The Perimeter Of A Rectangle Is To Be Between 140 And 220 Inches. FindThe Range Of Values For Its Length

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Understanding the relationship between the dimensions of a rectangle and its perimeter is fundamental in geometry. When given constraints on the perimeter, such as it being between specific values, it becomes important to determine the possible range of lengths and widths that satisfy this condition. This article aims to explore how to find the range of possible lengths of a rectangle when its perimeter is constrained between 140 and 220 inches, providing a detailed step-by-step explanation, relevant formulas, and practical examples.

Understanding the Perimeter of a Rectangle

What Is the Perimeter?

The perimeter of a rectangle is the total distance around the rectangle. It is calculated by summing the lengths of all four sides. Since opposite sides of a rectangle are equal, the perimeter formula simplifies to:

Perimeter (P) = 2 × (Length (L) + Width (W))

where:
- L = length of the rectangle
- W = width of the rectangle

Given Constraints on Perimeter

In this specific problem, the perimeter is constrained to be between 140 inches and 220 inches. Mathematically, this is expressed as:

$140 \leq P \leq 220$

Substituting the formula for perimeter:

$140 \leq 2 \times (L + W) \leq 220$

This inequality provides the basis for finding the possible range of the length, given certain conditions.

Deriving the Range of Lengths

Step 1: Expressing in Terms of Length and Width

Starting with the inequality:

$140 \leq 2(L + W) \leq 220$

Divide all parts of the inequality by 2:

$70 \leq L + W \leq 110$

This simplifies the problem to analyzing the sum of length and width.

Step 2: Understanding the Relationship Between Length and Width

Since both L and W are positive (a rectangle cannot have negative dimensions), and the sum of L and W must fall within the range [70, 110], we can analyze possible values for L.

If we fix a value of L, then W must satisfy:

$70 - L \leq W \leq 110 - L$

Furthermore, because W must be positive:

$W > 0$

Similarly, L must be positive:

$L > 0$

These conditions define the feasible values of L and W.

Step 3: Finding the Range of Lengths

To find the possible range of L, consider the constraints:

- L must be positive
- W must be positive
- W = sum - L, where sum is between 70 and 110

So:

$W = S - L$, where $70 \leq S \leq 110$

Since $W > 0$:

$S - L > 0 \Rightarrow L < S$

But S varies between 70 and 110, so for any fixed L:

- To ensure $W > 0$, L must be less than the maximum S, which is 110
- To ensure W is positive for the minimum S (70), L must be less than 70

Similarly, for L to be positive:

$L > 0$

Now, to cover all possible cases, L must satisfy:

$0 < L < 70$ (from the lower sum limit)

and

$L < 110$ (from the upper sum limit)

But since L cannot exceed 70 to keep W positive when S is at its minimum (70), the overall feasible range for L is:

$0 < L < 70$

In other words, the length L can be any positive value less than 70 inches.

Practical Interpretation and Summary

Based on the derivation, the possible lengths of the rectangle, given the perimeter constraint between 140 and 220 inches, are all positive values less than 70 inches. But to provide a more comprehensive understanding, let's analyze specific cases and bounds.

Case 1: Minimum Perimeter (140 inches)

When $P = 140$:

$140 = 2(L + W)$

which simplifies to:

$L + W = 70$

For $W > 0$:

$L < 70$

and for $L > 0$:

$W = 70 - L > 0 \Rightarrow L < 70$

Thus, the length can be any positive value less than 70 inches, with the width adjusting accordingly.

Case 2: Maximum Perimeter (220 inches)

When $P = 220$:

$220 = 2(L + W)$

which simplifies to:

$L + W = 110$

Similarly, for $W > 0$:

$L < 110$

and for $L > 0$:

$W = 110 - L > 0 \Rightarrow L < 110$

But since the maximum sum $L + W$ is 110, and W must be positive, the maximum L is just under 110 inches. However, considering the earlier constraints, the practical upper limit for L, when the perimeter is at its maximum, is just under 110 inches.

Important Note: Since the problem specifies that the perimeter is between 140 and 220 inches, the range of L is from just above 0 to less than 70 inches when considering the minimum perimeter, and up to just below 110 inches when considering the maximum perimeter.

Summary of the Range of Lengths

- For the perimeter to be at least 140 inches, the length L must be less than 70 inches.
- For the perimeter to be at most 220 inches, the length L can be as high as just below 110 inches, provided the width remains positive.

However, to satisfy both constraints simultaneously, the most practical range for the length L of the rectangle is:

$0 < L < 70$ inches

since any length less than 70 inches can be paired with an appropriate width to produce a perimeter within the specified range.

Additional Considerations and Real-World Applications

Designing a Rectangle Within Given Perimeter Limits

Suppose you are designing a garden, a room, or a display area with a maximum and minimum perimeter. Understanding the relationship outlined above allows you to choose dimensions that fit within these constraints.

Steps to determine feasible dimensions:
1. Choose a length L within the range (0, 70).
2. Calculate the corresponding width W = (S - L), where S varies between 70 and 110.
3. Ensure W remains positive.

This flexibility allows for a variety of configurations, accommodating different design needs.

Practical Example

Imagine you want the length of your rectangle to be 60 inches. What is the possible range of widths and perimeters?

- For L = 60 inches:

Perimeter bounds:

$70 \leq L + W \leq 110$

$\Rightarrow 70 \leq 60 + W \leq 110$

$\Rightarrow 10 \leq W \leq 50$

- When W = 10 inches:

Perimeter = 2(60 + 10) = 2(70) = 140 inches (minimum perimeter)

- When W = 50 inches:

Perimeter = 2(60 + 50) = 2(110) = 220 inches (maximum perimeter)

Thus, with a length of 60 inches, the width can vary from 10 to 50 inches, and the perimeter will stay within the specified range.

Conclusion

Determining the range of possible lengths for a rectangle with a perimeter constrained between 140 and 220 inches involves understanding the relationship between length and width, and how their sum influences the perimeter. By analyzing the inequalities and considering both the minimum and maximum perimeter cases, we find that the length can vary from just above 0 inches up to less than 70 inches, with the width adjusting accordingly to maintain the perimeter within the specified bounds.

This analysis not only provides mathematical clarity but also practical insights for designing rectangles within given perimeter constraints, whether for construction, landscaping, or other applications. Remember, always verify that the chosen dimensions satisfy all the conditions, including the positivity of width and length, to ensure realistic and feasible measurements.

Keywords: rectangle perimeter, range of length, geometric constraints, perimeter calculation, problem-solving, dimensions, practical applications

Frequently Asked Questions

What is the formula to find the perimeter of a rectangle?

The perimeter of a rectangle is given by the formula P = 2(length + width).

Given the perimeter is between 140 and 220 inches, how can we set up inequalities to find the length?

Let the length be L and the width be W. The inequalities are 140 < 2(L + W) < 220. Dividing all parts by 2 gives 70 < L + W < 110.

If the width is known, how do we find the range of possible lengths?

Subtract the width from the bounds: L > 70 - W and L < 110 - W, giving the range of L as (70 - W, 110 - W).

What assumptions are made if the width is not specified?

We assume the width W can be any positive value that makes the length L satisfy the perimeter constraints, so the range of L depends on possible values of W.

How can we express the range of length when the width is variable but positive?

The length L must be greater than 70 minus W and less than 110 minus W, for all W > 0, which means L is between just above 70 and just below 110, depending on W.

Is it possible to determine a specific range for the length without knowing the width?

No, because the length depends on the width; without a fixed width, we can only describe the range of possible lengths based on W.

Can the length be equal to the bounds of the range, such as exactly 70 or 110 inches?

Yes, if the conditions are inclusive; however, since the inequalities are strict (> and <), the length cannot be exactly 70 or 110 inches without additional information.

What is the overall strategy to find the range of lengths for the rectangle?

Express the perimeter inequalities, solve for the sum of length and width, and analyze how changing the width affects the possible lengths within the given perimeter range.

How does the variability of width influence the possible length values for the rectangle?

Since the perimeter depends on both length and width, changing the width shifts the feasible length values within the bounds determined by the perimeter constraints.

Additional Resources

Understanding the Range of Lengths for a Rectangle Based on Its Perimeter Constraints

When dealing with geometric figures, especially rectangles, one common problem involves determining the possible dimensions given certain conditions. A typical example is finding the range of possible lengths for a rectangle when its perimeter is constrained within a specific interval. In this case, the problem states:

The perimeter of a rectangle is to be between 140 inches and 220 inches. Find the range of values for its length.

This problem combines fundamental concepts of geometry and algebra, providing an excellent opportunity to explore how these mathematical tools work together to solve real-world-like scenarios.

Fundamentals of Rectangle Geometry

Before delving into the problem specifics, it’s essential to understand the basic properties of rectangles:

- Sides of a Rectangle: A rectangle has four sides, with opposite sides equal in length.
- Length and Width: Typically, the longer side is called the length, and the shorter side is called the width.
- Perimeter Formula: The perimeter P of a rectangle is calculated as:

$$
P = 2 \times (\text{length} + \text{width})
$$

This formula is fundamental to our analysis, as it relates the dimensions directly to the perimeter.

Understanding the Perimeter Constraints

Given the problem, the perimeter P must satisfy:

$$
140 \leq P \leq 220
$$

Using the perimeter formula:

$$
P = 2 \times (L + W)
$$

where:
- L = length of the rectangle
- W = width of the rectangle

Substituting into the inequalities:

$$
140 \leq 2 \times (L + W) \leq 220
$$

Dividing through by 2 to simplify:

$$
70 \leq L + W \leq 110
$$

This inequality provides a relationship between the length and width, constraining their sum to be within 70 and 110 inches.

Isolating the Length: Finding Its Range

The key goal is to express the possible values of L, the length, in terms of the width W. Since the problem asks specifically for the range of the length, we need to analyze how L can vary given the constraints.

Step 1: Express L in terms of W:

$$
L = (L + W) - W
$$

Given the inequality $70 \leq L + W \leq 110$, then:

$$
L \geq 70 - W
$$
$$
L \leq 110 - W
$$

Step 2: Recognize that the width W is a positive quantity (since a rectangle cannot have zero or negative width). So:

$$
W > 0
$$

Step 3: Determine the permissible values of L based on W. For each possible W, L must satisfy:

\[

70 - W \leq L \leq 110 - W
\]

Step 4: Establish bounds considering the physical constraints:

- Because \(L \) and \(W \) are positive:

\[
L > 0 \quad \text{and} \quad W > 0
\]

- For the inequalities to make sense (i.e., the bounds are positive), \(W \) must be less than the maximum value.

Determining the Range of Lengths Without Fixing Width

To find the range of possible lengths regardless of the specific width, consider the extreme cases:

Case 1: Maximize \(L \)

- To maximize \(L \), set \(W \) to its minimum, which is just above 0:

\[
W \rightarrow 0^+,
\]

then:

\[
L \leq 110 - W \rightarrow 110,
\]
\[
L \geq 70 - W \rightarrow 70.
\]

Since \(W \) approaches zero, the maximum possible \(L \):

\[
L_{\max} \approx 110
\]

and the minimum possible \(L \):

\[

$$
L_{\min} \approx 70
$$

Case 2: Minimize L

- To minimize L, set W to its maximum, which occurs when W approaches the value that still keeps L positive.

Suppose $W \rightarrow 110$, then:

$$
L \geq 70 - W \rightarrow 70 - 110 = -40,
$$

which is invalid because length cannot be negative. Therefore, the maximum W permissible is constrained by the physical reality that L must be positive:

$$
L \geq 0 \Rightarrow 70 - W \geq 0 \Rightarrow W \leq 70,
$$

Similarly, for the upper bound:

$$
L \leq 110 - W
$$

and since L must be positive, W must be less than 110.

Summary:

- The width W must satisfy:

$$
0 < W \leq 70
$$

- Correspondingly, the length L can vary:

$$
70 - W \leq L \leq 110 - W
$$

for each $W \in (0, 70]$.

Expressing the Range of Lengths Independently of W

To find the overall possible range of L, consider the extreme bounds:

- When W approaches 0 (the minimal width), L approaches:

$$
70 - 0 = 70 \quad \text{(minimum length)}
$$

- When W approaches 70 (maximal width), L approaches:

$$
70 - 70 = 0 \quad \text{(but length cannot be zero, so the minimum length is just above 0)}
$$

Similarly, for the maximum L:

- When W approaches 0, L approaches:

$$
110 - 0 = 110
$$

- When W approaches 70, L approaches:

$$
110 - 70 = 40
$$

Conclusion:

- The possible length range considering the entire feasible W is approximately:

$$
\boxed{
40 < L < 110
}
$$

- The lower bound (just above 40 inches) corresponds to the scenario where W is close to 70 inches, and the upper bound (up to 110 inches) occurs when W approaches zero.

Refining the Range: Physical and Practical Considerations

While the mathematical derivation suggests \(L \) can be anywhere between just above 40 inches and nearly 110 inches, practical constraints often influence the feasible dimensions of a rectangle:

- Minimum and maximum dimensions: For a rectangle, extremely narrow or elongated shapes may not be practical or desired.
- Design constraints: If the problem context involves a real object or construction, specific dimensions may be limited.

However, from a purely mathematical perspective, the range of possible lengths compatible with the perimeter constraints is:

\[
\boxed{
\text{Length } L \in (40, 110) \text{ inches}
}
\]

This means:

- The length can be just above 40 inches (approaching 40 from above).
- It can also approach 110 inches, but not equal to 110 unless the width is zero, which is not physically possible.

Implications and Practical Applications

Understanding the range of possible dimensions for a rectangle based on perimeter constraints has numerous applications:

- Design and manufacturing: Engineers and designers often need to ensure their products fit within certain size boundaries while maintaining specific perimeter or area requirements.

- Construction: When designing rooms, plots, or components, knowing the feasible dimensions helps in optimizing space usage and material costs.

- Educational purposes: Problems like this serve as excellent exercises in translating word problems into algebraic inequalities, reinforcing the link between geometry and algebra.

- Optimization problems: Sometimes, the goal is not just to find the possible range but to determine the optimal dimensions for strength, aesthetics, or function within these constraints.

Summary of Key Points

- The problem involves a rectangle with a perimeter between 140 and 220 inches.
- Using the perimeter formula \(P = 2(L + W) \), the constraints translate into \(70 \leq L + W \leq 110 \).
- The length \(L \) depends on \(W \), with inequalities \(70 - W \leq L \leq 110 - W \).
- Considering positive dimensions, the feasible width \(W \) ranges from just above 0 to 70 inches.
- Consequently, the possible length \(L \) can vary approximately between just above 40 inches and nearly 110 inches.
- The precise range of lengths is:

\[
\boxed{
\text{Length } L \in (

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