and B and C are units digits.

Given the context of deriving an explicit formula, the most mathematically consistent interpretation is that:

- A, B, and C are digits or constants.
- The terms $A B$ and $A C$ are two-digit numbers formed by placing A in the tens place, and B and C in the units place.

Example:

If $A=3$, $B=4$, $C=7$:
- First term: $A B = 34$
- Second term: $A C = 37$

This interpretation aligns well with common practice in algebraic contexts involving digits.

Formulating the Sequence: From Terms to Formula

Step 1: Expressing the Terms Algebraically

Assuming the above interpretation, the first two terms are:

$$
a_1 = 10A + B
$$
$$
a_2 = 10A + C
$$

where:
- A, B, C are digits (or constants).

Note: If A, B, C are variables, the formula remains similar, with these variables representing known or unknown constants.

Step 2: Determining the Common Difference d

Since the sequence is arithmetic, the common difference d is:

$$
d = a_2 - a_1
$$

Substituting the expressions:

$$
d = (10A + C) - (10A + B) = C - B
$$

Observation:
- The common difference depends solely on the difference between C and B.
- If $C > B$, the sequence increases; if $C < B$, it decreases; if $C = B$, the sequence is constant.

Step 3: Deriving the Explicit Formula

Recall that the explicit formula of an arithmetic sequence is:

$$
a_n = a_1 + (n - 1)d
$$

Substitute the known values:

$$
a_n = (10A + B) + (n - 1)(C - B)
$$

This formula allows the calculation of any term in the sequence directly, based on the initial parameters A, B, C.

Final explicit formula:

\[
\boxed{
a_n = 10A + B + (n - 1)(C - B)
}
\]

Special Cases and Further Considerations

Case 1: When \(C = B\)

If \(C = B\):
- \(d = 0\),
- The sequence is constant: all terms are \(10A + B\).

Explicit formula simplifies to:

\[
a_n = 10A + B
\]

This indicates a sequence where the first two terms are identical, and subsequent terms follow suit.

Case 2: When \(A, B, C\) Are Known Numbers

Suppose you are given actual digits, such as:
- \(A = 2\),
- \(B = 5\),
- \(C = 8\).

Then:
\[
a_1 = 25
\]
\[
a_2 = 28
\]
\[
d = 8 - 5 = 3
\]

Explicit formula:

$$
a_n = 25 + (n - 1) \times 3
$$

which simplifies to:

$$
a_n = 25 + 3(n - 1)
$$

Practical Applications and Insights

Understanding how to derive the explicit formula from the first two terms is more than an academic exercise; it has tangible applications:

- Financial Planning: Calculating future payments or savings with consistent increments.
- Computer Science: Generating sequences or patterns programmatically.
- Engineering: Modeling systems with linear progression.

The key takeaway is the importance of accurately interpreting the initial terms. If the terms are represented as digit concatenations, as in the case of $A B$ and $A C$, the explicit formula hinges on the numeric interpretation of these expressions.

Conclusion: Mastering the Derivation Process

In summary, given the first term $A B$ and the second term $A C$ of an arithmetic sequence, the process to find the explicit formula involves:

1. Interpreting the terms algebraically, often as two-digit numbers: $a_1 = 10A + B$, $a_2 = 10A + C$.
2. Computing the common difference: $d = C - B$.
3. Formulating the explicit expression:

$$
a_n = a_1 + (n - 1)d = 10A + B + (n - 1)(C - B)
$$

This formula empowers you to generate any term in the sequence directly, fostering a deeper understanding of linear progression. Whether you're tackling purely mathematical problems or applying these principles in real-world contexts, mastering the derivation of explicit formulas from initial terms is an invaluable skill that bridges theory and application seamlessly.

In essence, the explicit formula for an arithmetic sequence defined by the first term \(A B\) and second term \(A C\) is:

$$
\boxed{
a_n = 10A + B + (n - 1)(C - B)
}
$$

Armed with this knowledge, you can confidently analyze and construct arithmetic sequences grounded on minimal initial information, exemplifying the elegance and utility of mathematical reasoning.

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