The Radius Of A Circle Is 6 Kilometers. What Is The Area Of A Sector Bounded By A 126 Arc?Give The Exact

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Understanding the properties of circles and their sectors is fundamental in geometry, especially when dealing with real-world applications such as land measurement, engineering, and navigation. In this article, we will explore how to calculate the area of a sector given a circle's radius and the measure of the arc that bounds it. Specifically, we will analyze the case where the radius of the circle is 6 kilometers, and the sector is bounded by a 126-degree arc. Our goal is to determine the exact area of this sector using precise mathematical formulas.

Understanding the Basics of Circles and Sectors

Before delving into calculations, it's essential to understand some foundational concepts related to circles and their sectors.

What Is a Circle?

A circle is a set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center to any point on the circle is known as the radius, denoted by 'r'.

What Is a Sector of a Circle?

A sector of a circle resembles a "slice" of the circle, bounded by two radii and the arc between them. It is analogous to a pie slice and is characterized by:

- The radius (r) of the circle.
- The measure of the central angle (θ), usually in degrees or radians.
- The arc length (the curved part of the sector boundary).

Key Terms in Sector Calculations

- Radius (r): The distance from the center to any point on the circle.
- Central angle (θ): The angle subtended at the center of the circle by the sector.
- Arc length (L): The length of the curved boundary of the sector.
- Area of the sector (A): The surface area enclosed by the two radii and the arc.

Given Data and Objective

Let's specify the problem clearly:

- Radius of the circle, $r = 6$ kilometers
- Measure of the arc (central angle), $\theta = 126$ degrees

Objective: Calculate the exact area of the sector bounded by the 126-degree arc.

Mathematical Formulas for Sector Area

To compute the area of a sector, we use the following formulas depending on the units of the central angle:

1. Sector Area When the Central Angle Is in Degrees

$$
A = \frac{\theta}{360} \times \pi r^2
$$

Where:
- A = area of the sector
- θ = central angle in degrees
- r = radius of the circle

2. Sector Area When the Central Angle Is in Radians

$$
A = \frac{1}{2} r^2 \theta
$$

Where:
- θ = central angle in radians

For this problem, since the angle is given in degrees, we'll use the first formula.

Step-by-Step Calculation of the Sector Area

Let's proceed step by step to find the exact area.

Step 1: Confirm the Data

- Radius, $r = 6$ km
- Central angle, $\theta = 126^\circ$

Step 2: Convert Degrees to Radians (Optional)

While we can directly use degrees in the formula, sometimes converting to radians offers more flexibility.

Conversion formula:
$$
\text{Radians} = \frac{\pi}{180} \times \text{Degrees}
$$
Calculating:
$$
\theta_{radians} = \frac{\pi}{180} \times 126 = \frac{126\pi}{180} = \frac{7\pi}{10}
$$

However, since our area formula in degrees is straightforward, we'll proceed with degrees.

Step 3: Apply the Sector Area Formula

Using:
$$
A = \frac{\theta}{360} \times \pi r^2
$$
Plug in the known values:
$$
A = \frac{126}{360} \times \pi \times (6)^2
$$

Calculate each component:
- Simplify $\frac{126}{360}$:
$$
\frac{126}{360} = \frac{7}{20}
$$
- Compute r^2:
$$
6^2 = 36
$$
- Now, multiply:
$$
A = \frac{7}{20} \times \pi \times 36
$$

Simplify further:
$$
A = \frac{7}{20} \times 36 \pi

$$
A = \left(\frac{7 \times 36}{20}\right) \pi
$$

Calculate numerator:

$$
7 \times 36 = 252
$$

Divide:

$$
\frac{252}{20} = 12.6
$$

Thus, the exact area:

$$
A = 12.6 \pi \text{ square kilometers}
$$

Expressing the Exact Area of the Sector

The area of the sector bounded by a 126-degree arc in a circle with a radius of 6 km is:

$$
\boxed{
\textbf{Area} = 12.6 \pi \text{ km}^2
}
$$

This is the exact value in terms of π. If you prefer a numerical approximation, multiply by $\pi \approx 3.1416$:

$$
A \approx 12.6 \times 3.1416 \approx 39.56 \text{ km}^2
$$

However, for precise calculations, especially in mathematical contexts, leaving the answer in terms of π is preferable.

Additional Insights and Applications

Calculating the area of a sector has numerous practical applications:

- Land surveying: Determining the size of a pie-shaped plot.
- Engineering: Designing mechanical parts with curved sections.
- Navigation and GPS: Calculating the area covered within a certain angle and radius.
- Architecture: Planning curved structures or sections of circular designs.

Understanding how to derive the area in exact terms helps in accurate planning and avoids approximations that could lead to errors in critical applications.

Summary and Key Takeaways

- The area of a sector depends on the radius and the angle subtended at the center.
- When the angle is in degrees, the formula $A = \frac{\theta}{360} \times \pi r^2$ is straightforward.
- For a circle with radius 6 km and a 126-degree arc, the exact sector area is $12.6 \pi \text{ km}^2$.
- Numerical approximations provide a close estimate but are less precise than the symbolic form.

Final Remarks

Calculating the area of a sector is a fundamental skill in geometry with broad applications. By understanding the relationship between the radius, the central angle, and the sector area, you can solve various problems involving circles with confidence. Remember to always pay attention to units and conversions, especially between degrees and radians, to ensure accuracy in your calculations.

Whether you're a student, engineer, or land surveyor, mastering these formulas enhances your ability to work with circular segments precisely and efficiently.

Frequently Asked Questions

What is the formula to find the area of a sector in a circle?

The area of a sector is given by $(\theta/360) \times \pi r^2$, where θ is the central angle in degrees and r is the radius.

Given a circle with radius 6 km and a sector with a 126° arc,

how do you calculate the area of the sector?

Use the formula: $(\theta/360) \times \pi r^2$. Plug in $\theta = 126°$ and $r = 6$ km to find the exact area.

What is the value of π used in calculating the area of a circle sector?

The value of π is approximately 3.1416, but for exact calculations, keep it as π in the formula.

How do you convert the central angle of 126° into a fraction of the full circle?

Divide 126° by 360°, resulting in $126/360 = 7/20$ of the full circle.

What is the exact area of the sector with a 126° arc in a circle of radius 6 km?

The exact area is $(126/360) \times \pi \times 6^2 = (7/20) \times \pi \times 36 = (7/20) \times 36\pi = 126/20\ \pi = 6.3\pi$ square kilometers.

Can the area of the sector be expressed in terms of π?

Yes, the exact area is 6.3π square kilometers.

What is the approximate numerical value of the sector's area in square kilometers?

Using $\pi \approx 3.1416$, the area $\approx 6.3 \times 3.1416 \approx 19.78$ square kilometers.

Why is it important to keep π in the calculations for an exact answer?

Keeping π in the calculations ensures the result remains exact and not rounded prematurely, maintaining mathematical precision.

If the radius of a circle increases, how does the area of a sector change?

The area of a sector increases proportionally to the square of the radius, so increasing the radius results in a larger sector area.

What is the significance of knowing the sector's angle when calculating its area?

The sector's angle determines the proportion of the circle's total area that the sector occupies, directly affecting the area calculation.

Additional Resources

The Radius Of A Circle Is 6 Kilometers. What Is The Area Of A Sector Bounded By A 126° Arc? Give The Exact

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Introduction

Mathematics, particularly geometry, provides a foundational framework for understanding the physical and conceptual spaces around us. Among the fundamental shapes studied, the circle holds a special place due to its unique properties and widespread applications—ranging from navigation and engineering to art and architecture. A core component of circle geometry involves understanding sectors, which are portions of the circle enclosed by two radii and the corresponding arc.

This article delves into a classic problem: Given a circle with a radius of 6 kilometers, what is the exact area of the sector defined by a 126° arc? While seemingly straightforward, this problem encapsulates fundamental principles of circle geometry, including sector area calculations and angular measurements, which merit thorough exploration.

Background: Fundamental Concepts in Circle Geometry

Before addressing the problem directly, it is essential to review key concepts:

- Circle: The set of all points equidistant from a fixed point called the center.
- Radius (r): The distance from the center to any point on the circle.
- Circumference (C): The total distance around the circle, given by $C = 2\pi r$.
- Central Angle: The angle subtended at the center of the circle by a given arc.
- Arc: A continuous segment of the circle's circumference.
- Sector: A portion of the circle bounded by two radii and the arc between them.

Understanding these basic definitions is crucial for precise calculations and for interpreting the problem's parameters accurately.

The Given Data and Its Significance

The problem specifies:

- Radius (r): 6 kilometers
- Arc measure: 126 degrees

The goal is to determine the exact area of the sector defined by this arc.

Step-by-Step Approach to the Solution

To find the area of the sector, the following steps are necessary:

1. Convert the given arc measure into a proportion of the full circle.
2. Calculate the total area of the circle.
3. Determine the sector's area based on the proportion of the circle it represents.

Let's analyze each step thoroughly.

Step 1: Understanding the Central Angle and Its Proportion of the Circle

Since the sector is bounded by a 126° arc, the central angle of the sector is 126°.

- The total degrees in a circle = 360°.
- The fraction of the circle that the sector covers = 126° / 360° = 7 / 20.

This proportion will directly determine the sector's area relative to the entire circle.

Step 2: Calculating the Total Area of the Circle

The formula for the area of a circle is:

$$ A_{circle} = \pi r^2 $$

Given the radius:

$$ r = 6 \text{ km} $$

Thus,

$$ A_{circle} = \pi \times (6)^2 = 36\pi \text{ km}^2 $$

This exact form, involving π, preserves the precision needed for an exact answer.

Step 3: Calculating the Sector's Area

The area of a sector is proportional to its central angle relative to the full circle:

$$ A_{sector} = \left(\frac{\text{central angle}}{360^\circ} \right) \times A_{circle} $$

Substituting known values:

$$ A_{sector} = \frac{126^\circ}{360^\circ} \times 36\pi $$

Simplify the fraction:

$$ \frac{126}{360} = \frac{7}{20} $$

Therefore,

$$ A_{sector} = \frac{7}{20} \times 36\pi $$

Calculating:

$$ A_{sector} = \left(\frac{7 \times 36}{20} \right) \pi = \left(\frac{252}{20} \right) \pi $$

Simplify:

$$ \frac{252}{20} = \frac{63}{5} $$

Hence, the exact area of the sector is:

$$ \boxed{\frac{63}{5} \pi \text{ km}^2} $$

Final Expression and Interpretation

The exact area of the sector bounded by a 126° arc in a circle with a radius of 6 kilometers is:

$$
\boxed{\frac{63}{5} \pi \text{ km}^2}
$$

which, when expressed numerically (using $\pi \approx 3.14159$), approximates to:

$$
\frac{63}{5} \times 3.14159 \approx 39.478 \text{ km}^2
$$

However, providing the answer in terms of π preserves the mathematical exactness, which is often preferred in theoretical contexts.

Broader Implications and Applications

Understanding how to compute sector areas has multiple practical applications:

- Navigation and Geospatial Analysis: Determining the area covered by a certain angular segment, such as radar coverage or radio signal zones.
- Engineering and Design: Calculating material quantities for curved segments in construction.
- Physics and Astronomy: Estimating areas of celestial sectors or observational fields of view.
- Education and Pedagogy: Teaching geometrical concepts through real-world problems enhances spatial reasoning.

Additional Considerations

While this problem is straightforward, several related factors often complicate real-world calculations:

- Unit conversions: When measurements are in different units, conversions are essential.
- Non-uniform sectors: If the sector is not defined by a simple central angle or involves irregular shapes, more complex calculations are necessary.
- Approximation accuracy: Depending on application, the level of precision needed varies; sometimes, approximations are acceptable, but exact expressions are preferred in mathematics and engineering.

Conclusion

The problem of determining the area of a sector with a 126° arc in a circle of radius 6 kilometers exemplifies core principles of circle geometry. The exact area, expressed as $\frac{63}{5} \pi$ square kilometers, highlights the elegance of mathematical relationships and their applicability to real-world problems. Mastery of such calculations enhances our ability to analyze spatial domains, design precise structures, and interpret the geometric aspects of our environment.

Whether used in academic contexts, engineering projects, or navigation systems, understanding the derivation and significance of sector areas remains a vital component of mathematical literacy and spatial reasoning.

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