The Surface Area And Volume Of A Sphere Are In The Ratio Of 3:7cm.What Is The Circumference Of Its Great

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Understanding the geometric properties of a sphere is fundamental in mathematics, physics, and engineering. When given specific ratios involving surface area and volume, it becomes an interesting problem that tests our knowledge of formulas and relationships within three-dimensional shapes. In this article, we will explore how to determine the circumference of a sphere when the surface area and volume are given in a specific ratio, delving into the underlying formulas, solving step-by-step, and explaining the concepts in detail to enhance your comprehension.

Understanding the Basics: Surface Area, Volume, and Circumference of a Sphere

Before we proceed with the problem, it’s essential to review the basic formulas related to a sphere's surface area, volume, and circumference.

Surface Area of a Sphere

The surface area (A) of a sphere with radius r is given by:
- $A = 4\pi r^2$

This formula represents the total area covering the outer surface of the sphere.

Volume of a Sphere

The volume (V) of a sphere with radius r is:
- $V = (4/3)\pi r^3$

This formula calculates how much space the sphere occupies.

Great Circle Circumference

The greatest circle of the sphere, also known as the "great circle," has the same radius as the sphere itself. Its circumference (C) is:
- $C = 2\pi r$

This is analogous to the circumference of a circle with radius r.

Problem Statement and Approach

Given:
- The surface area and volume are in the ratio of 3:7 (assuming units are consistent, e.g., cm^2 and cm^3).

Find:
- The circumference of the sphere's great circle.

Approach:

1. Express the given ratio in terms of the formulas for surface area and volume.
2. Set up an equation based on the ratio to find the radius.
3. Use the radius to calculate the circumference.

Step-by-Step Solution

Step 1: Express Surface Area and Volume in Terms of r

- Surface area: $A = 4\pi r^2$
- Volume: $V = (4/3)\pi r^3$

Step 2: Write the Ratio Equation

Given the ratio:
- A : V = 3 : 7

Expressed in terms of r:
- $(4\pi r^2) / ((4/3)\pi r^3) = 3 / 7$

Simplify numerator and denominator:
- $(4\pi r^2) \div ((4/3)\pi r^3) = 3 / 7$

Step 3: Simplify the Ratio

Calculate the division:
- $(4\pi r^2) (3 / 4\pi r^3) = 3 / 7$

Simplify:
- $(4\pi r^2) (3 / 4\pi r^3) = (4\pi r^2 3) / (4\pi r^3) = (12\pi r^2) / (4\pi r^3)$

Further simplification:
- $(12\pi r^2) / (4\pi r^3) = (12 / 4) (\pi / \pi) (r^2 / r^3) = 3 1 (1 / r) = 3 / r$

So, the ratio simplifies to:
- $3 / r = 3 / 7$

Step 4: Solve for r

Set the simplified ratio equal:
- $3 / r = 3 / 7$

Cross-multiplied:
- $3 7 = 3 r$

Simplify:
- $21 = 3r$

Divide both sides by 3:
- $r = 7$ cm

Thus, the radius of the sphere is 7 cm.

Calculating the Circumference of the Great Circle

Recall:
- $C = 2\pi r$

Substitute $r = 7$ cm:
- $C = 2\pi 7 = 14\pi$ cm

Using an approximate value of $\pi \approx 3.1416$:
- $C \approx 14 3.1416 \approx 43.9824$ cm

Therefore, the circumference of the sphere's great circle is approximately 44 cm.

Summary and Key Takeaways

- The ratio of surface area to volume can be used to find the radius of a sphere by simplifying the ratio algebraically.
- The established formulas for surface area, volume, and circumference are crucial for solving such problems.
- Always ensure units are consistent and simplify ratios carefully to avoid errors.
- The calculated radius enables us to determine the circumference accurately.

Additional Tips for Solving Similar Problems

- **Understand the formulas:** Familiarize yourself with the fundamental formulas for spheres and circles.
- **Express ratios carefully:** Set up the ratio equations precisely, simplifying step-by-step.
- **Check units:** Keep units consistent throughout calculations to avoid confusion.
- **Use approximations wisely:** When necessary, approximate π for a numerical answer.
- **Practice various problems:** Applying these steps to different ratios enhances problem-solving skills.

Conclusion

By analyzing the ratio of surface area to volume, we found that the radius of the sphere must be 7 cm. Using this radius, we calculated the circumference of its great circle to be approximately 44 cm. This problem illustrates the importance of understanding the relationships between different geometric properties of spheres and demonstrates how algebraic manipulation and knowledge of formulas can lead to solutions in geometry problems. Whether you're a student preparing for exams or a professional dealing with spherical objects, mastering these concepts is essential for accurate calculations and effective problem-solving.

Frequently Asked Questions

Given that the ratio of the surface area to volume of a sphere is 3:7, how can we find the radius of the sphere?

Using the formulas for surface area ($4\pi r^2$) and volume $(4/3)\pi r^3$, set up the ratio $4\pi r^2 / (4/3)\pi r^3 = 3/7$, simplify to find r, and then calculate the radius.

How do the surface area and volume ratios relate to the radius of a sphere?

Surface area is proportional to r^2, while volume is proportional to r^3; thus, their ratio involves the radius, and given the ratio, we can determine the radius accordingly.

Once the radius of the sphere is known, how do we calculate its circumference?

Circumference of a sphere (great circle) is given by $2\pi r$, so after finding r, multiply it by 2π to get the circumference.

What is the formula to find the circumference of a sphere's great circle?

The circumference is $2\pi r$, where r is the radius of the sphere.

If the surface area and volume ratios are 3:7, what is the step-by-step process to find the circumference of the sphere?

First, set up the ratio of surface area to volume: $(4\pi r^2)/(4/3)\pi r^3 = 3/7$. Simplify to find r, then use r in the circumference formula $2\pi r$ to find the circumference.

Additional Resources

The Surface Area And Volume Of A Sphere Are In The Ratio Of 3:7cm. What Is The Circumference Of Its Great?

Understanding the properties of a sphere is fundamental in geometry, physics, engineering, and various scientific disciplines. Among the key attributes of a sphere are its surface area, volume, and circumference. When faced with a problem that presents the ratio of surface area to volume, such as "The surface area and volume of a sphere are in the ratio of 3:7 cm," it prompts a deeper exploration into how these measurements relate and how to determine the sphere's characteristics, such as the circumference of its great circle.

This article aims to dissect this problem thoroughly, providing clarity on the mathematical concepts involved, guiding you through the calculations step-by-step, and presenting a comprehensive understanding of the relationships between a sphere’s surface area, volume, and circumference.

Understanding the Basics of a Sphere's Properties

Before delving into the problem, it’s essential to revisit the fundamental formulas related to a sphere:

- Surface Area (A): The total area covering the sphere's surface, given by:
$A = 4\pi r^2$

- Volume (V): The space enclosed within the sphere, given by:
$V = (4/3)\pi r^3$

- Circumference of the Great Circle (C): The largest possible circle on the sphere, given by:
$C = 2\pi r$

Here, r represents the radius of the sphere.

The Core Problem: Ratio of Surface Area to Volume

The problem states:
The surface area and volume of a sphere are in the ratio of 3:7 cm.

Mathematically, this can be expressed as:

$$ \frac{A}{V} = \frac{3}{7} $$

Substituting the formulas for A and V:

$$ \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{3}{7} $$

Simplify the left side:

$$ \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{4\pi r^2 \times 3}{4 \pi r^3} $$

$$ = \frac{12 \pi r^2}{4 \pi r^3} $$

Cancel common factors:

$$ = \frac{12}{4 r} $$

$$ = \frac{3}{r} $$

So, the ratio simplifies neatly to:

$$ \frac{A}{V} = \frac{3}{r} $$

Given that the ratio of surface area to volume is 3:7, then:

$$ \frac{3}{r} = \frac{3}{7} $$

Cross-multiplied:

$$ 3 \times 7 = 3 \times r $$

$$ 21 = 3r $$

Divide both sides by 3:

$$ r = 7 \text{ cm} $$

Result: The radius of the sphere is 7 cm.

Calculating the Circumference of the Great Circle

Having determined the radius, calculating the circumference becomes straightforward.

Recall:

$$ C = 2\pi r $$

Substitute r = 7 cm:

$$ C = 2 \pi \times 7 $$

$$ C = 14 \pi $$

Using the approximation $\pi \approx 3.1416$:

$$ C \approx 14 \times 3.1416 $$

$$ C \approx 43.9824 \text{ cm} $$

Rounded to two decimal places, the circumference is approximately:

43.98 cm

Summary of Findings

- The radius of the sphere is 7 cm.
- The circumference of the great circle is approximately 43.98 cm.

Significance of the Relationship

This problem exemplifies how ratios involving surface area and volume can be utilized to determine a sphere's dimensions. The neat algebraic simplification demonstrates the power of fundamental formulas and ratio analysis in geometry.

Furthermore, knowing the radius directly allows calculation of other properties, like surface area and volume, which are crucial for applications ranging from engineering design to natural sciences. For instance, in planetary sciences, knowing a planet's radius helps determine its surface area and volume, which are vital for understanding its surface conditions and mass.

Additional Insights and Applications

Understanding the relationships between a sphere's surface area, volume, and circumference extends beyond academic exercises:

- Engineering: Designing spherical tanks or domes requires precise calculations of capacity and surface materials.
- Physics: Spherical models are used in celestial mechanics, quantum physics, and electromagnetism.
- Biology: The shape of cells and viruses often approximates a sphere, impacting surface-area-to-volume ratios crucial for biological functions.

Final Thoughts

The problem of determining the circumference of a sphere given the ratio of its surface area to volume illustrates how geometric formulas interconnect. By translating the ratio into an algebraic equation, solving for the radius, and then calculating the circumference, one can uncover meaningful properties of the sphere.

This process highlights the elegance of geometry and mathematical reasoning, emphasizing that fundamental formulas, when combined with ratio analysis, can unlock insights into the physical world. Whether for academic pursuits, engineering challenges, or scientific research, mastering these relationships enhances our ability to analyze and interpret spherical objects in various contexts.

In conclusion, given the ratio of surface area to volume as 3:7, the sphere's radius is 7 cm, and its great circle's circumference is approximately 43.98 cm. This problem exemplifies the practical application of geometric principles and mathematical ratios in understanding the characteristics of spherical objects.

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