What Is The Nuclear Binding Energy Per Mole If The Mass Defect Of A Certain Isotope Is 0.1303 U Per Atom

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Understanding nuclear binding energy is fundamental to grasping how atomic nuclei are held together and the energy associated with nuclear reactions. In this article, we will explore how to calculate the nuclear binding energy per mole for an isotope given its mass defect per atom, specifically 0.1303 atomic mass units (U). We will delve into the concepts of mass defect, nuclear binding energy, and the steps involved in converting atomic-level data into molar quantities, providing a comprehensive overview suitable for students, educators, and science enthusiasts alike.

Fundamentals of Nuclear Binding Energy

What Is Nuclear Binding Energy?

Nuclear binding energy is the energy required to disassemble a nucleus into its individual protons and neutrons. It is also the energy released when a nucleus is formed from free nucleons, making it a key indicator of nuclear stability. The greater the binding energy, the more stable the nucleus.

Mathematically, the binding energy (BE) for a nucleus with atomic number Z (number of protons) and neutron number N (number of neutrons) is:

$$ BE = (Z \times m_p + N \times m_n - M) \times c^2 $$

where:
- m_p = mass of a proton
- m_n = mass of a neutron
- M = actual mass of the nucleus
- c = speed of light in vacuum

However, because masses are often given in atomic mass units (u), and energy in MeV, a simplified approach uses the mass defect in atomic mass units and a conversion factor.

Mass Defect and Its Significance

The mass defect is the difference between the sum of the individual masses of protons and neutrons and the actual mass of the nucleus:

$$ \text{Mass defect} (\Delta m) = \left(Z \times m_p + N \times m_n \right) - M $$

This defect accounts for the mass converted into binding energy, according to Einstein's mass-energy equivalence:

$$ E = \Delta m \times c^2 $$

Expressed in energy units, the mass defect directly correlates with the nuclear binding energy.

Calculating the Nuclear Binding Energy Per Atom

Given that the mass defect per atom is 0.1303 U, where 1 U (atomic mass unit) is approximately 1.66053906660 × 10^{-27} kg, we can determine the energy associated with this defect.

Step 1: Convert Mass Defect from Atomic Mass Units to Energy (MeV)

The conversion factor from atomic mass units to energy in MeV is well-established:

$$ 1\, \text{u} \approx 931.5\, \text{MeV} $$

Therefore, the binding energy per atom:

$$ E_{atom} = \Delta m (u) \times 931.5\, \text{MeV} $$

Plugging in the given value:

$$ E_{atom} = 0.1303\, u \times 931.5\, \text{MeV} $$

$$ E_{atom} \approx 0.1303 \times 931.5 $$

$$ E_{atom} \approx 121.32\, \text{MeV} $$

This is the nuclear binding energy for a single atom of the isotope.

Step 2: Convert Energy per Atom to Energy Per Mole

A mole of any substance contains Avogadro's number of particles:

$$ N_A = 6.022 \times 10^{23} $$

The total binding energy per mole:

$$ E_{mole} = E_{atom} \times N_A $$

$$ E_{mole} = 121.32\, \text{MeV} \times 6.022 \times 10^{23} $$

Calculating this:

$$ E_{mole} \approx 7.297 \times 10^{25}\, \text{MeV} $$

Step 3: Expressing the Molar Binding Energy in Joules (Optional)

While MeV is convenient in nuclear physics, energy in joules is often more practical in chemistry and engineering contexts. To convert MeV to joules:

$$ 1\, \text{MeV} = 1.60218 \times 10^{-13}\, \text{J} $$

Thus,

$$ E_{mole} \approx 7.297 \times 10^{25}\, \text{MeV} \times 1.60218 \times 10^{-13}\, \text{J/MeV} $$

$$ E_{mole} \approx 1.169 \times 10^{13}\, \text{J} $$

This is the molar binding energy in joules, indicating the energy required to disassemble one mole of the isotope into individual protons and neutrons.

Understanding the Significance of These Calculations

Implications for Nuclear Stability

A higher binding energy per mole signifies a more stable nucleus. It reflects the strength of the nuclear forces holding the nucleus together. Isotopes with higher molar binding energies tend to be more stable and less likely to undergo radioactive decay.

Applications in Nuclear Physics and Engineering

- Nuclear Power: Knowing the binding energy helps in calculating the energy released during nuclear fission or fusion.
- Radioactive Decay: Isotopes with lower binding energies are generally more unstable, leading to decay processes.
- Nuclear Synthesis: Understanding binding energies guides scientists in creating new, stable isotopes.

Additional Factors to Consider

Mass Defect Variability

The mass defect varies among different isotopes, affecting their binding energies. The specific numbers provided are for a particular isotope; different isotopes will have different mass defects and, consequently, different binding energies.

Precision in Calculations

While the calculations above are straightforward, real-world measurements involve uncertainties in mass measurements, and the actual binding energy might slightly differ due to nuclear effects not accounted for in simplified models.

Summary of Key Points

- The nuclear binding energy per atom is calculated by multiplying the mass defect in atomic mass units by 931.5 MeV/u.
- For a mass defect of 0.1303 U, the binding energy per atom is approximately 121.32 MeV.
- Converting to molar quantities involves multiplying by Avogadro's number, resulting in roughly 7.297×10^{25} MeV per mole.
- The molar binding energy in joules is approximately 1.169×10^{13} J, indicating substantial energy associated with nuclear stability.
- Understanding these concepts is vital for applications in nuclear energy, medicine, and astrophysics.

Conclusion

Calculating the nuclear binding energy per mole from the atomic mass defect provides profound insights into the stability and energy dynamics of atomic nuclei. The process involves converting the mass defect into energy units, then scaling up to a molar quantity using Avogadro's number. Given the mass defect of 0.1303 U per atom, the resulting molar binding energy is approximately 1.169×10^{13} joules, highlighting the immense energy stored within atomic nuclei. Mastery of these calculations enables scientists and engineers to harness nuclear energy effectively and deepen our understanding of atomic structure and nuclear reactions.

Frequently Asked Questions

What is nuclear binding energy per mole when the mass defect per atom is 0.1303 u?

The nuclear binding energy per mole is approximately 7.78 kJ/mol when the mass defect per atom is 0.1303 u.

How do you calculate the nuclear binding energy from the mass defect per atom?

You multiply the mass defect per atom (in atomic mass units) by 931.5 MeV/u to get the energy per atom, then convert that energy to kJ/mol by multiplying by Avogadro's number and converting units.

Why is the atomic mass unit (u) used in calculating nuclear binding energy?

Because the mass defect is given in atomic mass units, which directly relate to the mass differences in atomic nuclei, facilitating energy calculations via Einstein's equation.

What is the significance of the mass defect in nuclear physics?

The mass defect represents the mass converted into binding energy, indicating the stability of the nucleus; a larger defect generally means a more stable nucleus.

How do you convert the energy from MeV to kJ when calculating binding energy per mole?

Use the conversion factor 1 MeV = 1.60218×10^{-13} Joules, then multiply by Avogadro's number (6.022×10^{23}) to get kJ/mol.

What is the approximate binding energy per nucleon for this isotope?

Given the mass defect per atom, the binding energy per nucleon can be estimated by dividing the total binding energy per atom by the number of nucleons, providing insight into nuclear stability.

Can the binding energy per mole be used to compare different isotopes?

Yes, higher binding energy per mole generally indicates a more stable nucleus and can be used to compare the stability of different isotopes.

What assumptions are involved in calculating the nuclear binding energy from mass defect?

Assumptions include that the mass defect is solely due to nuclear binding energy, and that relativistic effects and electron mass contributions are negligible or accounted for in atomic mass units.

Additional Resources

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Introduction to Nuclear Binding Energy

Nuclear binding energy is a fundamental concept in nuclear physics that quantifies the stability of an atomic nucleus. It essentially measures how much energy would be required to disassemble a nucleus into its individual protons and neutrons, or conversely, how much energy is released when a nucleus is formed from these nucleons. Understanding binding energy per mole provides insights into nuclear stability, nuclear reactions, and energy production processes such as fission and fusion.

In this discussion, we will analyze the nuclear binding energy per mole for an isotope given its mass defect per atom as 0.1303 atomic mass units (u). This involves converting the mass defect into energy using Einstein’s mass-energy equivalence and then scaling it to a molar basis.

Understanding Mass Defect

What is Mass Defect?

Mass defect refers to the difference between the sum of the individual masses of free nucleons (protons and neutrons) and the actual mass of the bound nucleus. It arises because:

- When nucleons bind together, some mass is converted into binding energy.
- The nucleus has a lower mass than the sum of its constituent particles because of this energy release, in accordance with Einstein’s mass-energy equivalence $E=mc^2$.

Mathematically, the mass defect (Δm) per atom is given as:

$$
\Delta m = \text{Mass of free nucleons} - \text{Mass of the nucleus}
$$

In our case, the mass defect per atom is provided as 0.1303 u.

Significance of Mass Defect

- Indicates the energy associated with nuclear binding.
- Larger mass defect generally correlates with more stable nuclei.
- Essential for calculating the binding energy, which helps predict nuclear stability and energy release potential.

Calculating the Nuclear Binding Energy Per Atom

Step 1: Convert Mass Defect to Energy

Using Einstein's relation:

$$
E = \Delta m c^2
$$

where:

- $\Delta m = 0.1303\, \text{u}$,
- $c = 3.00 \times 10^8\, \text{m/s}$.

However, since the mass defect is given in atomic mass units, it's more convenient to use the conversion factor:

$$
1\, \text{u} = 931.5\, \text{MeV}/c^2
$$

Thus, the binding energy per atom:

$$
E_{\text{atom}} = \Delta m \times 931.5\, \text{MeV}
$$

Calculating:

$$
E_{\text{atom}} = 0.1303 \times 931.5\, \text{MeV} \approx 121.3\, \text{MeV}
$$

\]

Result: The nuclear binding energy per atom is approximately 121.3 MeV.

Calculating the Binding Energy Per Mole

Step 2: Convert to Per Mole Basis

A mole of particles contains Avogadro’s number (\(N_A\)) of atoms:

\[
N_A = 6.022 \times 10^{23}\, \text{atoms/mol}
\]

The total binding energy per mole:

\[
E_{\text{mole}} = E_{\text{atom}} \times N_A
\]

Expressed:

\[
E_{\text{mole}} = 121.3\, \text{MeV} \times 6.022 \times 10^{23}
\]

But to express energy in standard units (joules), we need to convert MeV to joules:

\[
1\, \text{MeV} = 1.602 \times 10^{-13}\, \text{J}
\]

Therefore:

\[
E_{\text{atom}} = 121.3\, \text{MeV} \times 1.602 \times 10^{-13}\, \text{J/MeV} \approx 1.943 \times 10^{-11}\, \text{J}
\]

Now, the molar binding energy:

\[
E_{\text{mole}} \approx 1.943 \times 10^{-11}\, \text{J} \times 6.022 \times 10^{23} \approx 11.7\, \text{MJ/mol}
\]

Result: The nuclear binding energy per mole is approximately 11.7 megajoules.

Implications and Significance of the Result

Understanding Nuclear Stability

- The calculated binding energy per mole (~11.7 MJ/mol) indicates a relatively stable nucleus, as higher binding energy per nucleon correlates with stability.
- For comparison, iron-56, one of the most stable nuclei, has a binding energy per nucleon of about 8.8 MeV, which corresponds to a high total binding energy per nucleon. Our isotope's total energy per mole suggests it is energetically favorable compared to less stable nuclei.

Energy Release in Nuclear Reactions

- The magnitude of the binding energy per mole indicates the potential energy that could be released in nuclear reactions involving this isotope.
- If this isotope undergoes fission or fusion, the energy difference relative to reactants' binding energies can be calculated, providing insights into energy yields.

Applications in Nuclear Power

- Knowledge of binding energies helps in designing nuclear reactors and understanding fuel efficiency.
- High binding energy per mole often correlates with the energy released during nuclear decay or reactions, informing reactor fuel choices.

Additional Considerations

Effect of Isotope Variations

- Different isotopes of an element may have varying mass defects, affecting their binding energy per nucleon.
- Comparing isotopes helps identify the most stable forms, influencing nuclear astrophysics, decay chains, and isotope selection for applications.

Limitations of the Calculation

- The calculation assumes the mass defect per atom is accurate and that the isotope is isolated.
- Real-world factors such as nuclear excitation states and environmental effects are not considered here.

Potential for Further Exploration

- Investigating the binding energy per nucleon (binding energy divided by the mass number) for a more normalized comparison.
- Analyzing how the binding energy influences decay modes and half-lives.

Summary

To summarize, starting with a mass defect per atom of 0.1303 u, we calculated the nuclear binding energy per atom to be approximately 121.3 MeV. Scaling this up to a molar basis, the binding energy per mole is roughly 11.7 megajoules. This substantial energy reflects the stability of the nucleus and its potential energy release in nuclear processes.

Understanding these calculations provides a deeper appreciation of nuclear stability and energy dynamics, which are foundational for nuclear physics, astrophysics, and practical applications such as energy generation and medical isotope production.

Final Remarks

The process of determining the nuclear binding energy per mole from the mass defect involves a straightforward but insightful sequence of conversions. It exemplifies how tiny differences in atomic masses translate into enormous energies, underpinning the power of nuclear reactions. As nuclear science advances, such calculations remain pivotal for innovations in energy, medicine, and understanding the universe’s fundamental structure.

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