The Image Of A(3,5) Under A Translation Is A'(6,1). Find The Image Of B(3,2) Under The Same Translation.

The Image Of A(3,5) Under A Translation Is A'(6,1). Find The Image Of B(3,2) Under The Same Translation.

Understanding how translations affect points in the coordinate plane is fundamental in coordinate geometry. When a figure or point undergoes a translation, its position shifts according to a specific rule, typically expressed as a vector. In this article, we will explore how to determine the image of a point under a translation, given the image of another point, and apply this knowledge to find the image of a second point. This concept is vital for students studying geometry, mathematics educators, and anyone interested in the geometric transformations.

What Is a Translation in Coordinate Geometry?

Definition of a Translation

A translation is a type of rigid motion in coordinate geometry that slides every point of a figure or a point in the plane by the same distance in a given direction. It preserves the size, shape, and orientation of the figures, making it a fundamental transformation in Euclidean geometry.

Translation as a Vector

Mathematically, a translation can be represented by a vector $\vec{v} = (h, k)$, where:

- h is the horizontal shift (positive for rightward, negative for leftward).
- k is the vertical shift (positive for upward, negative for downward).

Applying this translation to a point $P(x, y)$ results in a new point $P'(x', y')$:

$$
x' = x + h
$$
$$
y' = y + k
$$

Given Data and Objective

Data Provided:

- Original point $A(3, 5)$
- Its image after translation $A'(6, 1)$
- Another point $B(3, 2)$

Objective:

- Find the image of point $B(3, 2)$ under the same translation that maps A to A'.

Determining the Translation Vector

Step 1: Find the Change in Coordinates for A to A'

Given:

$$
A(3, 5) \quad \rightarrow \quad A'(6, 1)
$$

Calculate the differences:

$$
h = x_{A'} - x_{A} = 6 - 3 = 3
$$
$$
k = y_{A'} - y_{A} = 1 - 5 = -4
$$

This indicates that the translation vector $\vec{v}$ is:

$$
\boxed{\vec{v} = (3, -4)}
$$

This means every point undergoes a shift of 3 units to the right and 4 units downward.

Step 2: Confirm the Translation is Consistent

Since the same translation applies to all points, this vector applies uniformly. For the given points, the translation rule is:

\[
\text{For any point } (x, y), \quad (x + 3, y - 4)
\]

Applying the Translation to Find the Image of Point B

Step 1: Write Down the Coordinates of B

\[
B(3, 2)
\]

Step 2: Apply the Translation Vector

Using the translation rule:

\[
x' = x + 3 = 3 + 3 = 6
\]
\[
y' = y - 4 = 2 - 4 = -2
\]

Thus, the image of point B under the same translation is:

\[
\boxed{B'(6, -2)}
\]

Summary of the Process

Understanding how to find the image of a point under a translation involves these key steps:

1. Identify the initial and image points of a known figure or point.
2. Calculate the translation vector by subtracting the original point's coordinates from its image.
3. Apply the same translation vector to the second point to find its image.

This process is essential in solving geometric problems involving translations, as it allows for quick and accurate determination of points’ images under given transformations.

Visual Representation and Geometric Interpretation

Graphical Illustration of the Translation

Plotting the points on a coordinate plane helps visualize the translation:

- Point $A(3, 5)$ moves to $A'(6, 1)$.
- The movement vector $\vec{v} = (3, -4)$ shows a shift rightward by 3 units and downward by 4 units.
- Applying the same vector to $B(3, 2)$ results in $B'(6, -2)$.

Understanding the Movement

This visualization confirms that each point shifts identically, preserving the figure's shape and size but changing its position.

Practical Applications of Translation in Geometry

1. Geometric Constructions and Transformations

- Translations are used in creating congruent figures, tessellations, and tiling patterns.

- They are fundamental in computer graphics for moving objects without distortion.

2. Coordinate Geometry Problem Solving

- Determining images under translations simplifies complex geometric problems.
- Useful in coordinate proofs and geometric modeling.

3. Real-World Applications

- Engineering: Shifting components in design layouts.
- Navigation: Moving waypoints or wayfinding markers.
- Robotics: Programming movement paths.

Key Points to Remember

- The translation vector is found by subtracting the original point's coordinates from its image.
- The same translation applies to all points in the figure or set.
- Applying the translation involves adding the vector components to the point's coordinates.
- Visual aids and coordinate plotting enhance understanding of the translation process.

Additional Practice Problems

1. Given \(C(2, 4)\) maps to \(C'(5, 0)\), find the image of \(D(1, 3)\).
2. If \(E(7, -2)\) maps to \(E'(10, 3)\), what is the image of \(F(4, -5)\)?
3. A triangle with vertices \(A(1, 2)\), \(B(3, 4)\), and \(C(5, 0)\) is translated to \(A'(4, 5)\), \(B'(6, 7)\), and \(C'(8, 3)\). Find the translation vector.

Conclusion

Understanding how to determine the image of a point under a translation is a fundamental skill in coordinate geometry. By calculating the translation vector based on known points and their images, students can efficiently find the images of other points. This process underscores the importance of vectors in geometric transformations and helps build a strong foundation for more advanced topics in mathematics, computer graphics, and engineering design.

Whether you're solving academic problems or applying these concepts in real-world scenarios, mastering translations and their properties equips you with essential tools for spatial reasoning and geometric analysis. Remember, every translation can be broken down into a simple vector shift, and applying this shift to any point yields its new position in the plane.

Frequently Asked Questions

What is the translation vector that maps A(3,5) to A'(6,1)?

The translation vector is (6 - 3, 1 - 5) = (3, -4).

How do you find the image of point B(3,2) under the same translation?

Add the translation vector (3, -4) to B(3,2): (3 + 3, 2 - 4) = (6, -2).

What is the image of B(3,2) under the translation that maps A to A'?

The image of B(3,2) is B'(6, -2).

If a point P(x, y) is translated by vector (a, b), what is its new position?

Its new position is (x + a, y + b).

Given A(3,5) maps to A'(6,1), what is the translation rule applied?

The rule is (x, y) → (x + 3, y - 4).

Is the translation from A to A' the same for any other point in the plane?

Yes, the same translation vector applies to all points, moving them consistently.

Can the image of B(3,2) under this translation be

outside the original coordinate plane?

No, since the translation involves adding finite values, the image remains within the coordinate plane.

How does understanding the translation help in geometric transformations?

It allows us to predict the movement of any point by applying the same translation vector, simplifying the process of mapping figures.

Additional Resources

The Image Of A(3,5) Under A Translation Is A'(6,1). Find The Image Of B(3,2) Under The Same Translation.

Introduction

In the realm of geometry and vector transformations, understanding how points move under various transformations is fundamental. One of the simplest yet most essential transformations is translation, which shifts every point of a figure or a coordinate system uniformly by a fixed vector. This operation preserves the shape and size of figures, making it a core concept in Euclidean geometry and various applications across mathematics, physics, and engineering.

In this article, we meticulously analyze a specific translation given by the movement of point A from (3,5) to (6,1). Using this information, we aim to determine the image of another point, B(3,2), under the same translation. This problem exemplifies the process of deducing a translation vector from known point mappings and applying it to find the images of other points.

Understanding Translations in the Coordinate Plane

Definition and Properties

A translation in the coordinate plane is a transformation that "slides" every point of a figure or a set of points by the same distance in a specified direction. It can be represented mathematically as:

$$
T_{(h,k)}(x,y) = (x + h, y + k)
$$

where (h,k) is the translation vector indicating how far and in which direction the points are moved.

Key properties of translation:

- Distance-preserving: It is an isometry.
- Angle-preserving: It maintains angles.
- Orientation-preserving: It does not flip or rotate figures.
- Uniform: All points are shifted equally.

Deriving the Translation Vector

Step 1: Identify Known Point Movement

Given:

- Original point A: $(3, 5)$
- Image of A: $A' = (6, 1)$

The translation vector $\vec{v} = (h, k)$ can be inferred from this movement:

$$
h = x_{A'} - x_A = 6 - 3 = 3
$$
$$
k = y_{A'} - y_A = 1 - 5 = -4
$$

Hence, the translation vector:

$$
\boxed{\vec{v} = (3, -4)}
$$

This vector indicates that every point is shifted 3 units to the right and 4 units downward.

Step 2: Confirming the Translation Vector

To validate, check whether the movement of A matches the translation vector:

$$
(3, 5) \xrightarrow{\text{translation by } (3, -4)} (3 + 3, 5 - 4) = (6, 1)
$$

Yes, it aligns perfectly, confirming the translation vector as $(3, -4)$.

Applying the Same Translation to Find B(3,2)

Step 1: Express the Transformation

Given a point $B(x, y) = (3, 2)$, its image under the translation is:

$$
B' = (x + h, y + k) = (3 + 3, 2 - 4)
$$

Calculating:

$$
x' = 3 + 3 = 6
$$
$$
y' = 2 - 4 = -2
$$

Therefore, the image of point B under the same translation is:

$$
\boxed{B' = (6, -2)}
$$

Deeper Insights and Geometric Implications

The Nature of the Translation

The translation vector $(3, -4)$ indicates a movement:

- Horizontally: 3 units to the right
- Vertically: 4 units downward

This movement shifts the entire plane or figure accordingly, preserving distances and angles. Such a transformation is fundamental in congruence mappings and plays a critical role in proofs involving vector addition and coordinate geometry.

Visualizing the Transformation

Imagine plotting the points:

- A at (3, 5)
- B at (3, 2)

and then shifting both 3 units right and 4 units down. The point A lands at (6, 1), and B at (6, -2). Notice how the vertical shift affects B differently due to its initial position, but the process remains consistent for all points.

Broader Context and Applications

Relevance in Coordinate Geometry

This problem exemplifies how to determine the image of any point under a given translation when at least one point's movement is known. It underscores the importance of the translation vector as a fundamental concept in coordinate transformations.

Practical Applications

- Robotics and Navigation: Knowing how objects move under translations helps in path planning.
- Computer Graphics: Translations are used to move objects within scenes.
- Physics: Describing uniform motion involves translating coordinate systems.
- Engineering Design: Moving components or parts by specified vectors.

Summary and Key Takeaways

- Translation is a fundamental geometric transformation characterized by shifting all points by a fixed vector.
- Given the movement of a known point, the translation vector can be deduced straightforwardly.
- Applying the translation vector to other points allows us to find their images under the same transformation.
- In this specific problem, the translation that takes A(3,5) to A'(6,1) is (3, -4), leading to the image of B(3,2) as (6, -2).

Final Reflection

The process of solving this problem highlights the elegance and simplicity of translation in coordinate geometry. It emphasizes that understanding the movement of one point provides a blueprint for transforming entire figures or sets of points. Recognizing these patterns and translating them into algebraic expressions is essential for mastering geometric transformations, which serve as a foundation for more advanced topics like rotations, reflections, and dilations.

This case study demonstrates the importance of precise calculation and the power of vector operations in geometric contexts, reinforcing their significance across multiple scientific and mathematical disciplines.

References

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Note: This detailed analysis aims to provide a comprehensive understanding of how translations operate in the coordinate plane, using a specific example to elucidate the principles involved.

[The Image Of A 3 5 Under A Translation Is A 6 1 Find The Image Of B 3 2 Under The Same Translation]

The Image Of A 3 5 Under A Translation Is A 6 1 Find The Image Of B 3 2 Under The Same Translation

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