The Radius Of Convergence Of The Power Series Representation Of 6x F(x) = 7x + 11 Is R= 7 11 Select One:

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Understanding the radius of convergence of a power series is fundamental in mathematical analysis, especially when dealing with functions represented as infinite series. In this article, we will explore the radius of convergence associated with the power series representation of the given function $6x F(x) = 7x + 11$, analyze what the value $R=7$ signifies, and clarify the process of determining this radius. This exploration is crucial for students and professionals working in fields such as calculus, complex analysis, and mathematical modeling.

Introduction to Power Series and Radius of Convergence

What Is a Power Series?

A power series is an infinite sum of the form:

$$
\sum_{n=0}^{\infty} a_n (x - c)^n
$$

where:
- a_n are the coefficients,
- c is the center of the series,
- x is the variable.

Power series are a powerful tool to represent functions locally around a point c. They enable us to approximate complex functions using polynomials within a certain radius of convergence.

The Radius of Convergence Defined

The radius of convergence R of a power series is the distance from the center c within which the series converges to the function it represents. Outside this radius, the series diverges or does not represent the function accurately.

Mathematically, R can be determined using tests such as the ratio test or root test applied to the coefficients a_n:

$$
\frac{1}{R} = \limsup_{n \to \infty} |a_n|^{1/n}
$$

This value indicates the maximum extent around c where the series converges.

Analyzing the Given Function and Its Power Series

Given Equation and Its Implications

The given relation is:

$$
6x F(x) = 7x + 11
$$

which expresses a connection between $F(x)$ and the polynomial $7x + 11$.

To analyze the power series of $F(x)$, rewrite the relation:

$$
F(x) = \frac{7x + 11}{6x}
$$

This simplifies to:

$$
F(x) = \frac{7x}{6x} + \frac{11}{6x} = \frac{7}{6} + \frac{11}{6x}
$$

Nature of the Function $F(x)$

From the expression:

$$
F(x) = \frac{7}{6} + \frac{11}{6x}
$$

we observe that $F(x)$ is a rational function with a potential singularity at $x=0$. The function is defined everywhere except at $x=0$.

Key point: The power series expansion of $F(x)$ around a point c will depend on the location of c relative to the singularity at $x=0$.

Determining the Radius of Convergence

Identifying the Center of Expansion

Typically, the power series is expanded around a specific point c. For simplicity, assume the expansion is around $c=0$. The singularity at $x=0$ affects the convergence radius.

Calculating the Radius of Convergence

Since $F(x)$ has a singularity at $x=0$, the radius of convergence for any power series expansion around 0 is limited by the distance to the singularity:

- Distance from $c=0$ to the singularity at $x=0$ is zero, which implies the power series centered at 0 does not converge (it's not valid).

However, if the series is expanded around some $c \neq 0$, the radius of convergence is determined by the distance from c to the nearest singularity.

Suppose the series is expanded around a point $c \neq 0$. The singularity at $x=0$ is at a distance $|c - 0| = |c|$. Therefore:

$$
R = |c|
$$

In particular, if the expansion is centered at $c = 7$, then the radius of convergence is:

$$
R = 7
$$

which aligns with the value given in the question.

Interpreting the Value R=7

Significance of R=7

- The radius $R=7$ indicates that the power series representation of $F(x)$ centered at $c=7$ converges for all x such that:

$$
|x - 7| < 7
$$

- Geometrically, this means the series is valid within the interval $(0, 14)$ along the real axis, excluding the boundary points where convergence may fail.

Why Is This Important?

Knowing the radius of convergence helps in:
- Approximating the function accurately within the interval.
- Understanding the domain where the power series provides a reliable representation.
- Avoiding divergence or inaccuracies outside this interval.

Choosing the Correct Option

Given the options:
- $R = 7$
- $R = 11$
- Other options

And considering the analysis above, the correct choice based on the singularity at $x=0$ and the center at $c=7$ is:

\[
\boxed{R=7}
\]

This aligns with the standard interpretation of the radius of convergence for a rational function expanded around a point away from its singularities.

Conclusion

Understanding the radius of convergence for a power series involves analyzing the function's singularities and the point of expansion. In the case of the function \(F(x) \) derived from the relation \(6x F(x) = 7x + 11 \), the key singularity at \(x=0 \) determines the maximum interval where the power series converges when expanded around \(c=7 \).

The value \(R=7 \) reflects the distance from the center of expansion to the singularity, confirming the significance of the radius as a measure of the series' validity domain. Recognizing these concepts is essential for mathematicians and scientists who rely on series expansions for approximation, modeling, and solving complex problems.

In summary:
- Power series converge within a radius determined by the nearest singularity.
- For \(F(x) \) expanded around \(c=7 \), the radius is \(R=7 \).
- This informs us about the domain of validity of the series representation and ensures accurate usage within that interval.

By mastering these principles, you can better analyze functions and their series representations, leading to more precise mathematical modeling and problem-solving.

Frequently Asked Questions

What is the power series representation of the function 6x F(x) = 7x + 11?

First, solve for F(x): F(x) = (7x + 11) / 6x. To express this as a power series, you can manipulate it into a form suitable for expansion, typically involving geometric series or partial fractions.

How do you determine the radius of convergence for the power series of F(x)?

The radius of convergence is determined by identifying the singularities of the function and finding the distance from the expansion point (usually x=0) to the nearest singularity in the complex plane.

What are the singularities of the function F(x) = (7x + 11) /

6x?

The singularity occurs at x=0 because the denominator 6x equals zero there, which makes the function undefined.

Why is the radius of convergence R=7 or 11 in this problem?

The problem states R=7 or 11, which suggests the nearest singularity to the expansion point determines the radius. Since the singularity is at x=0, the radius depends on the domain considered and the distance to the next singularity.

How is the radius of convergence related to the singularities of a function?

The radius of convergence of a power series centered at a point is equal to the distance from that point to the nearest singularity of the function in the complex plane.

In this context, what does the option R=7 or R=11 imply about the function’s behavior?

It implies that the function's power series converges within a radius of either 7 or 11 units from the center, depending on the location of the nearest singularity relative to the expansion point.

How do you select the correct radius of convergence when multiple options are given, such as R=7 or R=11?

You analyze the distance to the nearest singularity from the expansion point; the smaller of the two options (7 or 11) that corresponds to this distance is the correct radius.

What is the significance of choosing the correct radius of convergence in power series expansions?

Choosing the correct radius ensures the power series accurately represents the function within that domain and converges to the function's true values.

Can the radius of convergence be infinite for this type of function?

Yes, if the function has no singularities in the complex plane (entire functions), the radius of convergence can be infinite. However, in this case, the presence of singularities at x=0 limits the radius.

Additional Resources

The Radius Of Convergence Of The Power Series Representation Of 6x F(x) = 7x + 11 Is R= 7 11 Select One:

Understanding the radius of convergence of a power series is fundamental in mathematical analysis, especially when working with functions represented as infinite series. In the context of the given function $6xF(x) = 7x + 11$, the question centers around determining the radius of convergence of its power series representation and understanding the implications of this radius, particularly whether it is $R = 7$ or $R = 11$. This review aims to explore these concepts thoroughly, breaking down the derivation, the underlying principles, and the significance of these values.

Introduction to Power Series and Radius of Convergence

A power series is an infinite sum of the form:

$$
\sum_{n=0}^{\infty} a_n (x - x_0)^n,
$$

where a_n are coefficients, x_0 is the center of the series, and x is the variable. Power series are crucial because they can represent many functions within their radius of convergence, which is the distance from the center x_0 within which the series converges.

Radius of convergence (R):
The radius of convergence defines the interval $(x_0 - R, x_0 + R)$ within which the power series converges to the function it represents. Outside this interval, the series diverges, meaning it does not sum to the function.

Why is R important?
- It determines the domain where the power series provides a valid representation of the function.
- It helps in understanding the function's behavior and singularities.

Analyzing the Given Function: $6xF(x) = 7x + 11$

First, let's interpret the given expression properly. The equation:

$$
6xF(x) = 7x + 11,
$$

can be rearranged to express $F(x)$:

$$
F(x) = \frac{7x + 11}{6x}.
$$

This simplifies to:

$$
F(x) = \frac{7x}{6x} + \frac{11}{6x} = \frac{7}{6} + \frac{11}{6x}.
$$

Notice that $F(x)$ is a rational function with a potential singularity at $x=0$.

Power Series Representation of $F(x)$

To analyze the radius of convergence, we need to express $F(x)$ as a power series around a point, commonly around $x=0$ (Maclaurin series). The function:

$$
F(x) = \frac{7}{6} + \frac{11}{6x},
$$

can be expanded into a power series, but the presence of $\frac{1}{x}$ indicates a singularity at $x=0$. Therefore, the function cannot be expanded in a power series centered at $x=0$ that converges in a neighborhood containing $x=0$.

Alternatively, we can consider the function in a form suitable for expansion elsewhere or analyze the structure of the series for the rational part.

Key observations:

- The term $\frac{11}{6x}$ indicates a simple pole at $x=0$.
- The function is not analytic at $x=0$, so a power series centered at zero (Maclaurin series) does not exist for the entire function, but perhaps for the part $\frac{7}{6}$.

Understanding the Singularity and Its Impact on Radius of Convergence

The radius of convergence for a power series expansion about a point x_0 depends on the distance from x_0 to the nearest singularity of the function.

Singularities of $F(x)$:

- The function has a singularity at $x=0$ due to the $\frac{1}{x}$ term.
- If considering expansion around a point other than zero, the singularity's location relative to that point is crucial.

Suppose we attempt to expand $F(x)$ around some point $x_0 \neq 0$, say $x_0 = a$. The radius of convergence R will be the distance from a to the closest singularity, which in this case is at $x=0$.

Connection to the Given Options: R=7 or R=11

The options $R=7$ and $R=11$ suggest that the problem might involve the interval or distance related to these numbers, possibly arising from the coefficients or bounds of the series.

Potential interpretations:

- The numbers 7 and 11 could relate to the coefficients in the series or the distances to the nearest singularity.
- Since the original question mentions "Select One," and the expression involves constants 7 and 11, perhaps the radius of convergence is tied to these constants.

Analyzing the Constants 7 and 11 in the Context of the Series

Given the function:

$$
F(x) = \frac{7}{6} + \frac{11}{6x},
$$

the singularity at $x=0$ is at a distance zero from any point near zero, so the power series centered at zero cannot converge there.

However, if we consider the entire expression $6xF(x) = 7x + 11$, perhaps the problem is designed around a different interpretation—possibly involving the expansion of some related series or the coefficients.

Reconstructing the Power Series and Determining R

Suppose the problem involves the power series expansion of a related function, or perhaps the series expansion of $F(x)$ in a form that involves these constants.

Hypothetical scenario:

- Assume $F(x)$ is expanded around some point $x=a$, and the closest singularity determines the

radius \(R\).
- The constants 7 and 11 could relate to the coefficients in the series expansion, or perhaps to the bounds of the domain.

Key idea:

The radius of convergence \(R\) is determined by the distance from the expansion point to the nearest singularity.

Conclusion and Final Insights

Based on the analysis, the rational function \(F(x) = \frac{7}{6} + \frac{11}{6x}\) has a singularity at \(x=0\). If the power series is expanded around a point \(x=a \neq 0\), the radius of convergence is the distance from \(a\) to the nearest singularity, which is at zero.

Implications:

- If the series is expanded around \(x=0\), it diverges because of the singularity.
- If expanded around some \(x=a\), the radius \(R = |a - 0| = |a|\).

Given the options \(R=7\) or \(R=11\), and considering typical problem structures, the most straightforward interpretation is that the closest singularity is at a distance of 7 or 11 units from the expansion point, possibly related to the constants in the problem.

Therefore, the likely correct choice is R=7 or R=11, depending on the specific center of expansion and the problem's context.

Pros and Cons of Each Choice:

- R=7:
- Pros: If the expansion point is at \(x=0\), and the closest singularity is at \(x=-7\) or \(x=7\), then this is plausible.
- Cons: Without explicit information about the singularity's position, this remains speculative.

- R=11:
- Pros: Similarly, if the singularity is at \(x=-11\) or \(x=11\), then the radius of convergence is 11.
- Cons: Again, without explicit data, this is conjectural.

Final Remarks

Understanding the radius of convergence in power series involves analyzing the location of singularities relative to the expansion point. For the function $F(x) = \frac{7}{6} + \frac{11}{6x}$, the key takeaway is that the singularity at $x=0$ limits the domain of convergence if expanding around zero. The constants 7 and 11 hint at the distances or bounds involved, and selecting between the options $R=7$ or $R=11$ hinges on the specific point of expansion and the position of singularities.

In conclusion, the precise radius depends on the expansion point and the location of singularities, but the problem's choices suggest that either 7 or 11 could be the radius of convergence, with the context guiding the correct selection. Understanding these concepts is vital for advanced analysis and the effective application of power series in mathematics and physics.

Note: For a more accurate determination, additional context about the specific point of expansion or the full problem statement would be necessary.

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