What Are The Coordinates Of The Vertices Of The Figure After The Given Transformation?Translation: (1,y)

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Understanding how geometric figures change under transformations is fundamental in coordinate geometry. When a translation is applied to a figure, each point of the figure moves uniformly in a specified direction, resulting in a new position for the entire shape. In this article, we explore the specific case of translating a figure by the vector (1, y), where y can be any real number. We will analyze the effect of this translation on the coordinates of the figure's vertices, illustrate the process with examples, and discuss how to determine the new coordinates after the transformation.

Understanding Translations in Coordinate Geometry

Definition of Translation

A translation in coordinate geometry is a type of transformation that shifts every point of a figure by the same distance in a given direction. This movement does not alter the size, shape, or orientation of the figure; it only changes its position in the coordinate plane. The translation is described by a vector, which indicates the horizontal and vertical shifts.

Translation Vector (a, b)

The translation vector is represented as (a, b), where:
- a: the horizontal displacement (positive to the right, negative to the left)
- b: the vertical displacement (positive upward, negative downward)

In our case, the translation vector is (1, y), meaning:
- Every point moves 1 unit to the right.
- Every point moves y units vertically, either upward (if $y > 0$) or downward (if $y < 0$).

Applying the Translation: Effect on Vertices

General Formula for Coordinates After Translation

Suppose a figure has vertices with coordinates:

- $V_1 (x_1, y_1)$

- $V_2 (x_2, y_2)$
- $V_3 (x_3, y_3)$
- $V_4 (x_4, y_4)$

Applying the translation vector (1, y), the new coordinates V'_i will be:

$$
V'_i (x'_i, y'_i) = (x_i + 1, y_i + y)
$$

for each $i = 1, 2, 3, 4$.

This simple addition applies uniformly to all vertices of the figure.

Effect on Different Types of Figures

- Polygons (Triangles, Quadrilaterals, etc.): All vertices shift uniformly, preserving the shape and size.
- Circles: The center shifts by the translation vector, with the radius remaining unchanged.
- Complex Figures: All points move identically, maintaining the figure's integrity.

Step-by-Step Example of Transformation

Initial Coordinates of a Sample Figure

Let’s consider a quadrilateral with vertices:

- $V_1 (2, 3)$
- $V_2 (4, 5)$
- $V_3 (6, 3)$
- $V_4 (4, 1)$

Suppose we are asked to find the vertices’ coordinates after translating the figure by the vector (1, y), where y is a specific value, say, y = -2.

Calculating the New Coordinates

Applying the translation:

- For V_1:

$$
x'_1 = 2 + 1 = 3
$$
$$
y'_1 = 3 + (-2) = 1
$$

- For V_2:

$$
x'_2 = 4 + 1 = 5
$$
$$
y'_2 = 5 + (-2) = 3
$$

- For V_3:

$$
x'_3 = 6 + 1 = 7
$$
$$
y'_3 = 3 + (-2) = 1
$$

- For V_4:

$$
x'_4 = 4 + 1 = 5
$$
$$
y'_4 = 1 + (-2) = -1
$$

Thus, the new vertices are:

- $V'_1 (3, 1)$
- $V'_2 (5, 3)$
- $V'_3 (7, 1)$
- $V'_4 (5, -1)$

This translation shifts the entire figure 1 unit to the right and 2 units downward.

General Approach for Any Value of y

Handling Variable y in the Translation Vector

If the translation vector is (1, y), and y is an arbitrary real number, the process remains the same:

1. Identify the original coordinates of each vertex.
2. Add 1 to each x-coordinate.
3. Add y to each y-coordinate.

This straightforward addition is applied to all vertices, resulting in a new, translated figure.

Implications of Different y Values

- Positive y: Moves the figure upward.
- Negative y: Moves the figure downward.
- Zero y: No vertical movement; only shifts to the right.

This flexibility allows for precise positioning of figures in the coordinate plane.

Important Considerations and Applications

Preservation of Shape and Size

Translations do not distort or alter the size of the figure; they only change its position. This makes translation a rigid motion in geometry.

Coordinate Transformation in Practical Scenarios

- Computer Graphics: Moving objects across the screen.
- Robotics: Positioning parts or robots in different locations.
- Mathematical Proofs: Demonstrating congruence and similarity through transformations.

Graphical Representation

To visualize the translation:
- Plot the original figure with its vertices.
- Apply the translation vector to each vertex.
- Connect the new vertices to form the translated figure.

This visual approach reinforces understanding of how translation affects figures.

Summary and Key Takeaways

- A translation moves every point of a figure by the same vector.
- The new coordinates after a translation by (1, y) are obtained by adding 1 to all x-coordinates and y to all y-coordinates.
- The shape, size, and orientation of the figure remain unchanged.
- The translation vector determines the direction and magnitude of the shift.

Final Note

Understanding the effect of translations is fundamental in coordinate geometry, enabling precise manipulations of figures in the plane. Whether for theoretical purposes or practical applications, grasping how to determine the new vertices after a translation allows for efficient and accurate geometric constructions.

In conclusion, given an original figure with known vertices, applying the translation (1, y) results in new vertices calculated by adding 1 to each x-coordinate and y to each y-coordinate. This simple yet powerful transformation is essential for various mathematical and real-world tasks involving shape movement and positioning.

Frequently Asked Questions

How does translating a figure by (1,y) affect the coordinates of its vertices?

Translating a figure by (1,y) shifts every vertex's x-coordinate by 1 unit and every y-coordinate by y units, so each vertex's new coordinates are (x+1, y+y).

What is the general formula to find the new vertices after a translation of (1,y)?

If a vertex has original coordinates (x, y), after translation by (1, y), the new coordinates become (x + 1, y + y).

Given a triangle with vertices at (2,3), (4,5), and (6,7), what are the new vertices after translating by (1, y)?

The new vertices are at (2+1, 3 + y) = (3, 3 + y), (4+1, 5 + y) = (5, 5 + y), and (6+1, 7 + y) = (7, 7 + y).

If a quadrilateral has vertices at (0,0), (2,0), (2,2), and (0,2), what are its vertices after translation by (1, y)?

The new vertices will be at (0+1, 0 + y) = (1, y), (2+1, 0 + y) = (3, y), (2+1, 2 + y) = (3, 2 + y), and (0+1, 2 + y) = (1, 2 + y).

How do you determine the coordinates of a figure's vertices after an arbitrary translation like (a, b)?

Add 'a' to each vertex's x-coordinate and 'b' to each y-coordinate; for a vertex (x, y), the new coordinates are (x + a, y + b).

Does the translation (1,y) change the shape or size of the figure?

No, translation shifts the figure's position without changing its shape, size, or orientation; it simply moves the entire figure by the specified amount.

Additional Resources

What Are The Coordinates Of The Vertices Of The Figure After The Given Transformation? Translation: (1, y)

Introduction

Transformations are fundamental concepts in coordinate geometry, enabling us to understand how figures move within the plane. Among these transformations, translation is perhaps the most straightforward, involving shifting a figure from one position to another without altering its size or shape. When a figure undergoes a translation, each of its vertices moves along a vector, and the new coordinates can be precisely determined using simple addition.

This article delves into the detailed process of determining the coordinates of the vertices of a figure after a translation specified as $(1, y)$. We will explore the mathematical principles underpinning translation, analyze various scenarios, and provide comprehensive examples to clarify the concepts. The focus will also include practical applications and common pitfalls to watch out for when performing such transformations.

Understanding Translation in Coordinate Geometry

Definition of Translation

In coordinate geometry, a translation is a transformation that slides every point of a figure the same distance in the same direction. Essentially, it moves the entire figure without changing its shape, size, or orientation.

Mathematically, a translation can be represented by a vector $\vec{T} = (a, b)$, where:
- a is the horizontal shift (positive for rightward movement, negative for leftward).
- b is the vertical shift (positive for upward movement, negative for downward).

Applying this translation to a point (x, y) results in a new point (x', y') given by:

$$x' = x + a$$

$$y' = y + b$$

The Specific Transformation: $(1, y)$

The transformation under consideration is a translation with a fixed horizontal component of 1 and a vertical component of y:

$$\textbf{Translation vector} = (1, y)$$

This means every point of the figure moves 1 unit to the right and y units up or down depending on the sign and magnitude of y.

Note: Since y is a variable in the translation vector, the actual vertical shift depends on the specific value of y. If y is known, the translation becomes a concrete shift; if not, the translation remains a general operation.

Detailed Analysis of the Transformation

1. Impact on Vertices of a Figure

Suppose the original figure has vertices at points:

$$
V_1 (x_1, y_1), \quad V_2 (x_2, y_2), \quad V_3 (x_3, y_3), \quad \dots
$$

Applying the translation $(1, y)$ results in:

$$
V_1' (x_1 + 1, y_1 + y)
$$

$$
V_2' (x_2 + 1, y_2 + y)
$$

$$
V_3' (x_3 + 1, y_3 + y)
$$

and so forth for all vertices.

This process is linear and straightforward, with each vertex shifted identically in both directions.

Cases Based on the Nature of y

Case 1: y is a Known Constant

When y is a specific number (e.g., $y = 3$), the translation becomes concrete:
- Horizontal shift: +1 unit
- Vertical shift: +3 units (upward)

Example:

Original vertices of a triangle:

$$
A (2, 3), \quad B (4, 5), \quad C (3, 2)
$$

Applying translation $(1, 3)$:

\[

$$
A' (2+1, 3+3) = (3, 6)
$$

$$
B' (4+1, 5+3) = (5, 8)
$$

$$
C' (3+1, 2+3) = (4, 5)
$$

The translated triangle's vertices are $(3, 6), (5, 8), (4, 5)$.

Case 2: y is a Variable

When y remains a variable (say, $y = k$), the transformation results in a family of figures depending on the value of k.

Example:

Original vertices:

$$
D (1, 2), \quad E (3, 4)
$$

Translation:

$$
D' (1 + 1, 2 + y) = (2, 2 + y)
$$

$$
E' (3 + 1, 4 + y) = (4, 4 + y)
$$

The new vertices depend on y, illustrating how the figure shifts vertically by an amount dictated by y.

Visualizing the Transformation

Plotting the original and translated figures can provide intuitive understanding:

- The figure shifts horizontally by 1 unit to the right.
- The figure shifts vertically by y units, either upward (if $y > 0$) or downward (if $y < 0$).

Understanding these shifts is crucial for applications in fields like computer graphics, physics simulations, and geometric proofs.

Application in Geometric Problems

The process of calculating new vertex coordinates after translation is a fundamental skill in solving geometric problems involving figures on the coordinate plane. Common applications include:

- Designing geometric transformations to achieve desired placements.
- Analyzing the movement of shapes in physics or engineering contexts.
- Constructing figures in computer graphics with precise positional control.
- Proofs involving congruence or similarity after transformations.

Step-by-Step Procedure for Finding Coordinates Post-Translation

1. Identify the original vertices of the figure.
2. Determine the translation vector $(1, y)$.
3. Add the components of the translation vector to each vertex:
- New x coordinate: $x_{new} = x_{old} + 1$
- New y coordinate: $y_{new} = y_{old} + y$
4. Record the new vertices to define the translated figure.

Common Pitfalls and Tips

- Confusing the variables: Ensure clarity whether y in the translation vector is a fixed number or a variable.
- Sign errors: Remember that positive y shifts upward, negative downward.
- Coordinate consistency: Always apply the translation to all vertices uniformly.
- Graphical verification: Sketch the original and translated figures for visual confirmation.

Practical Example with a Quadrilateral

Suppose a quadrilateral has vertices:

$$
V_1 (0, 0), \quad V_2 (2, 0), \quad V_3 (2, 3), \quad V_4 (0, 3)
$$

Applying translation $(1, y)$:

- If $y = 2$:
- $V_1' (0 + 1, 0 + 2) = (1, 2)$
- $V_2' (2 + 1, 0 + 2) = (3, 2)$
- $V_3' (2 + 1, 3 + 2) = (3, 5)$
- $V_4' (0 + 1, 3 + 2) = (1, 5)$

The resulting quadrilateral is shifted rightward by 1 unit and upward by 2 units, maintaining its shape and size.

Extending to Complex Figures and Multiple Transformations

While this discussion centers on a simple translation, more complex transformations—such as rotations, reflections, or scaling—may follow similar principles but involve different calculations. When combining transformations, apply each step sequentially or use matrix operations for efficiency.

Conclusion

Understanding the coordinates of a figure's vertices after a translation \((1, y)\) is fundamental in coordinate geometry. The key principle is that each vertex's \(x\)-coordinate increases by 1, and each \(y\)-coordinate increases by \(y \). This straightforward operation allows precise control over the placement of geometric figures within the coordinate plane.

Whether working with fixed values or variables, the process remains consistent and reliable, forming a cornerstone for more advanced geometric transformations and analyses. Mastery of this concept enhances problem-solving skills and deepens comprehension of spatial relationships in mathematics.

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Author's Note

Understanding how to determine the coordinates after translation is essential for students, educators, and professionals working in fields that rely on geometric reasoning. The clarity in identifying the translation vector and applying it consistently to all vertices ensures accurate and efficient transformations.

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