Triangle ABC Has A Perimeter Of 22cm AB=8cm BC=5cmDeduce Whether Triangle Abc Is A Right Angled Triangle

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Understanding the properties of triangles and applying geometric principles are fundamental in solving problems related to their sides and angles. In this article, we will analyze the given information about triangle ABC, where the perimeter is 22 cm, with sides AB = 8 cm and BC = 5 cm, to determine whether the triangle is a right-angled triangle. This exercise involves calculating the third side, examining the Pythagorean theorem, and considering various geometric criteria to reach a conclusion.

Analyzing the Given Data

Before delving into the calculations, it is essential to organize and interpret the provided information. The key data points are:

- Perimeter of triangle ABC: 22 cm
- Length of side AB: 8 cm
- Length of side BC: 5 cm

However, the position of the sides and the naming convention need clarification. Typically, in triangle ABC:

- AB is the side between points A and B
- BC is the side between points B and C
- AC is the side between points A and C

Given that, the third side, AC, is unknown and needs to be deduced.

Calculating the Unknown Side AC

Since the perimeter is the sum of all three sides, we can write:

Perimeter = AB + BC + AC

Plugging in the known values:

22 cm = 8 cm + 5 cm + AC

Solving for AC:

AC = 22 cm - (8 cm + 5 cm) = 22 cm - 13 cm = 9 cm

Therefore, the sides of triangle ABC are:

- AB = 8 cm
- BC = 5 cm
- AC = 9 cm

This set of side lengths allows us to proceed with analyzing whether the triangle is right-angled.

Applying the Pythagorean Theorem

The most direct method to check whether a triangle is right-angled is to apply the Pythagorean theorem, which states that:

- In a right-angled triangle, the square of the hypotenuse (the longest side) equals the sum of the squares of the other two sides.

Given the side lengths, the longest side is AC = 9 cm. The other two sides are AB = 8 cm and BC = 5 cm.

Let's verify whether:

$(AC)^2 = (AB)^2 + (BC)^2$

Calculating:

$(9)^2 = 81$

$(8)^2 + (5)^2 = 64 + 25 = 89$

Since $81 \neq 89$, the Pythagorean theorem does not hold with AC as the hypotenuse.

Next, check whether any other side could serve as the hypotenuse:

- Is AB the hypotenuse?

$(AB)^2 = 64$

Sum of squares of other sides:

$(BC)^2 + (AC)^2 = 25 + 81 = 106$

No, $64 \neq 106$.

- Is BC the hypotenuse?

$(BC)^2 = 25$

Sum of squares of other sides:

$(AB)^2 + (AC)^2 = 64 + 81 = 145$

Again, no match.

Therefore, based on the side lengths, the Pythagorean theorem does not hold for any pairing, indicating that triangle ABC is not right-angled.

Using the Converse of the Pythagorean Theorem

The converse of the Pythagorean theorem states that if, for three sides, the square of the longest side equals the sum of the squares of the other two sides, then the triangle is right-angled.

Since:

- For AC (longest side), $81 \neq 89$
- For AB, $64 \neq 106$
- For BC, $25 \neq 145$

The triangle does not satisfy the Pythagorean condition for any pairing.

Therefore, triangle ABC is not a right-angled triangle.

Verifying via the Law of Cosines

Another approach to determine whether a triangle is right-angled involves the Law of Cosines, which states:

$c^2 = a^2 + b^2 - 2ab \cos(C)$

Where:

- c is the side opposite angle C
- a and b are the other two sides

If the triangle is right-angled at C, then $\cos(C) = 0$, and:

$c^2 = a^2 + b^2$

If the calculated value of $\cos(C)$ is zero, then the angle is a right angle.

Let's test for each angle:

At vertex C (opposite side AC = 9 cm):

- $c = 9$ cm
- $a = 8$ cm (AB)
- $b = 5$ cm (BC)

Calculate:

$\cos(C) = [a^2 + b^2 - c^2] / (2ab)$

Plug in the values:

$\cos(C) = [64 + 25 - 81] / (2\ 8\ 5) = (89 - 81) / (80) = 8 / 80 = 0.1$

Since $\cos(C) \neq 0$, angle C is not 90°.

At vertex A (opposite side AB = 8 cm):

- $c = 8$ cm
- $a = 5$ cm (BC)
- $b = 9$ cm (AC)

Calculate:

$\cos(A) = [a^2 + c^2 - b^2] / (2ac) = [25 + 81 - 81] / (2\ 5\ 9) = (25 + 0) / 90 = 25 / 90 \approx 0.2778$

Not zero, so angle A is not 90°.

At vertex B (opposite side BC = 5 cm):

- $c = 5$ cm
- $a = 8$ cm (AB)
- $b = 9$ cm (AC)

Calculate:

$\cos(B) = [a^2 + c^2 - b^2] / (2ac) = [64 + 25 - 81] / (2\ 8\ 5) = 8 / 80 = 0.1$

Again, not zero; angle B is not 90°.

Conclusion:

None of the angles are 90°, confirming that triangle ABC is not right-angled.

Summary of Findings

Based on the calculations and geometric principles applied:

- The sides of triangle ABC are 8 cm, 5 cm, and 9 cm.
- The Pythagorean theorem does not hold for any side combination.
- The Law of Cosines calculations indicate no angle measures 90°.

- Therefore, triangle ABC is an acute or obtuse triangle, but not right-angled.

Additional Considerations: Types of Triangle Based on Sides

While the triangle is not right-angled, understanding its classification by sides is valuable:

- Since all sides are of different lengths, ABC is a scalene triangle.
- The longest side is 9 cm, and the other sides are 8 cm and 5 cm.

To determine whether it is acute or obtuse:

- For a scalene triangle, compare c^2 with $a^2 + b^2$:

If $c^2 > a^2 + b^2$, the triangle is obtuse.

If $c^2 < a^2 + b^2$, the triangle is acute.

Calculate:

81 (c^2) vs. 64 + 25 = 89

Since 81 < 89, the triangle is acute.

Final conclusion: Triangle ABC is a scalene, acute triangle with sides of 8 cm, 5 cm, and 9 cm, and it is not a right-angled triangle.

Conclusion

In this comprehensive analysis, we've deduced that given the perimeter of 22 cm, sides of 8 cm and 5 cm, and the calculated third side of 9 cm, triangle ABC does not satisfy the criteria of a right-angled triangle. Applying the Pythagorean theorem and the Law of Cosines confirms that none of the angles measure 90°, and the triangle is therefore classified as an acute scalene triangle. This exercise demonstrates the importance of combining multiple geometric principles to analyze the properties of triangles accurately, a fundamental skill in geometry and related fields.

Frequently Asked Questions

What is the length of side AC in triangle ABC?

The length of side AC can be found by subtracting the sum of AB and BC from the perimeter: 22 cm - (8 cm + 5 cm) = 9 cm.

How do you determine if triangle ABC is a right-angled triangle?

Use the Pythagorean theorem: if the square of the longest side equals the sum of the squares of the other two sides, then the triangle is right-angled.

What is the length of side AC in triangle ABC?

AC is 9 cm long, calculated as 22 cm (perimeter) minus 8 cm (AB) and 5 cm (BC).

Which side in triangle ABC is the longest?

Side AB is 8 cm, BC is 5 cm, and AC is 9 cm, so AC is the longest side.

Is triangle ABC right-angled based on the given sides?

Check if $8^2 + 5^2$ equals 9^2: $64 + 25 = 89$, which does not equal 81, so triangle ABC is not right-angled.

What are the steps to verify if a triangle with given sides is right-angled?

Identify the longest side, then verify if the sum of the squares of the other two sides equals the square of the longest side using the Pythagorean theorem.

Can triangle ABC be a right-angled triangle with the given measurements?

No, because the Pythagorean theorem does not hold for the given sides, so triangle ABC is not right-angled.

Additional Resources

Triangle ABC Has A Perimeter Of 22cm: Deduce Whether Triangle ABC Is a Right-Angled Triangle

Understanding the properties of triangles is fundamental in geometry, especially when it comes to classifying triangles based on their sides and angles. In this detailed analysis, we will investigate whether triangle ABC, with a given perimeter of 22 cm, and specific side lengths AB and BC, qualifies as a right-angled triangle. This exploration involves a thorough examination of the given data, geometric principles, and the application of mathematical theorems.

Introduction to Triangle Classification

Before delving into the specifics of triangle ABC, it’s essential to understand the basic types of triangles based on their sides and angles:

- Based on sides:
- Equilateral triangle: all three sides are equal.
- Isosceles triangle: at least two sides are equal.
- Scalene triangle: all three sides are different.

- Based on angles:
- Acute triangle: all angles less than 90°.
- Right triangle: one angle exactly 90°.
- Obtuse triangle: one angle greater than 90°.

In our case, the primary goal is to determine whether triangle ABC is a right-angled triangle, which hinges on the relationship between the sides and the Pythagorean theorem.

Given Data and Initial Observations

Let's compile the essential data provided:

- Perimeter of Triangle ABC: 22 cm
- Side AB: 8 cm
- Side BC: 5 cm

However, the length of side AC is not directly given. To analyze the triangle comprehensively, we need to determine the possible length of AC, given the perimeter and known sides.

Key Points:

- The perimeter formula:
$$
AB + BC + AC = 22\,cm
$$
- Known sides:
$$
AB = 8\,cm, \quad BC = 5\,cm
$$
- Unknown side:
$$
AC = x\,cm
$$

From the perimeter:

$$
8 + 5 + x = 22 \Rightarrow x = 22 - 13 = 9\,cm
$$

So, the length of side AC is 9 cm.

Determining the Nature of Triangle ABC

With all side lengths identified, the sides are:

- AB = 8 cm
- BC = 5 cm
- AC = 9 cm

Now, to determine whether triangle ABC is right-angled, we examine the Pythagorean relationship among these sides.

Applying the Pythagorean Theorem

The Pythagorean theorem states that for a triangle to be right-angled (with the right angle between sides of lengths a and b), the following must hold:

$$
a^2 + b^2 = c^2
$$

where c is the length of the hypotenuse (the longest side).

Step-by-step analysis:

1. Identify the longest side:
$$
AC = 9\,cm
$$
$$
AB = 8\,cm
$$
$$
BC = 5\,cm
$$

The longest side is AC = 9 cm, so if the triangle is right-angled, the hypotenuse should be AC.

2. Calculate the sum of squares of the other two sides:
$$
AB^2 + BC^2 = 8^2 + 5^2 = 64 + 25 = 89
$$

3. Calculate the square of the hypotenuse:
$$
AC^2 = 9^2 = 81
$$

4. Compare the sums:
$$
AB^2 + BC^2 = 89
$$
$$
AC^2 = 81
$$

Since $89 \neq 81$, the Pythagorean theorem does not hold for these sides, indicating that triangle ABC is not a right-angled triangle with AC as the hypotenuse.

Alternative Configurations and Theorem Checks

While the initial analysis suggests that triangle ABC is not right-angled, for completeness, consider whether any other side could serve as a hypotenuse to satisfy the Pythagorean theorem.

Possible hypotenuse candidates:

- If AB = 8 cm is hypotenuse:
- Check if $AC^2 + BC^2 = AB^2$
- $9^2 + 5^2 = 81 + 25 = 106$
- $AB^2 = 8^2 = 64$

No, since $106 \neq 64$.

- If BC = 5 cm is hypotenuse:
- Check if $AB^2 + AC^2 = BC^2$
- $8^2 + 9^2 = 64 + 81 = 145$
- $BC^2 = 25$

No, since $145 \neq 25$.

Conclusion:

- None of the sides satisfy the Pythagorean relation for right angles.
- Therefore, triangle ABC is not a right-angled triangle regardless of which side is considered as a hypotenuse.

Geometric Constraints and Triangle Inequality

To ensure the sides form a valid triangle, the triangle inequality must hold:

$$
\text{Sum of any two sides} > the third side
$$

Check with the current side lengths:

- $AB + BC = 8 + 5 = 13 > 9$ (valid)
- $AB + AC = 8 + 9 = 17 > 5$ (valid)
- $BC + AC = 5 + 9 = 14 > 8$ (valid)

All inequalities are satisfied, confirming the sides can form a valid triangle.

Summary of Findings and Conclusions

Based on the detailed analysis:

- The sides of triangle ABC are 8 cm, 5 cm, and 9 cm.
- The perimeter matches the given data:
$$
8 + 5 + 9 = 22\,cm
$$
- The Pythagorean theorem does not hold for any side combination; thus, triangle ABC is not a right-angled triangle.
- The triangle inequalities are satisfied, confirming the sides form a valid triangle.

Final conclusion:

> Triangle ABC, with sides 8 cm, 5 cm, and 9 cm, and a perimeter of 22 cm, is not a right-angled triangle.

Additional Insights and Related Topics

While the primary question pertains to whether the triangle is right-angled, this analysis opens avenues for further exploration:

- Classification as an acute or obtuse triangle:
- Since the square of the longest side (81) is less than the sum of the squares of the other two sides

(89), the triangle is acute.
- Why?
For a triangle with sides a, b, c (with c the longest), if $c^2 < a^2 + b^2$, the triangle is acute.
Here, $81 < 89$, so triangle ABC is acute.

- Using Law of Cosines to find angles:
- To determine the measure of angles precisely, the Law of Cosines is useful:

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

- For angle C opposite side $c=9\,cm$:

$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

- Applying with $a=8\,cm$, $b=5\,cm$, $c=9\,cm$:

$$
\cos C = \frac{64 + 25 - 81}{2 \times 8 \times 5} = \frac{8}{80} = 0.1
$$

- $\Rightarrow C = \arccos(0.1) \approx 84.26^\circ$

- Since $C \approx 84.26^\circ < 90^\circ$, the triangle is indeed acute in the sense that no angle reaches or exceeds 90°.

Implications and Practical Applications

Understanding whether a triangle is right-angled has numerous implications in real-world scenarios:

- Construction and Engineering:
- Right triangles are fundamental in establishing orthogonal directions and creating accurate right angles.
- Knowing the side lengths and verifying the right-angle property helps in designing structures.

- Navigation and Surveying:
- Triangles with known side lengths and right angles facilitate distance measurements and land plotting.

- Mathematical Problem Solving:
- Recognizing the nature of triangles enhances problem-solving skills in geometry, trigonometry, and coordinate geometry.

- Educational Value:
- This

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