Use The Binomial Theorem And Substitution To Expand Each Binomial. Show Your Step.A) (3x+y)^4B) (x-2y)^6

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Introduction to Binomial Expansion

The binomial theorem is a powerful algebraic tool that allows us to expand expressions of the form $(a + b)^n$, where a and b are any algebraic expressions, and n is a non-negative integer. This method provides a systematic way to find each term in the expansion without performing lengthy multiplications repeatedly.

In this article, we will explore the application of the binomial theorem and substitution techniques to expand two specific binomials:

- $\mathbf{(3x + y)^4}$
- $\mathbf{(x - 2y)^6}$

We will carefully walk through each step, including how to determine the binomial coefficients, apply the theorem, and simplify the resulting expressions.

Understanding the Binomial Theorem

The Binomial Theorem Statement

The binomial theorem states that for any non-negative integer n:

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n - k} b^{k}
$$

where:

- $\binom{n}{k}$ is the binomial coefficient, read as "n choose k," calculated as:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

- a and b are algebraic expressions.

Significance of the Binomial Coefficients

The binomial coefficients $\binom{n}{k}$ form the entries of Pascal's triangle and determine the weight of each term in the expansion. They are symmetric and satisfy certain properties that simplify calculations.

Applying the Theorem to Specific Binomials

To expand a binomial like $(a + b)^n$:

1. Identify a, b, and n.
2. Determine the binomial coefficients for each term.
3. Compute each term by raising a and b to appropriate powers.
4. Sum all terms to get the expanded form.

Part A: Expanding $(3x + y)^4$

Step 1: Recognize the binomial components

- $a = 3x$
- $b = y$
- $n = 4$

Step 2: Write the general expansion

$$
(3x + y)^4 = \sum_{k=0}^{4} \binom{4}{k} (3x)^{4 - k} y^{k}
$$

This means we need to compute five terms (for $k=0,1,2,3,4$).

Step 3: Calculate binomial coefficients

Using $\binom{4}{k}$:

k	$\binom{4}{k}$	Calculation
0	1	$\frac{4!}{0!4!} = 1$
1	4	$\frac{4!}{1!3!} = 4$
2	6	$\frac{4!}{2!2!} = 6$
3	4	$\frac{4!}{3!1!} = 4$
4	1	$\frac{4!}{4!0!} = 1$

Step 4: Compute each term

Now, for each k:

- $k=0$:

$$
\binom{4}{0} (3x)^{4} y^{0} = 1 \times (3x)^4 \times 1 = (3x)^4
$$

Calculate $(3x)^4$:

$$
(3x)^4 = 3^4 x^4 = 81 x^4
$$

- $k=1$:

$$
\binom{4}{1} (3x)^3 y^{1} = 4 \times (3x)^3 \times y
$$

Calculate $(3x)^3$:

$$
(3x)^3 = 3^3 x^3 = 27 x^3
$$

So,

$$
4 \times 27 x^3 y = 108 x^3 y
$$

- $k=2$:

$$
\binom{4}{2} (3x)^2 y^2 = 6 \times (3x)^2 \times y^2
$$

Calculate $(3x)^2$:

$$
(3x)^2 = 3^2 x^2 = 9 x^2
$$

Thus,

$$
6 \times 9 x^2 y^2 = 54 x^2 y^2
$$

- $k=3$:

$$
\binom{4}{3} (3x)^1 y^3 = 4 \times 3x \times y^3
$$

Simplify:

$$
4 \times 3 x y^3 = 12 x y^3
$$

- $k=4$:

$$
\binom{4}{4} (3x)^0 y^4 = 1 \times 1 \times y^4 = y^4
$$

Step 5: Write the expanded form

Putting it all together:

$$
(3x + y)^4 = 81 x^4 + 108 x^3 y + 54 x^2 y^2 + 12 x y^3 + y^4
$$

This is the fully expanded form of $(3x + y)^4$.

Part B: Expanding $(x - 2y)^6$

Step 1: Recognize the binomial components

- $a = x$
- $b = -2y$
- $n = 6$

Step 2: Write the general expansion

$$
(x - 2y)^6 = \sum_{k=0}^{6} \binom{6}{k} x^{6 - k} (-2y)^{k}
$$

We will compute seven terms for $k=0$ to 6.

Step 3: Calculate binomial coefficients

Using Pascal's triangle or direct calculation:

k	$\binom{6}{k}$	Calculation
0	1	$\frac{6!}{0!6!} = 1$
1	6	$\frac{6!}{1!5!} = 6$
2	15	$\frac{6!}{2!4!} = 15$
3	20	$\frac{6!}{3!3!} = 20$
4	15	$\frac{6!}{4!2!} = 15$
5	6	$\frac{6!}{5!1!} = 6$
6	1	$\frac{6!}{6!0!} = 1$

Step 4: Compute each term

For each k:

- $k=0$:

$$
\binom{6}{0} x^{6} (-2y)^0 = 1 \times x^{6} \times 1 = x^{6}
$$

- $k=1$:

$$
6 \times x^{5} \times (-2 y) = 6 \times x^{5} \times -2 y = -12 x^{5} y
$$

- $k=2$:

$$
15 \times x^{4} \times (-2 y)^2
$$

Calculate $(-2 y)^2$:

$$
(-2 y)^2 = 4 y^2
$$

Thus,

$$
15 \times x^{4} \times 4 y^{2} = 60 x^{4} y^{2}
$$

- $k=3$:

$$
20 \times x^{3} \times (-2 y)^3
$$

Calculate $(-2 y)^3$:

$$
(-2 y)^3 = -8 y^{3}
$$

So,

\[
20 \times x^{3} \times -8 y^{3} = -160 x^{3} y^{3

Frequently Asked Questions

Use the Binomial Theorem and substitution to expand (3x + y)^4. Show your steps.

Step 1: Recall the binomial theorem: $(a + b)^n = \Sigma$ [n choose k] $a^{n-k} b^k$ for $k=0$ to n.
Step 2: Identify $a = 3x$, $b = y$, $n=4$.
Step 3: Calculate each term:

- For $k=0$: $C(4,0)\ (3x)^4\ y^0 = 1\ 81x^4\ 1 = 81x^4$
- For $k=1$: $C(4,1)\ (3x)^3\ y^1 = 4\ 27x^3\ y = 108x^3\ y$
- For $k=2$: $C(4,2)\ (3x)^2\ y^2 = 6\ 9x^2\ y^2 = 54x^2\ y^2$
- For $k=3$: $C(4,3)\ (3x)^1\ y^3 = 4\ 3x\ y^3 = 12x\ y^3$
- For $k=4$: $C(4,4)\ (3x)^0\ y^4 = 1\ 1\ y^4 = y^4$

Final expansion: $81x^4 + 108x^3 y + 54x^2 y^2 + 12x y^3 + y^4$.

Use the Binomial Theorem and substitution to expand $(x - 2y)^6$. Show your steps.

Step 1: Recall the binomial theorem: $(a + b)^n = \Sigma\ [n \text{ choose } k]\ a^{n-k} b^k$.
Step 2: Identify $a = x$, $b = -2y$, $n=6$.
Step 3: Calculate each term:
- For $k=0$: $C(6,0)\ x^6\ (-2y)^0 = 1\ x^6\ 1 = x^6$
- For $k=1$: $C(6,1)\ x^5\ (-2y) = 6\ x^5\ (-2y) = -12x^5\ y$
- For $k=2$: $C(6,2)\ x^4\ (-2y)^2 = 15\ x^4\ 4y^2 = 60x^4\ y^2$
- For $k=3$: $C(6,3)\ x^3\ (-2y)^3 = 20\ x^3\ (-8\ y^3) = -160x^3\ y^3$
- For $k=4$: $C(6,4)\ x^2\ 16\ y^4 = 15\ x^2\ 16\ y^4 = 240x^2\ y^4$
- For $k=5$: $C(6,5)\ x\ (-32\ y^5) = 6\ x\ (-32\ y^5) = -192x\ y^5$
- For $k=6$: $C(6,6)\ 1\ 64\ y^6 = 1\ 64\ y^6 = 64\ y^6$

Final expansion: $x^6 - 12x^5 y + 60x^4 y^2 - 160x^3 y^3 + 240x^2 y^4 - 192x y^5 + 64 y^6$.

Explain the step-by-step process for expanding $(3x + y)^4$ using the Binomial Theorem.

To expand $(3x + y)^4$:
1. Recognize the binomial theorem: $(a + b)^n = \Sigma\ [n \text{ choose } k]\ a^{n-k} b^k$.
2. Set $a = 3x$, $b = y$, $n=4$.
3. Compute each term:
- $k=0$: $C(4,0)\ (3x)^4\ y^0 = 1\ 81x^4\ 1 = 81x^4$
- $k=1$: $C(4,1)\ (3x)^3\ y = 4\ 27x^3\ y = 108x^3\ y$
- $k=2$: $C(4,2)\ (3x)^2\ y^2 = 6\ 9x^2\ y^2 = 54x^2\ y^2$
- $k=3$: $C(4,3)\ 3x\ y^3 = 4\ 3x\ y^3 = 12x\ y^3$
- $k=4$: $C(4,4)\ 1\ y^4 = y^4$

4. Sum all terms: $81x^4 + 108x^3 y + 54x^2 y^2 + 12x y^3 + y^4$.

Describe how substitution simplifies the process of expanding binomials with variables, with an example.

Substitution simplifies binomial expansion by allowing us to treat expressions with variables as if they are constants or simpler terms during calculation. For example, in $(3x + y)^4$, substituting $a = 3x$ and $b = y$ simplifies the expansion process by applying the binomial theorem directly to a and b. After expansion, substitute back the original expressions if needed. This approach makes complex variables manageable and reduces errors in calculations.

What are the key steps to systematically expand a binomial using the Binomial Theorem, especially for higher powers like (x - 2y)^6?

Key steps include:
1. Identify the binomial components (a and b) and the power n.
2. Write the binomial theorem formula: (a + b)^n = Σ [n choose k] a^{n-k} b^k.
3. Calculate each binomial coefficient C(n, k).
4. Compute each term by raising a and b to the appropriate powers.
5. Sum all terms to get the expanded form.
For higher powers like (x - 2y)^6, carefully compute each term, paying attention to signs and coefficients, to ensure accuracy.

How does understanding the binomial coefficients assist in expanding binomials such as (3x + y)^4 and (x - 2y)^6?

Understanding binomial coefficients (the 'n choose k' values) is essential because they determine the weight of each term in the expansion. They provide the exact number of combinations for choosing k elements from n, which directly corresponds to the coefficient of each term in the binomial expansion. Accurate calculation of these coefficients ensures the correct expansion, especially in higher powers like 4 and 6, where manual computation can be complex.

Additional Resources

Use The Binomial Theorem And Substitution To Expand Each Binomial: A Detailed Analysis

The Binomial Theorem stands as one of the fundamental principles in algebra, offering a systematic approach to expanding binomials raised to any positive integer power. Its utility extends beyond pure mathematics, finding applications in fields such as probability, combinatorics, and algebraic computations. When combined with substitution techniques, the theorem becomes even more powerful, enabling the expansion of complex binomials involving multiple variables. This article delves into the process of expanding specific binomials using the Binomial Theorem and substitution, illustrating each step with clarity and precision.

Understanding the Binomial Theorem

Before tackling specific examples, it is essential to grasp the foundation of the Binomial Theorem. The theorem provides a formula for expanding expressions of the form $(a + b)^n$, where a and b are any algebraic terms, and n is a non-negative integer.

The Binomial Theorem Formula:

$$
(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k
$$

where:

- $\binom{n}{k}$ is the binomial coefficient, calculated as $\frac{n!}{k!(n-k)!}$,
- a and b are terms within the binomial,
- $n!$ represents the factorial of n,
- k is the term index, ranging from 0 to n.

This formula allows for a systematic expansion by calculating each term's coefficient and multiplying appropriately.

Applying the Binomial Theorem with Substitution: An Overview

In many algebraic problems, binomials involve expressions with multiple variables. To expand such expressions efficiently, substitution becomes a strategic tool. Substitution involves replacing complex parts of the binomial with simpler variables, allowing the direct application of the Binomial Theorem. After expansion, the substituted variables are replaced back with the original expressions, yielding a fully expanded form.

The process typically involves:

1. Identifying the binomial expression.
2. Choosing appropriate substitution variables (if necessary).
3. Applying the Binomial Theorem to the simplified expression.
4. Reverting substitution to express the expansion in original variables.

This technique simplifies the algebraic manipulations, especially when dealing with higher powers or complex binomials.

Part A: Expanding $(3x + y)^4$ Using the Binomial Theorem

Let's now examine a specific binomial: $(3x + y)^4$. To expand this, we follow a systematic approach based on the Binomial Theorem.

Step 1: Recognize the structure

Here, $a = 3x$ and $b = y$, with $n = 4$.

Step 2: Write the general expansion

$$
(3x + y)^4 = \sum_{k=0}^4 \binom{4}{k} (3x)^{4 - k} y^{k}
$$

Step 3: Calculate binomial coefficients

$$
\binom{4}{0} = 1, \quad \binom{4}{1} = 4, \quad \binom{4}{2} = 6, \quad \binom{4}{3} = 4, \quad \binom{4}{4} = 1
$$

Step 4: Expand each term

Now, compute each term individually:

- Term 1: $k=0$

$$
\binom{4}{0} (3x)^{4} y^{0} = 1 \times (3x)^4 \times 1 = 1 \times 81x^4 \times 1 = 81x^4
$$

- Term 2: $k=1$

$$
\binom{4}{1} (3x)^3 y^{1} = 4 \times 27x^3 \times y = 4 \times 27x^3 y = 108x^3 y
$$

- Term 3: $k=2$

$$
\binom{4}{2} (3x)^2 y^{2} = 6 \times 9x^2 \times y^2 = 6 \times 9x^2 y^2 = 54x^2 y^2
$$

- Term 4: $k=3$

$$
\binom{4}{3} (3x)^1 y^{3} = 4 \times 3x \times y^3 = 4 \times 3x y^3 = 12x y^3
$$

- Term 5: $k=4$

$$
\binom{4}{4} (3x)^0 y^{4} = 1 \times 1 \times y^4 = y^4
$$

Step 5: Write the expanded form

Combining all terms, the expansion becomes:

$$
(3x + y)^4 = 81x^4 + 108x^3 y + 54x^2 y^2 + 12x y^3 + y^4
$$

Analysis:

This expansion showcases how the binomial theorem facilitates a straightforward calculation of each term based on binomial coefficients and powers. The coefficients grow systematically, and the powers of x and y adjust accordingly.

Part B: Expanding $(x - 2y)^6$ Using the Binomial Theorem

Next, we examine a binomial with a negative term: $(x - 2y)^6$. The approach remains consistent with the previous example but introduces considerations for negative signs.

Step 1: Recognize the structure

In this case,

$$
a = x, \quad b = -2y, \quad n = 6
$$

Step 2: Write the general expansion

$$
(x - 2y)^6 = \sum_{k=0}^6 \binom{6}{k} x^{6 - k} (-2y)^k
$$

Step 3: Calculate binomial coefficients

$$
\binom{6}{0} = 1, \quad \binom{6}{1} = 6, \quad \binom{6}{2} = 15, \quad \binom{6}{3} = 20, \quad \binom{6}{4} = 15, \quad \binom{6}{5} = 6, \quad \binom{6}{6} = 1
$$

Step 4: Expand each term

- Term 1: $k=0$

$$
\binom{6}{0} x^{6} (-2y)^0 = 1 \times x^6 \times 1 = x^6
$$

- Term 2: $k=1$

$$
6 \times x^{5} \times (-2y) = 6 \times x^5 \times (-2y) = -12 x^5 y
$$

- Term 3: $k=2$

$$
15 \times x^{4} \times (-2y)^2 = 15 \times x^4 \times 4 y^2 = 15 \times 4 x^4 y^2 = 60 x^4 y^2
$$

- Term 4: $k=3$

$$
20 \times x^{3} \times (-2 y)^3 = 20 \times x^3 \times (-8 y^3) = -160 x^3 y^3
$$

- Term 5: $k=4$

$$
15 \times x^{2} \times (-2 y)^4 = 15 \times x^2 \times 16 y^4 = 240 x^2 y^4
$$

- Term 6: $k=5$

$$
6 \times x \times (-2 y)^5 = 6 \times x \times (-32 y^5) = -192 x y^5
$$

- Term 7: $k=6$

$$

1 \times 1 \times (-2 y)^6 = 1 \times 1 \times 64 y^6 = 64 y^6
\]

Step 5: Write the expanded form

Putting all terms together:

\[
(x - 2 y)^6 = x^6 - 12 x^5 y + 60 x^4 y^2 - 160 x^3 y^3 + 240 x^2 y^4 - 192 x y^5 + 64 y^6
\]

Analysis:

The expansion highlights how negative signs influence the coefficients, with alternating signs appearing due to the powers of \(-2 y\). The binomial theorem handles these signs seamlessly, provided the powers and coefficients are correctly computed.

The Role of Sub

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