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Understanding probability distributions is fundamental in statistics and probability theory. One critical concept is the Probability Mass Function (PMF), which provides the probabilities of discrete random variables. In this article, we will analyze a specific PMF: $F(x) = (25 / (2x + 1))$ for $x = 0, 1, 2, 3, 4$. We will address three key questions: (a) How to find $P(X = 3)$, (b) How to compute $P(2x < 3.5)$, and (c) How to interpret P(X1), assuming it relates to the probability of a specific event involving X. This comprehensive guide aims to clarify the calculation process and deepen your understanding of discrete probability distributions.

Understanding the Given Probability Mass Function

Before diving into calculations, it is crucial to understand the nature of the PMF provided.

What is a Probability Mass Function (PMF)?

- The PMF assigns probabilities to each possible value of a discrete random variable.
- The sum of all probabilities in the PMF must equal 1.
- For a discrete variable X taking values x1, x2, ..., xn, the PMF f(x) satisfies:

\[

\sum_{i} f(x_i) = 1

\]

- The probability that X equals a specific value x is denoted as P(X = x).

Analyzing the Given PMF: F(x) = 25 / (2x + 1)

- The function F(x) = 25 / (2x + 1) is provided for x = 0, 1, 2, 3, 4.

- To qualify as a valid PMF, the sum of F(x) over these x values must be 1:

\[

\sum_{x=0}^4 F(x) = 1

\]

- First, verify whether the given function is normalized.

Verification of the PMF Normalization

Let's compute the sum of F(x) for x = 0 to 4:

- F(0) = 25 / (20 + 1) = 25 / 1 = 25
- F(1) = 25 / (21 + 1) = 25 / 3 □ 8.333
- F(2) = 25 / (22 + 1) = 25 / 5 = 5
- F(3) = 25 / (23 + 1) = 25 / 7 □ 3.571
- F(4) = 25 / (24 + 1) = 25 / 9 □ 2.778

Sum these values:

$$
25 + 8.333 + 5 + 3.571 + 2.778 \approx 44.682
$$

Since the total sum is approximately 44.682, not 1, the provided $F(x)$ does not represent a probability distribution as-is. To turn these into valid probabilities, we need to normalize each value:

$$
f(x) = \frac{F(x)}{\sum_{x=0}^4 F(x)} \approx \frac{F(x)}{44.682}
$$

Thus, the actual probability mass function is:

$$
f(x) = \frac{25 / (2x + 1)}{44.682}
$$

This normalization ensures the total probability equals 1.

Calculating P(X = 3)

Now, let's find $P(X = 3)$:

Step 1: Compute F(3)

- $F(3) = 25 / 7 \approx 3.571$

Step 2: Normalize F(3) to find P(X = 3)

- Total sum of F(x) ≈ 44.682
- Therefore,

$$
P(X=3) = f(3) = \frac{F(3)}{44.682} \approx \frac{3.571}{44.682} \approx 0.0799
$$

Answer: P(X=3) ≈ 0.0799

Calculating P(2x < 3.5)

Next, we evaluate the probability that 2x < 3.5.

Step 1: Determine the x values satisfying 2x < 3.5

- Solve for x:

$$
2x < 3.5 \Rightarrow x < 1.75
$$

- Since x is integer-valued (x=0,1,2,3,4), the x values satisfying this are:

$$
x = 0, 1
$$

Step 2: Sum the probabilities for x=0 and x=1

- Recall the normalized probabilities:

$$
f(0) = \frac{25 / 1}{44.682} \approx \frac{25}{44.682} \approx 0.5596
$$

$$
f(1) = \frac{8.333}{44.682} \approx 0.1864
$$

- Sum:

$$
P(2x < 3.5) = f(0) + f(1) \approx 0.5596 + 0.1864 = 0.746
$$

Answer: $P(2x < 3.5) \approx 0.746$

Interpreting P(X 1) — Clarification Needed

The notation P(X1) appears incomplete or possibly a typo. Typically, in probability contexts, notation may refer to:

- $P(X = x)$: Probability that the random variable X equals x.
- $P(X > x)$: Probability that X exceeds x.
- $P(X \in A)$: Probability that X falls within set A.

If "X1" refers to an individual observation or a specific value, clarification is needed. Assuming it might be a typo or shorthand for $P(X=1)$, then:

Calculating P(X=1)

- Recall F(1) = 8.333
- Normalize:

$$
P(X=1) = \frac{8.333}{44.682} \approx 0.1864
$$

Therefore, P(X=1) □ 0.1864

Summary of Key Results:

Calculation	Result	Explanation
P(X=3)	□ 0.0799	Probability that X equals 3 after normalization
P(2x < 3.5)	□ 0.746	Sum of probabilities for x=0 and x=1
P(X=1)	□ 0.1864	Probability that X equals 1

Conclusion

In this article, we've explored how to work with a discrete probability distribution specified by a given PMF. Since the initial function F(x) = 25 / (2x + 1) does not sum to 1 over the specified x values, normalization is essential to convert these into valid probabilities. After normalization, calculating specific probabilities such as P(X=3) and P(2x < 3.5) becomes straightforward.

Understanding how to handle such distributions is vital for statistical analysis, risk assessment, and data modeling. Remember always to verify that your PMF sums to 1 before performing probability calculations. Clarify any ambiguous notation, such as P(X1), to ensure accurate interpretation and computation.

Whether you're a student learning probability or a data professional modeling discrete variables, mastering the process of deriving and interpreting PMFs is a foundational skill that underpins many aspects of statistical reasoning.

Additional Resources:

- [Introduction to Probability Theory](https://www.khanacademy.org/math/statistics-probability/probability-library)
- [Discrete Probability Distributions](https://statisticsbyjim.com/probability/discrete-probability-distributions/)
- [Normalization of Probability Functions](https://en.wikipedia.org/wiki/Probability_mass_function)

Note: Always verify the domain and normalization of your PMF to ensure valid probability calculations.

Frequently Asked Questions

Given the probability mass function F(x) = 25(2x + 1) for x = 0, 1, 2, 3, 4, what is the value of P(X=3)?

First, verify that F(x) sums to 1: sum over x=0 to 4 of 25(2x + 1). Calculate: 25(1) + 25(3) + 25(5) + 25(7) + 25(9) = 25 + 75 + 125 + 175 + 225 = 625. Since total is 625, the pmf should be normalized. Therefore, P(X=3) = (25 (23 + 1)) / 625 = (25 7) / 625 = 175 / 625 = 0.28.

Using the pmf F(x) = 25(2x + 1), what is P(2x < 3.5)?

Solve for x: 2x < 3.5 => x < 1.75. Since x can only be 0, 1, 2, 3, 4, the relevant x values are 0 and 1. Compute P(X=0) and P(X=1): P(0) = 25(1)/625 = 25/625 = 0.04, P(1) = 25(3)/625 = 75/625 = 0.12. Sum: 0.04 + 0.12 = 0.16.

What is the probability P(X=1)?

P(X=1) = 25(21 + 1) / 625 = 25(3) / 625 = 75 / 625 = 0.12.

Calculate P(2X □ 6).

2X □ 6 implies X □ 3. The possible values are X=3 and X=4. P(X=3) = 175/625 = 0.28, P(X=4) = 25(9)/625= 225/625=0.36. Sum: 0.28 + 0.36= 0.64.

Find P(X □ 2) based on the pmf F(x) = 25(2x + 1).

P(X=0) = 25(1)/625 = 0.04, P(X=1)=75/625=0.12, P(X=2)=25(5)/625=125/625=0.20. Total: 0.04+0.12+0.20=0.36.

What is P(X □ 2)?

Since sum of probabilities for all x=0 to 4 is 1, P(X□2) = 1 - P(X=2). P(X=2)=125/625=0.20, so P(X□2)=0.80.

Determine P(X > 1) using the pmf.

X > 1 includes X=2, 3, 4. P(2)=125/625=0.20, P(3)=175/625=0.28, P(4)=225/625=0.36. Sum: 0.20+0.28+0.36=0.84.

What is the cumulative probability P(X □ 3)?

P(X=0)=0.04, P(1)=0.12, P(2)=0.20, P(3)=25(7)/625=175/625=0.28. Sum: 0.04+0.12+0.20+0.28=0.64.

Is the given function a valid probability mass function?

Sum of probabilities: (251+75+125+175+225)/625= (25+75+125+175+225)/625= 625/625=1, so yes, it is a valid pmf after normalization.

Additional Resources

In-Depth Analysis of the Probability Mass Function $F(x) = \frac{25}{2x + 1}$ for $x = 0, 1, 2, 3, 4$

Introduction

Probability Mass Functions (PMFs) are fundamental in the study of discrete random variables. They provide the probabilities that a discrete random variable takes specific values. When analyzing a PMF such as $F(x) = \frac{25}{2x + 1}$, it is essential to understand its properties, validity, and implications. This detailed review aims to thoroughly dissect the given PMF, interpret its meaning, and compute the specific probabilities requested:

- (a) $P(X=3)$
- (b) $P(2X < 3.5)$
- (c) $P(X \leq 1)$ (assuming the last part refers to the probability that X satisfies a certain condition, often a typo for $X \leq 1$)

Understanding the Given PMF

Definition and Domain

The probability mass function is specified as:

$$
F(x) = \frac{25}{2x + 1}
$$

for $x = 0, 1, 2, 3, 4$.

The domain is thus discrete and finite, consisting of these five points.

Validity of the PMF

To qualify as a PMF, the following conditions must be satisfied:

1. Non-negativity: $F(x) \geq 0$ for all x in the domain.
2. Sum to 1: $\sum_{x} F(x) = 1$.

Let's verify these conditions.

Verifying the Validity of the PMF

1. Non-negativity

Each $F(x)$ is positive because:

$$
2x + 1 > 0 \quad \text{for} \quad x \geq 0,
$$

and the numerator is 25, which is positive. So, all $F(x)$ are positive for the specified domain.

2. Sum of Probabilities

Calculate the sum:

$$
\sum_{x=0}^4 F(x) = \sum_{x=0}^4 \frac{25}{2x + 1}
$$

Compute each term:

- $x=0$: $F(0) = \frac{25}{2(0) + 1} = \frac{25}{1} = 25$
- $x=1$: $F(1) = \frac{25}{3} \approx 8.333$
- $x=2$: $F(2) = \frac{25}{5} = 5$
- $x=3$: $F(3) = \frac{25}{7} \approx 3.571$
- $x=4$: $F(4) = \frac{25}{9} \approx 2.778$

Sum:

$$
25 + \frac{25}{3} + 5 + \frac{25}{7} + \frac{25}{9}
$$

Expressed as fractions:

$$
25 + \frac{25}{3} + 5 + \frac{25}{7} + \frac{25}{9}
$$

To check if these sum to 1, we need to normalize the PMF:

$$
\text{Total sum} = 25 + \frac{25}{3} + 5 + \frac{25}{7} + \frac{25}{9}
$$

Calculating:

- Convert all to a common denominator or decimal:

\[
25 + 8.333 + 5 + 3.571 + 2.778 \approx 44.682
\]

Since the total sum is approximately 44.68, the probabilities do not sum to 1 as-is.

Conclusion: The given PMF \(F(x) = \frac{25}{2x+1}\) does not satisfy the total probability condition unless it is normalized.

Normalizing the PMF

To make the PMF valid, define:

\[
\tilde{F}(x) = \frac{F(x)}{S}
\]
where

\[
S = \sum_{x=0}^4 F(x) \approx 44.682
\]

Therefore, the normalized PMF:

\[

\boxed{

\hat{F}(x) = \frac{25/(2x+1)}{44.682}

}

\]

This ensures:

\[

\sum_{x=0}^4 \hat{F}(x) = 1

\]

Computing the Specific Probabilities

Now, with the normalized PMF, we can proceed to compute each requested probability.

(a) Calculating \(P(X=3)\)

Using the normalized PMF:

\[

P(X=3) = \hat{F}(3) = \frac{\frac{25}{2(3) + 1}}{S} = \frac{\frac{25}{7}}{44.682}

\]

Numerator:

\[

\frac{25}{7} \approx 3.571

\]

Therefore:

$$
P(X=3) \approx \frac{3.571}{44.682} \approx 0.0799
$$

Result:

$$
\boxed{
P(X=3) \approx 0.08
}
$$

(b) Calculating $P(2X < 3.5)$

First, interpret the inequality:

$$
2X < 3.5 \Rightarrow X < \frac{3.5}{2} = 1.75
$$

Since X is discrete and takes integer values:

$$
X \in \{0, 1\}
$$

Thus,

$$
P(2X < 3.5) = P(X=0) + P(X=1)
$$

Calculate each:

- $P(X=0)$:

$$
\hat{F}(0) = \frac{25/1}{44.682} = \frac{25}{44.682} \approx 0.559
$$

- $P(X=1)$:

$$
\hat{F}(1) = \frac{25/3}{44.682} = \frac{8.333}{44.682} \approx 0.186
$$

Sum:

$$
P(2X < 3.5) \approx 0.559 + 0.186 = 0.745
$$

Result:

$$
\boxed{
P(2X < 3.5) \approx 0.75
}
$$

\]

(c) Calculating \(P(X \leq 1)\)

This is similar to the previous calculation; the probability that \(X\) is 0 or 1.

\[
P(X \leq 1) = P(X=0) + P(X=1) \approx 0.559 + 0.186 = 0.745
\]

Result:
\[
\boxed{
P(X \leq 1) \approx 0.75
}
\]

Additional Insights and Considerations

1. Behavior of the Distribution

- The probabilities decrease as \(x\) increases, which is consistent with the denominators \(2x + 1\) increasing.
- The largest probability is at \(x=0\), which is typical for many discrete distributions that favor lower values.

2. Implications of the Normalization

- The normalization factor $S \approx 44.68$ significantly scales down the raw $F(x)$ values.
- Without normalization, the sum exceeds 1, violating the fundamental property of probabilities.
- This process highlights the importance of verifying the total sum of PMF values before use.

3. Potential Applications

- Such a distribution could model scenarios where probabilities are inversely related to a linear function of the discrete variable.
- For example, in systems where smaller values are more likely, and the likelihood decreases with increasing x.

Final Remarks

This detailed analysis underscores the crucial aspects of handling probability mass functions:

- Validation: Always verify whether the given PMF sums to 1; if not, normalization is necessary.
- Interpretation: Carefully interpret inequalities involving discrete variables.
- Calculation: Use precise calculations or fractions for accuracy, especially when dealing with probability sums.

By thoroughly understanding these principles, one can confidently analyze and compute probabilities from any discrete distribution, whether standard or custom-defined.

Summary of Results

Calculation	Approximate Value	Explanation

| \(P(X=3)\) | 0.08 | Probability at \(x=3\) after normalization. |

| \(P(2X<3.5)\) | 0.75 | Sum of probabilities at \(x=0,1\). |

| \(P(X \leq 1)\) | 0.75 | Same as above, probability \(X\) up to 1. |

Final Thoughts

Understanding the structure and properties of the PMF \(F(x)

The Probability Mass Function Is F X 25 2x 1 X 0 1 2 3 4 Find A P X 3 B P 2x Lt 3 5 C P X1

The Probability Mass Function Is F X 25 2x 1 X 0 1 2 3 4 Find A P X 3 B P 2x Lt 3 5 C P X1

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