# The Number Of Combinations On N Items Taken 3 At A Time Is 6 Times The Number Of Combinations Of N Items

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Understanding the relationship between different combinations is fundamental in combinatorics, a branch of mathematics concerned with counting, arrangement, and combination of objects. The statement, "The number of combinations on N items taken 3 at a time is 6 times the number of combinations of N items," presents an intriguing mathematical relationship that invites further exploration. This article aims to clarify this concept thoroughly, providing detailed explanations, formulas, and practical examples to help you grasp the underlying principles of combinations, specifically focusing on the relationship between combinations of N items taken three at a time and the total number of combinations of N items.

## Fundamentals of Combinations

Before delving into the specific relationship, it is essential to understand the basic concepts of combinations and how they are calculated.

### What Are Combinations?

Combinations refer to the selection of items from a larger set where order does not matter. For example, choosing 3 fruits from a basket containing apples, oranges, bananas, and grapes involves combinations because the order in which you pick the fruits does not matter—apple, banana, orange is the same as orange, apple, banana.

### Combination Formula

The number of ways to choose r items from a set of n distinct items is given by the combination formula:

$$
C(n, r) = \frac{n!}{r!(n - r)!}
$$

where:
- $n!$ (n factorial) is the product of all positive integers up to n.
- $r!$ is the factorial of r.
- $(n - r)!$ is the factorial of (n minus r).

This formula calculates the total number of unique combinations without regard to order.

## Understanding the Relationship: C(n, 3) versus Total Combinations

The core of the discussion revolves around the comparison between $C(n, 3)$ – the number of combinations of n items taken three at a time – and the total number of combinations of n items, which is the sum over all possible r values:

$$
\text{Total combinations} = \sum_{r=0}^{n} C(n, r) = 2^n
$$

\]

This is a fundamental combinatorial identity, asserting that the total number of subsets of an n-element set is \( 2^n \).

The statement suggests that:

\[
C(n, 3) = 6 \times \text{(some total number of combinations)}
\]

which implies a specific proportionality relationship. To clarify, we need to analyze the precise mathematical relationship between \( C(n, 3) \) and \( 2^n \).

## Deriving the Relationship: When Does \( C(n, 3) = 6 \times 2^m \)?

Let’s analyze the actual formulas:

\[
C(n, 3) = \frac{n!}{3!(n - 3)!} = \frac{n(n-1)(n-2)}{6}
\]

and the total number of all possible subsets of an n-item set:

\[
2^n
\]

The question becomes: For which values of n does the following hold?

$$
C(n, 3) = 6 \times 2^k
$$

for some integer k.

Rearranged:

$$
\frac{n(n-1)(n-2)}{6} = 6 \times 2^k
$$

Multiply both sides by 6:

$$
n(n-1)(n-2) = 36 \times 2^k
$$

This equation indicates that for certain values of n and k, the triple product $n(n-1)(n-2)$ must be a multiple of 36, specifically 36 times a power of two.

Key observation: To find specific n satisfying this condition, we analyze the divisibility and factorization properties.

## Practical Calculation and Examples

Let's evaluate $C(n, 3)$ for small n:

- For $n=3$:

$$
C(3, 3) = 1
$$

Total subsets:

$$
2^3 = 8
$$

- For $n=4$:

$$
C(4, 3) = \frac{4 \times 3 \times 2}{6} = 4
$$

Total subsets:

$$
2^4 = 16
$$

- For $n=6$:

$$
C(6, 3) = \frac{6 \times 5 \times 4}{6} = 20
$$

Total subsets:

$$
2^6 = 64
$$

Now, check if $C(n, 3)$ is six times some total:

- For $n=4$:

$$
C(4, 3) = 4
$$

Is 4 equal to 6 times some number? No, because $6 \times 1 = 6$, $6 \times 0.66$, etc.

- For $n=6$:

$$
20 \neq 6 \times 64
$$

Similarly, 20 is not six times 64.

Hence, the initial statement is likely a specific case or a misinterpretation unless the phrase refers to the relationship between individual combinations and total combinations under specific conditions.

## Interpreting the Original Statement Correctly

The phrase, "The number of combinations on N items taken 3 at a time is 6 times the number of combinations of N items," may be understood differently. It could be paraphrased as:

- The number of 3-element combinations from a set of N items is six times the total number of combinations of N items taken 1 at a time.

Let's verify this hypothesis.

Recall:

$$
C(n, 1) = n
$$

$$
C(n, 3) = \frac{n(n-1)(n-2)}{6}
$$

Set:

$$
C(n, 3) = 6 \times C(n, 1)
$$

which gives:

$$
\frac{n(n-1)(n-2)}{6} = 6 \times n
$$

Multiply both sides by 6:

$$
n(n-1)(n-2) = 36 n
$$

Divide both sides by n (assuming $n \neq 0$):

$$
(n-1)(n-2) = 36
$$

Now, expand the left:

$$
n^2 - 3n + 2 = 36
$$

Bring all to one side:

$$
n^2 - 3n + 2 - 36 = 0
$$

$$
n^2 - 3n - 34 = 0
$$

Solve quadratic:

$$
n = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-34)}}{2}
$$

$$
n = \frac{3 \pm \sqrt{9 + 136}}{2}
$$

$$
n = \frac{3 \pm \sqrt{145}}{2}
$$

\]

Since \( \sqrt{145} \) is approximately 12.04, the solutions are:

\[
n \approx \frac{3 + 12.04}{2} \approx 7.52
\]
\[
n \approx \frac{3 - 12.04}{2} \approx -4.52
\]

Only positive integer solutions make sense in the context of combinations:

- \( n \approx 7.52 \) — not an integer
- \( n \approx -4.52 \) — discard

Thus, no integer value of n satisfies this exact relationship.

Conclusion: The initial statement might refer to a specific mathematical identity or a particular case, possibly involving other combinatorial relationships, or it could be a conceptual statement rather than an exact equality.

## Practical Applications and Implications

Understanding the relationships between different types of combinations has significant applications in various fields such as probability theory, statistics, computer science, and operational research.

### Applications in Probability and Statistics

- Calculating the likelihood of selecting specific combinations in random sampling.
- Designing experiments where the number of possible groupings influences the outcome.

### Applications in Computer Science

- Algorithms involving subset generation.
- Analyzing complexity when dealing with combinatorial structures.

### Applications in Operations Research

- Resource allocation problems.
- Optimization of selection processes in logistics.

## Key Takeaways and Summary

- The number of combinations $C(n, r)$ is calculated using the factorial formula.
- The total number of subsets of an n-element set is $2^n$.
- The relationship between $C(n, 3)$ and total combinations depends on specific values of n.
- The initial statement suggests a proportionality or a special case, but does not hold as a general rule for all n.
- Understanding these relationships helps in problem-solving across various disciplines.

## Final Thoughts

While the statement "The number of combinations on N items taken 3 at a time is

## Frequently Asked Questions

### What does the equation C(n, 3) = 6 C(n, 1) imply about the value of n?

It suggests a relationship between the number of ways to choose 3 items and 1 item from n, leading to an equation that can be solved for n.

### How do you express the combination formulas involved in this problem?

C(n, 3) = n(n-1)(n-2)/6 and C(n, 1) = n, which can be used to set up the equation n(n-1)(n-2)/6 = 6n.

### What is the process to solve for n in the equation C(n, 3) = 6 C(n, 1)?

Substitute the formulas into the equation, simplify, and solve the resulting algebraic equation for n.

### What are the possible integer solutions for n based on this combinatorial problem?

The solutions are n = 7 and n = 0, but since n must be at least 3 for C(n, 3) to be defined, n = 7 is the valid solution.

### Why is n = 7 the only relevant solution in this context?

Because for combinations to be meaningful in this context, n must be at least 3, and solving the equation yields n = 7 as the positive integer solution.

### What does this problem illustrate about the relationship between different combination counts?

It demonstrates how the number of combinations of different sizes relate mathematically and how solving such equations can identify specific values of n.

### Can this combinatorial relationship be generalized to other ratios besides 6 times?

Yes, similar equations can be formed for other ratios, leading to different values of n depending on the ratio used, illustrating the broader principle of combinatorial relationships.

## Additional Resources

The Number Of Combinations On N Items Taken 3 At A Time Is 6 Times The Number Of Combinations Of N Items

Mathematics often unveils fascinating relationships between seemingly simple quantities. Among these, combinations — a fundamental concept in combinatorics — reveal how items can be grouped and counted in diverse ways. One particularly intriguing assertion is that the number of combinations when selecting 3 items from a set of N elements is exactly six times the total number of combinations when choosing just 1 item from the same set. This relationship not only underscores the combinatorial structure but also invites a deeper exploration into how these counts are derived and what they signify.

In this article, we'll delve into the mathematical foundations of this statement, analyze the formulas involved, and explore the implications of such a relationship. We will examine how the binomial coefficient functions, interpret the combinatorial meanings, and clarify the conditions under which this relationship holds or appears to hold.

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## Understanding Combinations and Binomial Coefficients

To comprehend the statement fully, it's essential to first understand what combinations are and how they are quantified mathematically.

### What Are Combinations?

In combinatorics, a combination refers to a way of selecting items from a larger set where the order of selection does not matter. For example, choosing teams from a group of people, selecting lottery numbers, or forming subsets are all instances of combinations.

Key points about combinations:

- Order Does Not Matter: The combination {A, B, C} is considered the same as {C, B, A}.
- No Repetition: Usually, each item can only appear once in each combination.

### The Binomial Coefficient: The Formal Count

The number of combinations of N items taken k at a time is given by the binomial coefficient, denoted as:

\[

\binom{N}{k} = \frac{N!}{k!(N - k)!}

\]

where:

- \( N! \) (N factorial) is the product of all positive integers up to N.
- \( k! \) and \( (N - k)! \) are similarly factorial functions.

This formula counts the total number of unique subsets of size k that can be formed from a set of N distinct elements.

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## Dissecting the Relationship: Is the Statement Correct?

The core claim states:

> "The number of combinations on N items taken 3 at a time is 6 times the number of combinations of N items."

At face value, this suggests:

\[

\binom{N}{3} = 6 \times \binom{N}{1}

\]

But does this equality hold universally? Let's explore this step-by-step.

## Expressing Both Sides Using the Binomial Formula

Using the binomial coefficient formula:

$$
\binom{N}{3} = \frac{N!}{3!(N - 3)!}
$$

and

$$
\binom{N}{1} = \frac{N!}{1!(N - 1)!} = N
$$

since $1! = 1$, and the factorial terms simplify accordingly.

Now, rewrite the relationship:

$$
\frac{N!}{3!(N - 3)!} \stackrel{?}{=} 6 \times N
$$

Recall that $3! = 6$, so:

$$
\frac{N!}{6(N - 3)!} = 6N
$$

Multiply both sides by $6(N - 3)!$:

\[

N! = 36N (N - 3)!

\]

Express \( N! \) as:

\[

N! = N \times (N - 1)!

\]

Similarly, rewrite \( (N - 1)! \):

\[

(N - 1)! = (N - 1)(N - 2)!

\]

and so forth, but perhaps a more straightforward approach is to express \( N! \) directly in terms of \( (N - 3)! \):

\[

N! = N \times (N - 1) \times (N - 2) \times (N - 3)!

\]

Plugging this into the previous equation:

\[

N \times (N - 1) \times (N - 2) \times (N - 3)! = 36N (N - 3)!

\]

Divide both sides by \( (N - 3)! \) (assuming \( N \geq 3 \)):

\[

N \times (N - 1) \times (N - 2) = 36N

\]

Divide both sides by N (again assuming \( N \neq 0 \)):

\[

(N - 1) \times (N - 2) = 36

\]

This quadratic in N simplifies to:

\[

(N - 1)(N - 2) = 36

\]

\[

N^2 - 3N + 2 = 36

\]

\[

N^2 - 3N - 34 = 0

\]

Solve for N:

\[

N = \frac{3 \pm \sqrt{9 - 4 \times 1 \times (-34)}}{2} = \frac{3 \pm \sqrt{9 + 136}}{2} = \frac{3 \pm \sqrt{145}}{2}

\]

Since \( \sqrt{145} \) is irrational, the solutions are irrational numbers approximately:

\[

$$N \approx \frac{3 \pm 12.042}{2}$$

which gives:

- $N \approx \frac{3 + 12.042}{2} \approx 7.521$
- $N \approx \frac{3 - 12.042}{2} \approx -4.521$

Given that the number of items N must be a positive integer, the only approximate feasible value is around 7.52, which is not an integer.

Conclusion:

- The initial relationship $\binom{N}{3} = 6 \times \binom{N}{1}$ is not generally valid for all N.
- It only holds approximately when N is around 7.5, which is not possible since N must be an integer.

## Reinterpreting the Relationship

The above derivation shows that:

$$\binom{N}{3} = 6N$$

only holds approximately near $N \approx 7.5$.

But what if we frame the statement differently? Could the intended meaning involve other combinatorial relationships, or perhaps a different ratio?

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## Alternative Interpretations and Clarifications

Given the initial analysis, it seems unlikely that the original statement is universally valid as written. However, some interpretations might shed light on similar relationships.

## Possible Misinterpretation: Comparing Different Combinatorial Counts

One common misconception is confusing the relationship between permutations and combinations or misreading ratios.

For example:

- The number of permutations of N items taken 3 at a time is:

$$
P(N, 3) = \frac{N!}{(N - 3)!} = N \times (N - 1) \times (N - 2)
$$

- The number of combinations is:

$$
\binom{N}{3} = \frac{N!}{3!(N - 3)!}
$$

Notice that:

$$
P(N, 3) = \binom{N}{3} \times 3!
$$

Similarly, the number of combinations of N items taken 1 at a time is:

$$
\binom{N}{1} = N
$$

and the relation between permutations and combinations is:

$$
P(N, 3) = \binom{N}{3} \times 6
$$

which is consistent with the factorial relationships.

Key insight: The number of permutations of 3 items from N is 6 times the number of combinations of 3 items from N:

$$
P(N, 3) = 6 \times \binom{N}{3}
$$

and

$$
P(N, 1) = N = \binom{N}{1}
$$

But note: The statement involving "6 times the number of combinations of N items" aligns with permutations, not combinations.

Rephrased, the relationship is:

$$
P(N, 3) = 6 \times \binom{N}{3}
$$

and

$$
\binom{N}{3} = \frac{P(N, 3)}{6}
$$

Similarly,

$$
\binom{N}{1} = N
$$

which leads us to see that:

$$
P(N, 3) = 6 \times \binom{N}{3}
$$

and the ratio between permutations and combinations for 3 items is always 6, which reflects the factorial $3!$.

---

## Implications and Practical Applications

Understanding these relationships is vital in fields ranging from probability theory to statistical mechanics and computer science.

### Probability and Statistics

- Sampling without replacement: Calculating the likelihood of certain combinations.

- Permutation-based problems: When order

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