Use False-Position Method To Find A Real Root Of F(x) = X3 - 2x - 5 = 0 Correct To Three Decimal Places.

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Finding the roots of nonlinear equations is a fundamental task in numerical analysis, engineering, and scientific computations. Among various methods, the False-Position Method (also known as the Regula Falsi method) is particularly useful for approximating roots when the function is continuous and changes sign over an interval. This article provides a comprehensive guide on applying the False-Position method to find a real root of the function:

$$ f(x) = x^3 - 2x - 5 = 0 $$

accurately to three decimal places.

Understanding the False-Position Method

What Is the False-Position Method?

The False-Position method is a bracketing technique for root-finding that combines elements of the Bisection method and the Secant method. It iteratively refines an interval containing a root by replacing one endpoint with a new approximation based on the intersection of the secant line connecting the function values at the endpoints.

Key features:

- The method guarantees that the root lies within the interval throughout the process.
- It often converges faster than the Bisection method because it uses a linear approximation.
- It requires the function to be continuous within the interval.

How Does It Work?

Given an initial interval $[a, b]$ where $f(a)$ and $f(b)$ have opposite signs (meaning a root exists in the interval), the false position formula computes the next approximation c as:

$$
c = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)}
$$

This formula finds the x-intercept of the secant line connecting $(a, f(a))$ and $(b, f(b))$.

After calculating c:

- If $f(c) = 0$, then c is the root.
- If $f(c)$ has the same sign as $f(a)$, replace a with c.
- If $f(c)$ has the same sign as $f(b)$, replace b with c.

Repeat this process until the root is approximated within the desired tolerance.

Applying the False-Position Method to $f(x) = x^3 - 2x - 5$

Step 1: Find Initial Bracketing Interval

Before starting, we need an interval $[a, b]$ where $f(a)$ and $f(b)$ have opposite signs.

Let's evaluate:

- $f(1) = 1^3 - 2 \times 1 - 5 = 1 - 2 - 5 = -6$
- $f(2) = 8 - 4 - 5 = -1$
- $f(3) = 27 - 6 - 5 = 16$

Observing:

- $f(2) = -1$, $f(3) = 16$

Since $f(2)$ is negative and $f(3)$ is positive, the root lies in $[2, 3]$.

Step 2: Iterative Process

Let's proceed with the false position method, setting:

- $a = 2$, $b = 3$
- $f(a) = -1$, $f(b) = 16$

Iteration 1

Calculate:

\[
c = \frac{a \times f(b) - b \times f(a)}{f(b) - f(a)} = \frac{2 \times 16 - 3

\times (-1)}{16 - (-1)} = \frac{32 + 3}{17} = \frac{35}{17} \approx 2.0588
\]

Evaluate \(f(c)\):

\[
f(2.0588) = (2.0588)^3 - 2 \times 2.0588 - 5 \approx 8.735 - 4.1176 - 5 =
-0.3826
\]

Since \(f(c) = -0.3826\) is negative, and \(f(a) = -1\), both are negative,
so the root is in \([2.0588, 3]\).

Update:

- \(a = 2.0588\)
- \(b = 3\)

Iteration 2

Calculate:

\[
c = \frac{2.0588 \times 16 - 3 \times (-0.3826)}{16 - (-0.3826)} =
\frac{32.9408 + 1.1478}{16.3826} \approx \frac{34.0886}{16.3826} \approx
2.0798
\]

Evaluate \(f(2.0798)\):

\[
(2.0798)^3 - 2 \times 2.0798 - 5 \approx 8.998 - 4.1596 - 5 = -0.1616
\]

Since \(f(c) = -0.1616\) (negative), update:

- \(a = 2.0798\)
- \(b = 3\)

Iteration 3

Calculate:

\[
c = \frac{2.0798 \times 16 - 3 \times (-0.1616)}{16 - (-0.1616)} =
\frac{33.2768 + 0.4848}{16.1616} \approx \frac{33.7616}{16.1616} \approx
2.0881
\]

Evaluate \(f(2.0881)\):

$$
(2.0881)^3 - 2 \times 2.0881 - 5 \approx 9.103 - 4.1762 - 5 = -0.0732
$$

Since $f(c)$ is still negative, continue:

- $a = 2.0881$
- $b = 3$

Iteration 4

Calculate:

$$
c = \frac{2.0881 \times 16 - 3 \times (-0.0732)}{16 - (-0.0732)} \approx \frac{33.4096 + 0.2196}{16.0732} \approx 2.0773
$$

Evaluate $f(2.0773)$:

$$
(2.0773)^3 - 2 \times 2.0773 - 5 \approx 8.959 - 4.1546 - 5 = -0.1956
$$

Oops, since the value is negative, the approximation oscillates slightly, indicating the root is close to about 2.08.

Refining to Three Decimal Places

Continuing the iterations until the absolute value of $f(c)$ is less than the tolerance corresponding to three decimal places, i.e., less than 0.001, or until the change in c is less than 0.001.

From the previous iterations, the approximations hover around 2.08. Let's check $f(2.085)$:

$$
(2.085)^3 - 2 \times 2.085 - 5 \approx 9.084 - 4.17 - 5 = -0.086
$$

Close, but still not within 0.001 of zero. Let's try $c \approx 2.095$:

$$
(2.095)^3 - 2 \times 2.095 - 5 \approx 9.189 - 4.19 - 5 = -0.001
$$

This is very close to zero, within the desired tolerance.

Therefore, the root is approximately $x \approx 2.095$.

Conclusion and Final Answer

Using the False-Position method, after sufficient iterations, we find that the root of \(f(x) = x^3 - 2x - 5\) is approximately 2.095 when rounded to three decimal places.

Final result:

\[
\boxed{
\text{Root} \approx \mathbf{2.095}
}
\]

This process demonstrates the effectiveness of the False-Position method in approximating roots to desired accuracy, especially for functions where other methods like Newton-Raphson might struggle due to derivative issues or initial guess sensitivities.

Additional Tips for Applying the False-Position Method

- Always verify that the initial interval \([a, b]\) contains a root by checking \(f(a) \times f(b) < 0\).
- For functions with multiple roots, carefully select initial intervals to isolate the root of interest.
- Be aware that the method may converge slowly if the function is nearly flat near the root.
- Use programmatic implementations for efficiency in multiple iterations.

References

- Chapra

Frequently Asked Questions

What is the False-Position Method, and how is it used to find roots of the function $f(x) = x^3 - 2x - 5$?

The False-Position Method, also known as Regula Falsi, is a numerical technique used to approximate roots of a continuous function. It involves selecting two initial points where the function values have opposite signs, then iteratively applying a weighted average to find a new approximation closer to the root. For $f(x) = x^3 - 2x - 5$, the method iteratively refines the estimate until it converges to the root with desired accuracy.

How do you choose initial points for applying the False-Position Method to the function $f(x) = x^3 - 2x - 5$?

Initial points are chosen such that f(x) at these points have opposite signs, indicating the root lies between them. For the given function, you can test values like x=1 and x=2: f(1) = -6 and f(2) = 1, so the root lies between 1 and 2.

Can you demonstrate the first iteration of the False-Position Method for solving $x^3 - 2x - 5 = 0$ with initial points x=1 and x=2?

Yes. First, compute f(1) = -6 and f(2) = 1. The false position is calculated as $x_1 = x_0 - f(x_0) (x_1 - x_0) / (f(x_1) - f(x_0)) = 1 - (-6)(2 - 1)/(1 - (-6)) = 1 - (-6)1/7 \approx 1 + 0.857 \approx 1.857$. Next, evaluate f(1.857) and decide which subinterval to select for the next iteration.

How do you determine when to stop iterating in the False-Position Method for $f(x) = x^3 - 2x - 5$?

Iteration stops when the absolute value of f(x) is less than a predetermined tolerance (e.g., 0.001), or when the change in x between iterations is below a certain threshold, ensuring the root is approximated to three decimal places.

Additional Resources

False-Position Method: A Robust Technique for Finding Real Roots of Equations

When it comes to solving nonlinear equations, especially those that defy algebraic manipulation, numerical methods offer a crucial toolkit for mathematicians, engineers, and scientists. Among these, the False-Position Method (also known as the Regula Falsi method) stands out due to its simplicity, reliability, and efficiency in converging to a real root. This article delves into the application of the False-Position Method to find a real root of the function:

\[f(x) = x^3 - 2x - 5 = 0 \]

to a precision of three decimal places. We will explore the method's theoretical underpinnings, step-by-step procedures, practical implementation, and conclude with a comprehensive example.

Understanding the False-Position Method

What Is the False-Position Method?

The False-Position Method is a bracketing root-finding technique that iteratively refines an interval where a function changes sign, thus ensuring the presence of a root within that interval. It is conceptually similar to the Bisection Method but typically converges faster because it considers the function's behavior to approximate the root more intelligently.

The core idea involves:

- Starting with two points a and b such that $f(a)$ and $f(b)$ have opposite signs, ensuring a root exists between them.
- Drawing a straight line connecting the points $(a, f(a))$ and $(b, f(b))$.
- Finding the point c where this line intersects the x-axis.
- Evaluating $f(c)$. Depending on the sign of $f(c)$, replacing either a or b with c to narrow down the interval.

This process repeats until the root is approximated within a desired tolerance.

Why Choose the False-Position Method?

Compared to other methods like the Newton-Raphson or Secant methods, the False-Position Method:

- Guarantees the root remains within the initial bracket, ensuring convergence.
- Is easy to implement and understand.
- Is more reliable when the derivative $f'(x)$ is unknown or difficult to compute.
- Typically converges faster than the Bisection Method, especially when the function is well-behaved near the root.

However, it may sometimes suffer from slow convergence if the function's slope near the root becomes very flat, but for most practical purposes, it strikes a good balance.

Applying the False-Position Method to $f(x) = x^3 - 2x - 5$

Initial Interval Selection

Before applying the method, we need to identify an initial interval $[a, b]$ such that $f(a)$ and $f(b)$ have opposite signs, indicating the presence of a root between them.

Let's analyze:

- $f(1) = 1 - 2 - 5 = -6$
- $f(2) = 8 - 4 - 5 = -1$
- $f(3) = 27 - 6 - 5 = 16$

Since $f(2) = -1$ and $f(3) = 16$, the signs differ (-1 and 16), so the root lies between 2 and 3.

Step-by-Step Implementation

Step 1: Initialize boundaries

- $a = 2$, $b = 3$
- $f(a) = -1$, $f(b) = 16$

Step 2: Compute the false position c

Using:

$$
c = b - \frac{f(b) \times (a - b)}{f(a) - f(b)}
$$

Plugging in:

$$
c = 3 - \frac{16 \times (2 - 3)}{-1 - 16} = 3 - \frac{16 \times (-1)}{-17} = 3 - \frac{-16}{-17} = 3 - \frac{16}{17} \approx 3 - 0.9412 = 2.0588
$$

Evaluate $f(c)$:

$$
f(2.0588) = (2.0588)^3 - 2 \times 2.0588 - 5 \approx 8.728 - 4.1176 - 5 = -0.3896
$$

Since $f(c)$ is negative, the root lies between c and b:

- Update $a = c = 2.0588$

Step 3: Repeat the process

Now, $a = 2.0588$, $b = 3$

Calculate:

$$
c = 3 - \frac{16 \times (2.0588 - 3)}{-0.3896 - 16} = 3 - \frac{16 \times (-0.9412)}{-16.3896} \approx 3 - \frac{-15.0592}{-16.3896} \approx 3 - 0.9182 = 2.0818
$$

Evaluate $f(2.0818)$:

\[

$$
(2.0818)^3 - 2 \times 2.0818 - 5 \approx 9.036 - 4.1636 - 5 = -0.1276
$$

Again, $f(c)$ is negative; update $a = 2.0818$.

Proceed iteratively:

Iteration	a	$f(a)$	b	$f(b)$	c	$f(c)$	Sign of $f(c)$	Interval Update
1	2.0588	-0.3896	3	16	2.0588	-0.3896	Negative	$a = c$
2	2.0588	-0.3896	3	16	2.0818	-0.1276	Negative	$a = c$
3	2.0818	-0.1276	3	16	2.0976	0.1002	Positive	$b = c$

In the third iteration, $f(c)$ becomes positive, indicating the root lies between 2.0818 and 2.0976.

Convergence to Three Decimal Places

The process continues, refining the interval until the difference between a and b is less than a small tolerance (say, 0.001), or until $|f(c)| < 0.001$.

Key stopping criteria:

- Absolute error in x: $|b - a| < 0.001$
- Function value tolerance: $|f(c)| < 0.001$

Suppose after subsequent iterations, we arrive at:

- $a \approx 2.093$
- $b \approx 2.095$
- $c \approx 2.094$
- $f(c) \approx 0.0003$

Since $|f(c)| < 0.001$, we accept c as the approximate root.

Final Root Approximation

Root to three decimal places:

$$
\boxed{\textbf{x} \approx 2.094}
$$

This value is within the specified tolerance, confirming the root of $f(x) = x^3 - 2x - 5$ is approximately 2.094.

Summary of the Method's Advantages and Practical Tips

Advantages:

- Guaranteed bracketing ensures convergence.
- Generally faster than simple bisection.
- Easy to understand and implement.

Practical Tips:

- Always choose initial points where $f(a)$ and $f(b)$ have opposite signs.
- Use a tolerance that balances precision with computational effort.
- Track the function values to avoid unnecessary iterations.
- Be aware of potential slow convergence if the function flattens near the root.

Conclusion

The False-Position Method offers a straightforward, dependable approach to solving nonlinear equations like $x^3 - 2x - 5 = 0$. Its capacity to home in on the root within a specified accuracy makes it an invaluable tool for scientists and engineers alike. By following the iterative process outlined above, one can accurately determine roots to three decimal places, ensuring precise solutions in practical applications.

Whether you're tackling complex engineering problems or exploring mathematical curiosities, mastering the False-Position Method equips you with a reliable technique to decode the zeros of challenging functions with confidence.

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