6. A Line Goes Through These Two Points, (-4, -1) Anti (-9,-5).A. Find An Equation For This Line In Point

6. A Line Goes Through These Two Points, (-4, -1) Anti (-9,-5).A. Find An Equation For This Line In Point

Understanding how to find the equation of a line that passes through two points is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams or a mathematician working on a problem, mastering this process is essential. In this article, we will explore step-by-step how to determine the equation of a line passing through the points (-4, -1) and (-9, -5). We will cover the concepts of slope, point-slope form, and slope-intercept form, providing clear explanations and example calculations to enhance your understanding.

Introduction to Line Equations in Coordinate Geometry

Coordinate geometry involves analyzing geometric figures using algebraic equations. The equation of a line is one of the most basic and important concepts within this field. When given two points, the goal is to find a line that passes through both, which requires calculating the slope and then formulating the equation accordingly.

Key points to remember:

- A line can be uniquely determined by two distinct points.
- The slope indicates the steepness and direction of the line.
- The point-slope form and slope-intercept form are common ways to express the line’s equation.

Understanding the Given Points

The problem provides two points through which the line passes:

- Point A: (-4, -1)
- Point B: (-9, -5)

Before diving into calculations, it’s crucial to understand what these points represent:

- The x-coordinate of Point A is -4, and the y-coordinate is -1.
- The x-coordinate of Point B is -9, and the y-coordinate is -5.

These coordinates will serve as the basis for calculating the slope and deriving the line's equation.

Calculating the Slope of the Line

The slope (m) of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Step-by-step calculation:

1. Assign the points:

$$
(x_1, y_1) = (-4, -1)
$$
$$
(x_2, y_2) = (-9, -5)
$$

2. Compute the differences:

$$
y_2 - y_1 = -5 - (-1) = -5 + 1 = -4
$$

$$
x_2 - x_1 = -9 - (-4) = -9 + 4 = -5
$$

3. Calculate the slope:

$$
m = \frac{-4}{-5} = \frac{4}{5}
$$

Result: The slope of the line is $\boxed{\frac{4}{5}}$.

Formulating the Equation of the Line

Once the slope is known, the next step is to find the equation of the line. There are two common forms:

- Point-Slope Form
- Slope-Intercept Form

Both forms are useful, and choosing between them depends on the context.

Using the Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

Using Point A $(-4, -1)$ and $m = \frac{4}{5}$:

$$
y - (-1) = \frac{4}{5}(x - (-4))
$$

Simplify:

$$
y + 1 = \frac{4}{5}(x + 4)
$$

This is the point-slope form of the line passing through the given points.

Converting to Slope-Intercept Form

To express the line in slope-intercept form $y = mx + b$, we solve the above equation:

$$
y + 1 = \frac{4}{5}x + \frac{4}{5} \times 4
$$

\]

Calculate:

\[
\frac{4}{5} \times 4 = \frac{4 \times 4}{5} = \frac{16}{5}
\]

Now, subtract 1 from both sides (note that 1 = \(\frac{5}{5}\)):

\[
y = \frac{4}{5}x + \frac{16}{5} - 1
\]
\[
y = \frac{4}{5}x + \frac{16}{5} - \frac{5}{5}
\]
\[
y = \frac{4}{5}x + \frac{16 - 5}{5}
\]
\[
y = \frac{4}{5}x + \frac{11}{5}
\]

Final equation in slope-intercept form:

\[
\boxed{y = \frac{4}{5}x + \frac{11}{5}}
\]

This equation describes the line passing through the points (-4, -1) and (-9, -5).

Verification of the Equation

To ensure the equation is correct, substitute the second point (-9, -5) into the derived equation:

\[
y = \frac{4}{5}x + \frac{11}{5}
\]

Substitute \(x = -9\):

\[
y = \frac{4}{5} \times (-9) + \frac{11}{5} = -\frac{36}{5} + \frac{11}{5} = -\frac{36 - 11}{5} = -\frac{25}{5} = -5
\]

The result matches the y-coordinate of the second point, confirming the correctness of the equation.

Additional Tips for Finding the Equation of a Line

When working with coordinate points, keep these tips in mind:

- Always calculate the slope carefully, paying attention to signs.
- Use the point-slope form when you have a specific point and the slope.
- Convert to slope-intercept form for easier graphing and interpretation.
- Verify your equation by substituting the known points.

Applications of Line Equations in Real Life

Understanding how to derive and interpret line equations has numerous practical applications:

- Engineering: Designing roads, bridges, and structures.
- Economics: Modeling cost versus production relationships.
- Physics: Describing motion with linear equations.
- Computer Graphics: Rendering lines and shapes efficiently.

Conclusion

Finding the equation of a line passing through two points is a foundational skill in mathematics. By calculating the slope accurately and applying the appropriate algebraic forms, you can derive the equation of any line given two points. In this article, we demonstrated this process step-by-step using the points (-4, -1) and (-9, -5). The key takeaway is that the line passing through these points has the equation:

\[
\boxed{y = \frac{4}{5}x + \frac{11}{5}}
\]

Mastering this technique enhances your understanding of coordinate geometry and prepares you for more complex mathematical challenges.

Meta Description: Learn how to find the equation of a line passing through two points, specifically for points (-4, -1) and (-9, -5). Step-by-step guide on calculating slope, formulating point-slope and slope-intercept equations, with verification and practical tips.

Frequently Asked Questions

How do I find the equation of a line passing through the points (-4, -1) and (-9, -5)?

First, calculate the slope using $m = (y_2 - y_1) / (x_2 - x_1)$, then use point-slope form $y - y_1 = m(x - x_1)$ with one of the points to find the equation.

What is the slope of the line passing through (-4, -1) and (-9, -5)?

The slope $m = (-5 - (-1)) / (-9 - (-4)) = (-4) / (-5) = 4/5$.

How do I write the point-slope form of the line passing through (-4, -1) with slope 4/5?

Use $y - y_1 = m(x - x_1)$: $y - (-1) = (4/5)(x - (-4))$. Simplify to $y + 1 = (4/5)(x + 4)$.

What is the slope-intercept form of the line passing through the two points?

Starting from $y + 1 = (4/5)(x + 4)$, expand to $y + 1 = (4/5)x + (16/5)$, then subtract 1 to get $y = (4/5)x + (16/5) - 1$, which simplifies to $y = (4/5)x + (11/5)$.

Can I verify the line passes through both points?

Yes, substitute $x = -4$ and $x = -9$ into $y = (4/5)x + (11/5)$. For $x = -4$, $y = (4/5)(-4) + (11/5) = -16/5 + 11/5 = -5/5 = -1$. For $x = -9$, $y = (4/5)(-9) + (11/5) = -36/5 + 11/5 = -25/5 = -5$. Both points satisfy the equation.

What is the significance of the slope in this line equation?

The slope (4/5) indicates that for every 5 units increase in x, y increases by 4 units, showing the line's steepness and direction.

How do I write the final equation of the line passing through (-4, -1) and (-9, -5)?

The equation is y = (4/5)x + (11/5).

Can I convert the equation to standard form?

Yes, multiply both sides by 5 to clear fractions: 5y = 4x + 11, then rearranged to 4x - 5y = -11.

Additional Resources

Understanding the Equation of a Line Passing Through Two Points: A Step-by-Step Analytical Approach

In the realm of coordinate geometry, the task of finding the equation of a line that passes through two specific points is fundamental yet profoundly insightful. This process not only consolidates our understanding of algebraic concepts but also enhances spatial reasoning skills. In this article, we explore the problem: "A line goes through these two points, (-4, -1) and (-9, -5). Find an equation for this line in point-slope form." We will dissect each step with detailed explanations, ensuring clarity for learners and enthusiasts alike.

Foundations: The Significance of Line Equations in Coordinate Geometry

Before delving into the specifics of the problem, it is essential to understand why lines and their equations are pivotal in mathematics.

What Is a Line in the Coordinate Plane?

A line in the coordinate plane is a straight, infinitely extending set of points that satisfy a linear equation. Its properties, such as slope and intercepts, provide vital information about its orientation and position.

Why Find the Equation of a Line?

Having the equation of a line allows us to predict, analyze, and understand relationships between variables. Whether in physics, engineering, or economics, lines serve as models representing constant rates, trends, or relationships.

Types of Line Equations

- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Slope-Intercept Form: $y = mx + b$
- Standard Form: $Ax + By = C$

Our focus will be on deriving an equation in point-slope form based on two given points.

Step 1: Identifying the Given Data and Its Significance

The problem provides two points:

- Point 1: $(-4, -1)$
- Point 2: $(-9, -5)$

Each point is expressed as (x, y). Our goal is to find the equation of the line passing through both points, expressed in point-slope form.

Why these points?

These specific points serve as the foundation for calculating the line's slope and then formulating its equation.

Key Concepts:

- Coordinates: The positions of points in the plane.
- Pair of points: Defines a unique line unless they are identical.

Step 2: Calculating the Slope of the Line

The slope, denoted as m, measures the steepness and direction of the line. It is calculated as the ratio of the change in y to the change in x:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying the formula:

$$
m = \frac{-5 - (-1)}{-9 - (-4)} = \frac{-5 + 1}{-9 + 4} = \frac{-4}{-5} =
$$

\frac{4}{5}
\]

Result: The slope of the line is \(\boxed{\displaystyle \frac{4}{5}}\).

Significance of the slope:

- A positive slope (\(\frac{4}{5}\)) indicates the line rises as it moves from left to right.
- The fractional form \(\frac{4}{5}\) reflects a gentle incline.

Step 3: Choosing a Point for the Equation

To write the equation in point-slope form, we select either of the given points. Typically, choosing the point with simpler coordinates simplifies calculations.

Selected Point:

\[
(x_1, y_1) = (-4, -1)
\]

This point will serve as a reference point in the point-slope formula.

Step 4: Deriving the Equation in Point-Slope Form

The general point-slope form of a line's equation is:

\[
y - y_1 = m(x - x_1)
\]

Substituting the known values:

\[
y - (-1) = \frac{4}{5}(x - (-4))
\]

Simplify the expression:

$$
y + 1 = \frac{4}{5}(x + 4)
$$

This is the equation of the line in point-slope form passing through $(-4, -1)$ with slope $\frac{4}{5}$.

Step 5: Interpreting and Analyzing the Equation

The derived equation:

$$
\boxed{ y + 1 = \frac{4}{5}(x + 4) }
$$

serves as a complete description of the line in point-slope form.

Features:

- Point: The line passes through $(-4, -1)$, as seen directly in the equation.
- Slope: The line rises at a rate of $\frac{4}{5}$ units vertically for every 5 units horizontally.

Geometric interpretation:

- The equation graphically represents a line inclined upward to the right.
- The point $(-4, -1)$ is on the line, ensuring the equation's accuracy.

Step 6: Optional Conversions and Further Analysis

While the question emphasizes the point-slope form, it’s often useful to convert the equation into other forms for broader analysis.

Slope-Intercept Form:

Starting from:

$$
y + 1 = \frac{4}{5}(x + 4)
$$

Distribute:

$$
y + 1 = \frac{4}{5}x + \frac{4}{5} \times 4 = \frac{4}{5}x + \frac{16}{5}
$$

Subtract 1 from both sides:

$$
y = \frac{4}{5}x + \frac{16}{5} - 1 = \frac{4}{5}x + \left(\frac{16}{5} - \frac{5}{5} \right) = \frac{4}{5}x + \frac{11}{5}
$$

Slope-intercept form:

$$
\boxed{ y = \frac{4}{5}x + \frac{11}{5} }
$$

Standard Form:

Multiply through by 5 to clear fractions:

$$
5y + 5 = 4x + 11
$$

Rearranged:

$$
4x - 5y = -6
$$

This form is useful in certain contexts like solving systems of equations.

Additional Considerations: Validating the Equation

To ensure the derived equation accurately represents the line passing through the two points, verify the second point:

- For $(-9, -5)$,

Substitute into the point-slope form:

\[

$$
y + 1 = \frac{4}{5}(x + 4)
$$

$$
-5 + 1 = \frac{4}{5}(-9 + 4)
$$

$$
-4 = \frac{4}{5}(-5)
$$

$$
-4 = -4
$$

The point satisfies the equation, confirming correctness.

Summary and Conclusions

The process of determining the equation of a line passing through two points involves several systematic steps:

1. Calculating the slope using the difference quotient.
2. Choosing a reference point for the point-slope form.
3. Applying the point-slope formula to derive the equation.
4. Optional conversions to slope-intercept and standard forms.
5. Verification by substituting the second point.

In our specific case, the line passing through $(-4, -1)$ and $(-9, -5)$ has a slope of $\frac{4}{5}$, and the point-slope form of its equation is:

$$
\boxed{ y + 1 = \frac{4}{5}(x + 4) }
$$

This comprehensive approach not only provides the required equation but also deepens understanding of the geometric and algebraic principles involved.

Implications and Real-World Applications

Understanding how to derive and interpret line equations has vast applications:

- Physics: Modeling constant velocity motion.
- Economics: Analyzing cost and revenue functions.
- Engineering: Designing linear components and systems.
- Data Science: Fitting linear models to data points.

Mastering these techniques enhances problem-solving skills and supports more advanced mathematical pursuits.

Final Thoughts

In conclusion, finding the equation of a line through two points is a foundational skill that combines algebra, geometry, and analytical reasoning. By following a structured approach—calculating the slope, choosing a point, applying the point-slope formula, and verifying—students and professionals can confidently model linear relationships. The specific example of points $(-4, -1)$ and $(-9, -5)$ exemplifies the elegance and utility of these methods, reinforcing their importance across numerous scientific and mathematical disciplines.

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