The Water In A Drainpiipe Is 18 Cm Deep. The Width Of The Surface Of The Water Is 48 Cm. Find The Radius

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Understanding the problem of finding the radius of the water surface in a drainpipe involves applying fundamental principles of geometry and mathematics. This scenario presents an interesting challenge often encountered in practical problems involving circles and their segments, which can be solved using formulas related to circle geometry. In this article, we will explore step-by-step methods to determine the radius of the drainpipe's water surface based on the given measurements.

Introduction to the Problem

Imagine a circular drainpipe in which water is accumulated, forming a segment of a circle. You're provided with two key measurements:

- The depth of the water, which is 18 centimeters.
- The width of the water's surface, which is 48 centimeters.

The goal is to determine the radius of the drainpipe, which is essentially the radius of the circle from which this water segment is a part.

This problem is a classic example of circle segment geometry, requiring the application of formulas involving the circle's radius, the chord length (width of the water surface), and the segment height (depth of water).

Understanding the Geometry of the Water Surface

Before calculating the radius, it's important to understand the geometric setup:

- The circle represents the cross-section of the drainpipe.
- The water surface corresponds to a chord of the circle, with a length of 48 cm.
- The depth of the water (18 cm) corresponds to the sagitta (or segment height), which is the distance from the midpoint of the chord to the circle's circumference along the perpendicular bisector of the chord.

This configuration can be visualized as a circle with a chord of length 48 cm, and a segment of the circle protruding inward, with a height of 18 cm.

Key Concepts and Formulas

To solve this problem, several geometric principles and formulas are used:

1. Circle and Chord Relationship

- Chord length (c): The straight-line distance across the circle at the water surface, given as 48 cm.
- Segment height (h): The distance from the midpoint of the chord to the circle's edge (the depth of

water), given as 18 cm.

- Radius (R): The radius of the circle, which we need to find.
- Distance from the circle's center to the chord (d): The perpendicular distance from the circle's center to the chord, related to the radius and the segment height.

2. Relationship between Radius, Chord, and Segment Height

The key formula connecting these variables is:

$$
h = R - \sqrt{R^2 - \left(\frac{c}{2}\right)^2}
$$

where:

- h is the segment height (18 cm),
- R is the radius (unknown),
- c is the chord length (48 cm),
- $\frac{c}{2}$ is half the chord length (24 cm).

Rearranged, the formula becomes:

$$
\sqrt{R^2 - \left(\frac{c}{2}\right)^2} = R - h
$$

Squaring both sides:

$$
R^2 - \left(\frac{c}{2}\right)^2 = (R - h)^2
$$

Expanding the right side:

$$
R^2 - \left(\frac{c}{2}\right)^2 = R^2 - 2Rh + h^2
$$

Subtract R^2 from both sides:

$$
- \left(\frac{c}{2}\right)^2 = - 2Rh + h^2
$$

Multiplying through by -1:

$$
\left(\frac{c}{2}\right)^2 = 2Rh - h^2
$$

Finally, solving for R:

$$
2Rh = \left(\frac{c}{2}\right)^2 + h^2
$$

$$
R = \frac{\left(\frac{c}{2}\right)^2 + h^2}{2h}
$$

This formula allows us to compute the radius directly.

Step-by-Step Calculation of the Radius

Let's now proceed with the actual calculation using the given measurements:

Step 1: List Known Values

- Chord length $c = 48 \text{ cm}$
- Segment height $h = 18 \text{ cm}$

Step 2: Calculate $\left(\frac{c}{2}\right)^2$

$$
\frac{c}{2} = \frac{48}{2} = 24 \text{ cm}
$$

$$
\left(\frac{c}{2}\right)^2 = 24^2 = 576 \text{ cm}^2
$$

Step 3: Calculate numerator for R

$$
\left(\frac{c}{2}\right)^2 + h^2 = 576 + 18^2 = 576 + 324 = 900 \text{ cm}^2
$$

Step 4: Calculate the denominator

$$
2h = 2 \times 18 = 36 \text{ cm}
$$

Step 5: Compute R

$$
R = \frac{900}{36} = 25 \text{ cm}
$$

Conclusion: The Radius of the Drainpipe

Based on the calculations, the radius of the drainpipe is 25 centimeters.

This means the cross-section of the pipe has a radius of 25 cm, and the water's surface forms a chord of length 48 cm at a depth of 18 cm from the top of the circle.

Practical Applications of This Calculation

Understanding how to find the radius of a circle segment has numerous real-world applications, especially in engineering, plumbing, and design:

- Pipe design and maintenance: Knowing the radius helps in assessing the capacity and flow rate of the pipe.
- Hydraulics engineering: Calculations of water flow, pressure, and capacity depend on understanding the cross-sectional geometry.
- Construction and renovation: Accurate measurements ensure proper fitting and functioning of drainage systems.

- Educational purposes: Teaching circle geometry concepts through practical problems.

Additional Tips and Common Mistakes

When solving similar problems, keep these points in mind:

- Always ensure units are consistent throughout calculations.
- Remember that the segment height (h) is measured from the chord to the circle's perimeter, not the midpoint of the circle.
- Double-check the application of formulas, especially the rearranged versions, to avoid algebraic errors.
- Visualize the problem to understand the geometric relationships better.

Summary

- The problem involves a circle with a chord (water surface) of 48 cm and a segment height of 18 cm.
- Using geometric relationships, the radius R can be calculated with the formula:

$$
R = \frac{\left(\frac{c}{2}\right)^2 + h^2}{2h}
$$

- Substituting the known values:

$$
R = \frac{576 + 324}{36} = 25 \text{ cm}
$$

- Therefore, the radius of the drainpipe is 25 centimeters.

Final Thoughts

Mastering the geometry of circles, including understanding chords, segments, and radii, is essential for solving various practical and theoretical problems. This specific problem exemplifies how geometry principles directly apply to real-world situations like plumbing and pipeline design. With practice, these calculations become intuitive, enabling engineers, students, and professionals to analyze and interpret similar problems efficiently.

By grasping the relationship between the water surface's width, depth, and the circle's radius, you can confidently approach and solve diverse problems involving circular segments.

Frequently Asked Questions

How do you find the radius of the water surface in a drainpipe given its depth and surface width?

You can find the radius by using the formula for the diameter of the circle (surface width) and dividing it by 2, since the surface is circular. If the surface width is 48 cm, then the radius is 48/2 = 24 cm.

Is the depth of the water in the drainpipe relevant for calculating the radius of the water surface?

In this context, the depth of the water (18 cm) helps determine the shape of the cross-section, but if only the surface width is given, the radius of the surface circle is simply half of that width, which is 24 cm.

What assumptions are made when calculating the radius of water in a drainpipe from surface width?

The main assumption is that the water surface is a perfect circle and that the surface width given corresponds to the diameter of that circle.

Can you determine the radius of the water in the drainpipe solely from the surface width?

Yes, if the surface width represents the diameter of the circular water surface, then the radius is half of that, so 24 cm.

How does the depth of 18 cm influence the calculation of the water's radius in the drainpipe?

The depth indicates the vertical extent of the water but does not directly affect the calculation of the radius of the surface unless additional information about the cross-sectional shape is provided.

What is the significance of knowing the surface width of water in a drainpipe?

The surface width helps determine the radius of the water surface, which can be useful for calculating volume, flow rate, or understanding the pipe's capacity.

If the surface of water forms a circle with a width of 48 cm, what is its radius?

The radius is half of the surface width, so it is 48 cm / 2 = 24 cm.

Are there any common mistakes to avoid when calculating the radius of water in a drainpipe?

Yes, a common mistake is confusing the diameter with the radius. Always divide the surface width (diameter) by 2 to find the radius. Also, ensure that the surface is indeed circular before applying this calculation.

Additional Resources

The Water In A Drainpipe Is 18 Cm Deep. The Width Of The Surface Of The Water Is 48 Cm. Find The Radius is a classic problem that combines principles of geometry and algebra to find an unknown radius based on given dimensions. This problem is often encountered in mathematics classes, especially those focusing on geometry, and serves as an excellent example for understanding how to manipulate formulas involving areas of circles and the properties of shapes. It challenges students and enthusiasts to translate real-world measurements into mathematical expressions and solve for the unknown, making it both practical and educational.

Understanding the Problem

Before diving into the solution, it’s important to analyze what the problem states and what is being asked.

Problem Restatement:

- The depth of water in a drainpipe is 18 cm.
- The width (diameter of the water surface) is 48 cm.
- The goal is to find the radius of the water surface, which is essentially the radius of the circular cross-section of the water at the surface.

Key points:

- The depth of water is 18 cm, which implies the water occupies a segment of the circular cross-section.
- The width of the water surface is 48 cm, which is the diameter at the top of the water segment.
- The problem appears to involve a segment of a circle, where the water surface forms a chord at a certain height.

This scenario is common in geometrical problems involving circles, segments, and chords, and understanding the relationship between the depth and the surface width is critical in deriving the radius.

Breaking Down the Geometry

To find the radius, we need to understand the geometric setup.

Visualizing the Cross-Section of the Pipe

Imagine looking at the cross-section of the drainpipe from the side. The cross-section is a circle, and the water forms a segment of this circle.

- The diameter of the water surface (chord) is 48 cm.

- The depth of water (distance from the water surface to the bottom of the pipe) is 18 cm.

The key is to relate these measurements to the radius of the circle.

Circle Properties and Segment Geometry

In a circle:

- The radius (R) is the distance from the center to any point on the circle.
- A chord is a line segment connecting two points on the circle.
- The depth of the segment (h) is the distance from the chord to the circle's center along a perpendicular line.

In this problem:

- The chord length (water surface width) = 48 cm.
- The segment depth (from the water surface to the bottom of the pipe) = 18 cm.

The key is to determine the radius R, given these measurements.

Mathematical Approach to the Solution

Step 1: Relate the chord length to the radius and the depth

Let:

- R = radius of the circle
- c = length of the chord (water surface) = 48 cm

- h = distance from the chord to the bottom of the pipe = 18 cm (which is the depth)

The distance from the circle's center to the chord (perpendicular) is called d.

Using right triangle relationships:

- The radius R, the distance d from the center to the chord, and half the chord length form a right triangle.

Half of the chord length:

- $c/2 = 48/2 = 24$ cm

By the Pythagorean theorem:

- $R^2 = d^2 + (c/2)^2$

Since the depth h is the distance from the chord to the bottom, and the circle's radius extends from the center to the circumference, we need to relate h to R and d.

Step 2: Express the relationship between d, R, and h

The distance from the circle's center to the chord (d) relates to the depth h as follows:

- The distance from the bottom of the pipe to the center of the circle is $(R - h)$.

Since the water surface is at a depth h from the bottom:

- The center of the circle is located at a distance R from the top (or from the bottom, depending on reference point).

But because the problem provides the depth of water (18 cm) as the distance from the water surface to the bottom, and the water surface is a chord at the top, the distance from the circle's center to the chord (d) can be expressed as:

$d = R - h$

Thus:

- $d = R - 18$

Using the earlier relation:

- $R^2 = d^2 + (c/2)^2$

Substitute d:

- $R^2 = (R - 18)^2 + 24^2$

Step 3: Set up the quadratic equation and solve for R

Expanding:

- $R^2 = (R - 18)^2 + 576$
- $R^2 = (R^2 - 36R + 324) + 576$

Simplify:

- $R^2 = R^2 - 36R + 900$

Subtract R^2 from both sides:

- $0 = -36R + 900$

Solve for R:

- $36R = 900$
- $R = 900 / 36$
- $R = 25$ cm

Final Answer and Verification

The radius of the circle, which corresponds to the radius of the water surface in the drainpipe, is 25 cm.

Verification:
- Half chord length: 24 cm
- $R = 25$ cm
- $d = R - h = 25 - 18 = 7$ cm

Check the Pythagorean relation:
- $R^2 = d^2 + (c/2)^2$
- $25^2 = 7^2 + 24^2$
- $625 = 49 + 576$
- $625 = 625$

The calculations match, confirming the solution is correct.

Features and Practical Implications

Understanding this problem and its solution has several features that are beneficial beyond academic exercises:

Pros:
- Demonstrates real-world applications of circle geometry.
- Reinforces problem-solving skills involving algebra and geometry.
- Helps in designing or analyzing systems involving pipes, tanks, or any cylindrical structures.

- Enhances visualization skills for three-dimensional objects.

Cons:

- Requires understanding of multiple geometric concepts.

- May be challenging for beginners to set up the equations correctly.

- Assumes perfect circles and uniform measurements, which may vary in real-world applications.

Conclusion

The problem of calculating the radius of a circle given the depth of a segment and the width of its chord is a classic example illustrating the intersection of geometry and algebra. By visualizing the circle, applying Pythagoras' theorem, and carefully setting up the relationships, we determined that the radius of the water surface in the drainpipe is 25 cm. This exercise not only sharpens mathematical skills but also provides insights into how geometric principles underpin many practical engineering and design tasks. Whether for designing piping systems, analyzing water flow, or simply honing problem-solving abilities, mastering such problems is invaluable for students and professionals alike.

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