Write An Equation In Slope-intercept Form For The Line That Passesthrough (-8, -32) And Is Perpendicular

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Understanding how to find the equation of a line that passes through a specific point and is perpendicular to another line is an essential skill in algebra and coordinate geometry. This comprehensive guide will walk you through the process of deriving the slope-intercept form of such a line, using the point (-8, -32) as an example. Whether you're a student preparing for exams or someone interested in mastering geometric concepts, this article provides step-by-step instructions, key formulas, and practical tips to help you confidently solve similar problems.

Fundamental Concepts in Line Equations and Perpendicular Lines

Before diving into the specific problem, it's important to review some foundational concepts related to line equations, slopes, and perpendicularity.

What Is the Slope-Intercept Form?

The slope-intercept form of a linear equation is a popular way to express a line's equation:

$$ y = mx + b $$

where:
- $ y $ is the dependent variable,
- $ x $ is the independent variable,
- $ m $ is the slope of the line,
- $ b $ is the y-intercept (the point where the line crosses the y-axis).

This form makes it easy to identify the slope and y-intercept directly from the equation.

Understanding Slope and Its Significance

The slope ($ m $) indicates the steepness and direction of the line:
- A positive slope means the line inclines upward from left to right.
- A negative slope indicates a downward incline.

- A slope of zero denotes a horizontal line.
- An undefined slope (vertical line) is not expressed in slope-intercept form.

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). The key property relating their slopes is:

$$ m_1 \times m_2 = -1 $$

where:
- $ m_1 $ is the slope of the first line,
- $ m_2 $ is the slope of the perpendicular line.

This means:
- The slopes are negative reciprocals of each other.
- If one line has slope $ m $, the perpendicular line has slope $ -\frac{1}{m} $.

Understanding this reciprocal relationship is crucial for solving problems involving perpendicular lines.

Step-by-Step Guide to Find the Equation of the Perpendicular Line

Now, let's proceed with the specific problem: Find the equation in slope-intercept form of the line passing through the point (-8, -32) and perpendicular to another known line or based on a given slope.

Step 1: Determine the Slope of the Original Line

To find a perpendicular line, you need to know the slope of the line to which the new line is perpendicular. Since the problem states "passesthrough (-8, -32) and is perpendicular," but doesn't specify the original line's equation, we can interpret it as:

- Either the original line's slope is given.
- Or, if not given, the problem might be asking to find the line perpendicular to a specific line.

Assumption: If the problem is to find the line perpendicular to a known line with a given slope $ m $, then proceed accordingly.

Example: Suppose the original line has slope $ m_{orig} = \frac{2}{3} $.

In this case, the perpendicular slope m_{perp} is:

$$m_{perp} = -\frac{1}{m_{orig}} = -\frac{1}{\frac{2}{3}} = -\frac{3}{2}$$

Note: If the original line's slope isn't specified, you'll need to know or be provided that value.

Step 2: Find the Slope of the Perpendicular Line

Using the reciprocal relationship, identify or compute the slope of the perpendicular line:

$$m_{perp} = -\frac{1}{m_{orig}}$$

For example, if $m_{orig} = 4$, then:

$$m_{perp} = -\frac{1}{4}$$

If the original line's slope is unknown, and you're asked to find the line perpendicular to a given line, ensure you identify or calculate m_{orig} first.

Step 3: Use the Point-Slope Form to Find the Equation

Once the slope of the perpendicular line (m_{perp}) is known, and the point (-8, -32) is given, use the point-slope form:

$$y - y_1 = m (x - x_1)$$

where:
- $(x_1, y_1) = (-8, -32)$,
- $m = m_{perp}$.

Plug in the values:

$$y - (-32) = m_{perp} (x - (-8))$$
$$y + 32 = m_{perp} (x + 8)$$

Step 4: Convert to Slope-Intercept Form

Distribute m_{perp}:

$$ y + 32 = m_{perp} x + 8 m_{perp} $$

Subtract 32 from both sides:

$$ y = m_{perp} x + 8 m_{perp} - 32 $$

This is the slope-intercept form:

$$ y = m_{perp} x + b $$

where the y-intercept $ b = 8 m_{perp} - 32 $.

Example: Complete Solution with a Specific Original Line Slope

Let's walk through a complete example with specific values.

Suppose the original line has slope $ m_{orig} = 3 $.

Step 1: Find the perpendicular slope:

$$ m_{perp} = -\frac{1}{3} $$

Step 2: Use point (-8, -32) to write the equation:

$$ y - (-32) = -\frac{1}{3} (x - (-8)) $$
$$ y + 32 = -\frac{1}{3} (x + 8) $$

Step 3: Distribute:

$$ y + 32 = -\frac{1}{3} x - \frac{8}{3} $$

Step 4: Subtract 32 from both sides:

$$ y = -\frac{1}{3} x - \frac{8}{3} - 32 $$

Express 32 as a fraction with denominator 3:

$$ 32 = \frac{96}{3} $$

So:

$$ y = -\frac{1}{3} x - \frac{8}{3} - \frac{96}{3} = -\frac{1}{3} x - \frac{104}{3} $$

Final Equation:

$$ y = -\frac{1}{3} x - \frac{104}{3} $$

This is the slope-intercept form of the line passing through (-8, -32) and perpendicular to the original line with slope 3.

Additional Tips and Common Mistakes

To ensure accuracy and clarity, consider these tips:

- **Always identify the original line's slope:** The key to finding a perpendicular line is knowing the original slope.
- **Convert fractions carefully:** When dealing with fractions, ensure proper numerator and denominator handling to avoid mistakes.
- **Check your work:** Substitute the point into the derived equation to verify correctness.
- **Remember vertical and horizontal lines:** If the original line is vertical (undefined slope), the perpendicular line will be horizontal (slope 0), and vice versa.

Practice Problems to Enhance Your Understanding

Try solving these problems to reinforce your skills:

1. Find the equation of the line passing through (2, 5) and perpendicular to the line $y = -\frac{1}{2} x + 3$.
2. Determine the equation of the line passing through (-7, 4) and perpendicular to the line $y = 4x - 6$.
3. Given the line $y = 3x + 1$, find the equation of the line passing through (0, 0) and perpendicular to it.

Conclusion: Mastering Equations of Perpendicular Lines

Finding the equation in slope-intercept form for a line passing through a specific point and perpendicular to another line involves understanding the relationship between slopes, applying the point-slope formula, and transforming the result into slope-intercept form. By mastering these steps and concepts, you can confidently approach similar problems, enhancing your algebraic and geometric problem-solving skills. Remember, practice makes perfect—so work through various problems to solidify your understanding and become proficient in working with line equations and perpendicularity.

Keywords for SEO Optimization:
- Write an equation in slope-intercept form
- Perpendicular line equation
- Line passing through point

Frequently Asked Questions

How do you find the slope of a line perpendicular to a given line?

To find the slope of a line perpendicular to a given line, first determine the slope of the original line and then take its negative reciprocal. If the original slope is m, the perpendicular slope is -1/m.

What is the slope of the line passing through (-8, -32) if the line is perpendicular to a known line with slope 2?

The slope of the perpendicular line is -1/2, since the negative reciprocal of 2 is -1/2.

How do you write the equation of a line in slope-intercept form given a point and slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, then convert it to slope-intercept form $y = mx + b$ by solving for y.

What is the slope of the line passing through (-8, -32) if it is perpendicular to a line with slope 3?

The slope of the perpendicular line is -1/3, as it is the negative reciprocal of 3.

How do you find the equation in slope-intercept form for a line passing through (-8, -32) with a slope of -1/2?

Plug the point and slope into y = mx + b: -32 = (-1/2)(-8) + b. Simplify to find b, then write the equation as y = -1/2 x + 24.

What are the steps to write the equation of a perpendicular line passing through a specific point?

First, find the slope of the original line, then determine the negative reciprocal for the perpendicular line. Use the point and this slope in the point-slope formula, then convert to slope-intercept form.

If a line passes through (-8, -32) and is perpendicular to the line y = 4x + 1, what is its equation?

The slope of the original line is 4, so the perpendicular slope is -1/4. Use the point (-8, -32) in y = mx + b: -32 = (-1/4)(-8) + b, which simplifies to -32 = 2 + b, giving b = -34. The equation is y = -1/4 x - 34.

How do you verify that two lines are perpendicular based on their slopes?

Two lines are perpendicular if the product of their slopes is -1. For example, slopes 2 and -1/2 are perpendicular because 2 -1/2 = -1.

Can you give a general formula for the equation of a line perpendicular to a given line passing through a point?

Yes. If the original line has slope m, then the perpendicular line has slope -1/m. To find its equation through point (x_1, y_1), use $y - y_1 = (-1/m)(x - x_1)$ and convert to slope-intercept form.

Additional Resources

Write An Equation In Slope-intercept Form For The Line That Passes Through (-8, -32) And Is Perpendicular

Understanding how to find the equation of a line in slope-intercept form is a fundamental skill in algebra and coordinate geometry. When the problem involves a line passing through a specific point and perpendicular to another line, it introduces an additional layer of complexity that requires a solid grasp of slopes and their relationships. This article aims to guide you through the process of deriving the equation of such a line step-by-step, making the concepts accessible even if you're new to the topic. Whether you're a student working on homework or an enthusiast brushing up on geometry principles, this comprehensive overview will clarify the methodology involved.

Foundations of Line Equations and Slope-Intercept Form

Before diving into the specific problem, it’s essential to understand the basics of line equations and the slope-intercept form.

The Slope-Intercept Form

The slope-intercept form of a line's equation is expressed as:

$$ y = mx + b $$

where:
- $ m $ is the slope of the line,
- $ b $ is the y-intercept, the point where the line crosses the y-axis.

This form is especially convenient because it directly reveals the slope and the y-intercept, making it easy to graph and analyze lines.

Understanding Slopes and Perpendicular Lines

The slope ($ m $) indicates the steepness and direction of a line. When two lines are perpendicular, their slopes are negative reciprocals of each other.

- If one line has a slope of $ m $,
- The line perpendicular to it has a slope of $ -\frac{1}{m} $.

For example:
- If a line has slope $ 2 $,
- The perpendicular line has slope $ -\frac{1}{2} $.

This relationship is critical in solving the problem at hand, as it allows us to determine the slope of the required line once the slope of the original line is known.

Step 1: Determining the Slope of the Original Line

The problem specifies a line that is perpendicular to another line passing through some point (which is not given here but is implied in the problem context). To find the perpendicular line's equation, we need to understand the slope of the original line.

Identifying the Original Line's Slope

Suppose the original line's equation is known or can be deduced from context. If the problem states that the line is perpendicular to a line passing through a certain point and with a known slope, you can find the slope of the original line directly.

If the original line's slope isn't explicitly given, but the line passes through a point and is perpendicular to some other line, then you need that other line's slope to proceed.

For illustration purposes, assume the problem involves a line passing through a point (x_1, y_1) and perpendicular to a line with slope m'. The slope of the perpendicular line would then be:

$$ m = -\frac{1}{m'} $$

Note: Since the original prompt mentions a specific point (-8, -32) but not the slope of the original line, it's reasonable to assume that the line in question is perpendicular to a known or given line, or that the slope of the original line is provided elsewhere. For this example, let's proceed with an example where the original line has a known slope, say, $m' = 2$. Then the perpendicular line's slope would be:

$$ m = -\frac{1}{2} $$

This assumption allows us to demonstrate the process clearly.

Step 2: Calculating the Slope of the Perpendicular Line

Given the original line's slope m', the perpendicular line's slope m is:

$$ m = -\frac{1}{m'} $$

Using our example where $m' = 2$:

$$ m = -\frac{1}{2} $$

This slope indicates that for every increase of 2 units in x, y decreases by 1 unit.

Step 3: Applying the Point-Slope Formula

Once the slope of the perpendicular line is known, the next step involves using the point-slope form of a line equation to find the specific line passing through the point $(-8, -32)$.

The Point-Slope Form

The point-slope form is:

$$ y - y_1 = m (x - x_1) $$

where:
- (x_1, y_1) is a point on the line,
- m is the slope.

Substituting the known point and slope:

$$ y - (-32) = -\frac{1}{2} (x - (-8)) $$
$$ y + 32 = -\frac{1}{2} (x + 8) $$

Expanding and Simplifying

Distribute the slope:

$$ y + 32 = -\frac{1}{2} x - 4 $$

Subtract 32 from both sides to solve for y:

$$ y = -\frac{1}{2} x - 4 - 32 $$
$$ y = -\frac{1}{2} x - 36 $$

This is the line's equation in slope-intercept form.

Step 4: Final Equation in Slope-Intercept Form

The resulting equation representing the line passing through $(-8, -32)$ and perpendicular to the original line with slope 2 is:

$$ y = -\frac{1}{2} x - 36 $$

This form clearly shows:
- The slope of the line: $-\frac{1}{2}$,
- The y-intercept: -36.

This equation can now be used for graphing, further analysis, or solving related problems.

Generalization and Additional Considerations

While the above example assumes a specific slope of 2 for the original line, real-world problems may vary. Here's how to adapt the process for different scenarios:

- Original line slope known: Use the negative reciprocal to find the perpendicular slope.
- Original slope unknown: Additional information is needed, such as the original line's equation or its slope.

- Line passing through different points: Plug in the corresponding \((x_1, y_1) \) into the point-slope form.

Important tips:

- Always verify the slope relationship for perpendicular lines.
- Double-check your calculations, especially signs, as they are critical in coordinate geometry.
- Simplify equations where possible to maintain clarity.

Summary and Practical Applications

Finding the equation of a line in slope-intercept form that passes through a specific point and is perpendicular to another line involves a clear sequence:

1. Determine the slope of the original line.
2. Calculate the negative reciprocal to find the perpendicular slope.
3. Use the point-slope formula with the known point and perpendicular slope.
4. Simplify to slope-intercept form.

This process is crucial in various fields, including engineering, architecture, computer graphics, and navigation, where precise line equations are necessary for design, analysis, and problem-solving.

Conclusion

Mastering how to write an equation in slope-intercept form for a line passing through a given point and perpendicular to another line enhances your understanding of geometric relationships and algebraic skills. By systematically applying the principles of slopes, reciprocals, and point-slope equations, you can confidently tackle similar problems with accuracy and clarity. Whether for academic purposes or practical applications, these foundational concepts form a vital part of the mathematical toolkit.

Note: The specific example used here assumes the original line's slope is 2. If the actual original line's slope differs, replace \(m' \) with the correct slope and follow the same steps to find the perpendicular line's equation.

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