The Product Of 2 Consecutive Positive Odd Integers Is 483. Write The Equation That Would Be Used To Find

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Introduction to the Problem

Mathematics often presents real-world and abstract problems that require translating words into algebraic expressions. One such intriguing problem involves finding two consecutive positive odd integers whose product equals 483. This problem not only tests your understanding of algebra but also enhances your problem-solving skills. In this article, we will explore how to formulate the appropriate equation to solve such a problem, interpret it, and find the solution step-by-step.

Understanding the Concept of Consecutive Positive Odd Integers

What Are Consecutive Odd Integers?

Consecutive odd integers are odd numbers that follow each other sequentially, with a difference of 2 between them. For example:

- 3 and 5
- 11 and 13
- 99 and 101

They are characterized by their oddness and their immediate succession in the sequence of odd numbers.

Properties of Consecutive Odd Integers

- They are separated by exactly 2 units.
- Both are odd numbers, which means they are not divisible evenly by 2.

- They can be expressed algebraically using a variable to represent the first integer, and then defining the second based on it.

Setting Up the Algebraic Representation

Defining Variables

To translate the problem into an algebraic equation, we need to assign a variable to represent the first odd integer. Let:

1. **Let x** be the first positive odd integer.

2. The second positive odd integer, being consecutive, would then be **x + 2**.

Expressing the Product

The problem states that the product of these two integers is 483. Therefore, the algebraic expression becomes:

$$x \quad (x + 2) = 483$$

The Equation To Find

The key equation that embodies the problem is:

$$x(x + 2) = 483$$

This is the primary equation we will analyze and solve to find the values of the integers.

Formulating the Equation Step-by-Step

Step 1: Write the algebraic expression

As established, the product of the two consecutive odd integers is:

$$x(x + 2) = 483$$

Step 2: Expand the expression

Distribute the x:

```
x^2 + 2x = 483
```

Step 3: Rearrange into standard quadratic form

Bring all terms to one side:

```
x^2 + 2x - 483 = 0
```

This quadratic equation is what we will solve to find the value(s) of x.

Solving the Quadratic Equation

Method 1: Factoring

Attempt to factor the quadratic:

```
x^2 + 2x - 483 = 0
```

Look for two numbers that multiply to -483 and add to 2.

- Factors of 483: 1 and 483, 3 and 161, 11 and 43
- Test sums with signs to find the pair that adds to 2

Considering 21 and -23:

```
21  (-23) = -483, and 21 + (-23) = -2
```

Since the sum is -2, not 2, adjust signs accordingly. Let's try -21 and 23:

```
-21  23 = -483, and -21 + 23 = 2
```

This matches our middle coefficient. So, the factors are:

```
(x - 21)(x + 23) = 0
```

Method 2: Applying the Quadratic Formula

If factoring is challenging, the quadratic formula offers a straightforward solution:

```
x = [-b ± √(b^2 - 4ac)] / 2a
```

Where a = 1, b = 2, c = -483. Plugging in these values:

```
x = [-2 ± √(2^2 - 41(-483))] / (21)
```

```
x = [-2 ± √(4 + 1932)] / 2
```

$$x = [-2 \pm \sqrt{1936}] / 2$$

$$x = [-2 \pm 44] / 2$$

Now, compute both solutions:

- $x = (-2 + 44) / 2 = 42 / 2 = 21$
- $x = (-2 - 44) / 2 = -46 / 2 = -23$

Determining the Valid Solution

Considering the Positivity and Oddness

The problem specifies positive odd integers, so we discard the negative solution.

Thus, the first integer is:

$$x = 21$$

And the second integer is:
$x + 2 = 23$

Verification

Check the product:

$$21 \quad 23 = 483$$

which confirms our solution is correct.

Conclusion and Final Answer

Summary of the Process

1. Identify variables to represent the integers.
2. Express the relationship between the integers algebraically.
3. Formulate the quadratic equation.
4. Solve the quadratic equation using factoring or the quadratic formula.
5. Choose the positive solution based on the problem's context.

6. Verify the solution by substituting back into the original problem.

The Final Answer

The two positive odd integers whose product is 483 are **21 and 23**. The equation used to find these integers was:

$$x(x + 2) = 483$$

which expanded to:

$$x^2 + 2x - 483 = 0$$

and upon solving, yielded $x = 21$ (positive odd integer), confirming our solution.

Additional Applications

This approach can be applied to similar problems involving:

- Finding consecutive integers with other properties (even, multiples).
- Solving for the product or sum of integers with specific differences.
- Modeling real-world scenarios such as budgeting, scheduling, or geometric problems.

Final Thoughts

Transforming word problems into algebraic equations is a foundational skill in mathematics. Recognizing the pattern of consecutive odd integers, setting up the correct variables, and solving the resulting quadratic are essential techniques. Practice with various numbers and conditions will enhance problem-solving proficiency and deepen understanding of algebraic concepts.

Frequently Asked Questions

What is the general form of two consecutive positive odd integers?

If the first odd integer is n, the next consecutive odd integer is n + 2.

How do you set up an equation to find two consecutive odd integers whose product is 483?

Let the first odd integer be n; then the second is n + 2. The product is n(n + 2) = 483.

What quadratic equation represents the problem of finding two consecutive odd integers with a product of 483?

n(n + 2) = 483, which simplifies to n^2 + 2n - 483 = 0.

How can we solve for n in the equation n^2 + 2n - 483 = 0?

Use the quadratic formula: n = [-b ± sqrt(b^2 - 4ac)] / 2a, where a=1, b=2, c=-483.

What are the steps to find the actual integers once n is determined?

Solve for n using the quadratic formula, then add 2 to n to find the second integer.

Are both solutions from the quadratic formula valid for the problem?

Only the positive odd integer solution is valid since the problem specifies positive integers.

What is the significance of the equation n^2 + 2n - 483 = 0 in solving the problem?

It models the relationship between the two integers, allowing us to find their values algebraically.

Can you verify the solution after finding the two integers?

Yes, by multiplying the two integers to check if the product equals 483.

Additional Resources

Mathematics: Unlocking the Mystery of Consecutive Odd Integers and Their Product

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Introduction: The Fascination with Consecutive Odd Integers

Mathematics often presents intriguing puzzles that challenge our reasoning and problem-solving skills. One such captivating problem involves understanding the relationship between consecutive odd integers and their products. These problems are not just abstract exercises; they serve as excellent tools to sharpen algebraic thinking, develop logical reasoning, and appreciate the beauty of patterns within numbers.

Imagine being handed a scenario where the product of two consecutive positive odd integers equals a specific number—say, 483. The question naturally arises: How can we formulate an equation to find these integers? This article delves into this problem, exploring the underlying principles, the step-by-step process of deriving the equation, and the methods to solve it efficiently.

Understanding the Problem: Key Concepts and Definitions

Before jumping into the mathematics, it's essential to clarify some key concepts:

- Positive integers: Whole numbers greater than zero (1, 2, 3, ...).
- Odd integers: Integers not divisible by 2 (1, 3, 5, ...).
- Consecutive odd integers: Odd integers that follow each other in sequence, with a difference of 2 between them (e.g., 3 and 5, 7 and 9).

The problem specifies:

- The integers are positive (greater than zero).
- They are consecutive odd integers.
- Their product is 483.

Representing Consecutive Odd Integers Algebraically

The core challenge is translating the word problem into a mathematical expression. To do this, we need a variable representation that captures the essence of "consecutive positive odd integers."

Let's define:

- First odd integer: n
- Next consecutive odd integer: $n + 2$

Why $n + 2$? Because odd integers follow a pattern where each successive odd number is 2 greater than the previous one. For example:

- 3 and 5: 3 + 2 = 5
- 7 and 9: 7 + 2 = 9

This pattern holds universally for consecutive odd integers.

Note: Since the integers are positive, we will consider $n \geq 1$, but the specific value will be determined as part of solving the equation.

Formulating the Equation: Step-by-Step Approach

Now, the goal is to express the condition: "The product of these two integers is 483." Using the variables:

$$
\text{Product} = n \times (n + 2) = 483
$$

This leads to the algebraic equation:

$$
n(n + 2) = 483
$$

which simplifies to:

$$
n^2 + 2n = 483
$$

To make it a standard quadratic form, subtract 483 from both sides:

$$
n^2 + 2n - 483 = 0
$$

This is the key quadratic equation that we will analyze and solve to find the values of n.

Solving the Quadratic Equation

The quadratic equation:

$$
n^2 + 2n - 483 = 0
$$

can be approached using various methods: factoring, completing the square, or the quadratic formula. Given that the coefficients are not straightforward factors, the quadratic formula provides a reliable approach:

$$
n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:

- $a = 1$
- $b = 2$
- $c = -483$

Calculating the discriminant:

$$
D = b^2 - 4ac = (2)^2 - 4 \times 1 \times (-483) = 4 + 1932 = 1936
$$

Since the discriminant is positive and a perfect square (since $44^2 = 1936$), the solutions will be real and rational.

Finding the square root of the discriminant:

$$
\sqrt{1936} = 44
$$

Applying the quadratic formula:

$$
n = \frac{-2 \pm 44}{2}
$$

This gives two solutions:

1. $n = \frac{-2 + 44}{2} = \frac{42}{2} = 21$
2. $n = \frac{-2 - 44}{2} = \frac{-46}{2} = -23$

Interpreting the Solutions

Recall that n represents the first positive odd integer in the pair. Since the problem specifies positive integers, we discard the negative solution $n = -23$. The only valid solution is:

$$
n = 21

\]

The consecutive odd integer following 21 is:

\[
n + 2 = 23
\]

Thus, the two integers are 21 and 23.

Verifying the Solution

Always verify in real-world problems to ensure the solution makes sense:

\[
21 \times 23 = 483
\]

Calculating:

\[
21 \times 23 = (20 + 1) \times (20 + 3) = 20 \times 20 + 20 \times 3 + 1 \times 20 + 1 \times 3 = 400 + 60 + 20 + 3 = 483
\]

The verification confirms the correctness of the solution.

Summary of the Process in a Structured Form

1. Identify Variables: Let n be the first positive odd integer.
2. Express the Consecutive Integer: The next odd integer is $n + 2$.
3. Formulate the Equation: Set the product equal to 483:

$$
n(n + 2) = 483
$$

4. Expand and Rearrange:

$$
n^2 + 2n - 483 = 0
$$

5. Solve the Quadratic Equation: Use the quadratic formula to find n:

\[

$$
n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

6. Determine Valid Solution: Discard negative solutions if only positive integers are relevant.

7. Verify the Result: Confirm that the product matches the given value.

Additional Insights and Variations

While this problem focuses on two consecutive positive odd integers, similar approaches can be extended to:

- Consecutive even integers: Replace $n + 2$ with $n + 2$ where n is even.
- More than two integers: For example, the product of three consecutive odd integers, which would involve variables like n, $n + 2$, and $n + 4$, leading to more complex equations.
- Different products: Adjusting the value (e.g., 483) to explore how solutions change.

Understanding how to translate word problems into algebraic equations is a vital skill in mathematics, allowing for systematic problem-solving and reasoning.

Conclusion: The Power of Algebra in Problem Solving

This problem exemplifies how algebraic techniques serve as powerful tools to decode number puzzles. By representing consecutive odd integers with variables, forming an equation based on the problem's conditions, and solving systematically, we can uncover solutions that might seem hidden at first glance.

The key takeaways include:

- Recognizing patterns in number sequences.
- Formulating algebraic expressions from word problems.
- Applying quadratic formulas to solve equations.
- Validating solutions by substitution.

Whether you're a student enhancing your math skills or an enthusiast appreciating the elegance of number patterns, mastering these techniques opens doors to a deeper understanding of mathematics and its applications.

In essence, the equation used to find two consecutive

positive odd integers whose product is 483 is:

$$
\boxed{n^2 + 2n - 483 = 0}
$$

with the solution $n = 21$, leading to the integers 21 and 23 — a perfect illustration of algebra's power in solving real-world-like number puzzles.

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