A 4.2-kg Mass Is Placed At (3.0, 4.0) M. Where Can An 8.4-kg Mass Be Placed So That The Moment Of Inertia

A 4.2-kg Mass Is Placed At (3.0, 4.0) M. Where Can An 8.4-kg Mass Be Placed So That The Moment Of Inertia

Understanding the principles of rotational dynamics is essential when analyzing how mass distribution affects an object's moment of inertia. In this article, we explore the scenario where a 4.2-kg mass is positioned at point (3.0, 4.0) meters, and we seek to determine the locations where an 8.4-kg mass can be placed so that the combined moment of inertia exceeds a specific threshold. This analysis is vital in fields such as mechanical engineering, physics, and robotics, where balancing rotational stability and optimizing mass distribution are critical.

Fundamentals of Moment of Inertia

What Is Moment of Inertia?

The moment of inertia (I) is a measure of an object's resistance to angular acceleration about a specific axis. It depends on how the mass is distributed relative to the axis of rotation. The general formula for point masses is:

- $I = \sum m_i r_i^2$

where:

- m_i is the mass of the i-th point mass,

- r_i is the perpendicular distance from the mass to the axis of rotation.

In our context, the axis of rotation is typically chosen as the origin (0,0), unless specified otherwise.

Significance in Design and Analysis

The distribution of mass impacts the rotational inertia, influencing how easily an object can be spun up or slowed down. Engineers often manipulate mass placement to achieve desired dynamic performance, stability, or energy efficiency.

Scenario Description and Objectives

Suppose a 4.2-kg mass is fixed at point (3.0, 4.0) meters in a coordinate plane. We are to find possible positions for an 8.4-kg mass such that the combined moment of inertia about the origin is at least a certain value—say, exceeding 1000 kg·m².

This involves:

- Calculating the contribution to the moment of inertia from the known mass,

- Determining the required position(s) of the second mass,

- Ensuring the combined I meets the threshold.

Calculating the Moment of Inertia for the Known Mass

Distance of the 4.2-kg Mass from the Origin

The position of the first mass is at (3.0, 4.0) meters. Its distance from the origin is:

$$
r_1 = \sqrt{(3.0)^2 + (4.0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.0 \text{ m}
$$

The contribution to the moment of inertia from this mass is:

$$
I_1 = m_1 r_1^2 = 4.2 \times 5.0^2 = 4.2 \times 25 = 105 \text{ kg·m}^2
$$

This value is fixed, regardless of where the second mass is placed.

Determining Placement of the 8.4-kg Mass

Variables and Constraints

Let's denote the position of the 8.4-kg mass as (x, y). Its distance from the origin is:

$$
r_2 = \sqrt{x^2 + y^2}
$$

The contribution to the moment of inertia from this mass is:

$$
I_2 = 8.4 \times r_2^2
$$

\]

The total moment of inertia is:

\[
I_{total} = I_1 + I_2 = 105 + 8.4 \times (x^2 + y^2)
\]

Our goal is to find positions where:

\[
I_{total} \geq 1000 \, \text{kg·m}^2
\]

which simplifies to:

\[
105 + 8.4 (x^2 + y^2) \geq 1000
\]

Solving for the Distance

Subtract 105 from both sides:

\[
8.4 (x^2 + y^2) \geq 895
\]

Divide both sides by 8.4:

\[

$$
x^2 + y^2 \geq \frac{895}{8.4} \approx 106.55
$$

This indicates that the second mass must be placed at a distance from the origin of at least:

$$
r_2 = \sqrt{x^2 + y^2} \geq \sqrt{106.55} \approx 10.33\, \text{m}
$$

Thus, the 8.4-kg mass must be located at a point at least approximately 10.33 meters from the origin.

Visualizing Possible Placement Regions

Circle of Possible Locations

The set of all points where the 8.4-kg mass can be placed to meet the moment of inertia requirement forms a circle centered at the origin with radius approximately 10.33 meters.

- Minimum distance: 10.33 meters from the origin
- Any position outside this circle will satisfy the inertia condition
- Positions inside this circle will result in a total inertia less than 1000 kg·m²

Implications for Deployment

- To increase the moment of inertia beyond 1000 kg·m², place the 8.4-kg mass further from the origin.
- For practical applications, the mass placement might be constrained by physical or design limitations, but mathematically, any point on or beyond the circle of radius 10.33 meters meets the criterion.

Extensions and Additional Considerations

Impact of Different Axes of Rotation

While our analysis assumes rotation about the origin, changing the axis of rotation modifies the distances involved, thereby altering the moment of inertia calculations. For example, if rotation occurs about a different point, distances should be measured relative to that point.

Effect of Multiple Masses

Adding more masses, either fixed or movable, complicates the calculation but follows the same principles: sum of individual contributions based on squared distances from the axis.

Design Optimization Tips

- To maximize the moment of inertia with minimal mass, place the mass as far as possible from the axis.
- For balancing, position masses symmetrically relative to the axis.
- Consider physical constraints and stability when designing mass distributions.

Practical Applications and Examples

Robotics and Mechanical Design

Designers often need to optimize the placement of components to ensure stability and efficient motion. By understanding how mass distribution affects moment of inertia, engineers can prevent excessive energy consumption or mechanical failure.

Structural Engineering

In constructing rotating machinery or vehicles, balancing mass distribution ensures smooth operation and reduces wear and tear.

Spacecraft and Satellite Engineering

Precise placement of mass is critical when designing spacecraft to control angular momentum and orientation during operation.

Summary

In conclusion, the placement of masses relative to the axis of rotation critically influences the moment of inertia. For the given scenario, placing the 8.4-kg mass at a distance of at least approximately 10.33 meters from the origin guarantees that the combined moment of inertia is no less than 1000 kg·m².
This insight allows engineers and designers to make informed decisions about mass placement, ensuring optimal performance, stability, and safety in mechanical systems.

Remember: Always consider physical constraints, safety factors, and specific application requirements when applying these principles to real-world problems.

Frequently Asked Questions

How do you calculate the moment of inertia for a mass placed at a specific point relative to an axis?

The moment of inertia is calculated by multiplying the mass by the square of its distance from the axis: $I = m r^2$, where m is the mass and r is the perpendicular distance to the axis.

Given a 4.2-kg mass at (3.0, 4.0) m, how can I determine where to place an 8.4-kg mass to achieve a specific moment of inertia?

You need to know the axis of rotation and the desired total moment of inertia. Using the formula $I = m r^2$ for each mass, you can set up an equation to find the position of the 8.4-kg mass that satisfies the total moment of inertia requirement.

What factors influence the placement of a mass to achieve a certain moment of inertia in a system?

Factors include the mass values, their distances from the axis, and the desired total moment of inertia. The placement must satisfy the equation $I_{total} = \sum m_i r_i^2$ for all masses.

Can the position of the 8.4-kg mass be directly calculated using the given coordinates of the 4.2-kg mass?

Yes, if the axis of rotation and the desired total moment of inertia are known, you can solve for the position of the 8.4-kg mass by setting up the moment of inertia equation and solving for its distance from the axis.

What assumptions are typically made when calculating moments of

inertia with discrete masses placed at specific points?

Assumptions often include point masses, rigid bodies, fixed axes of rotation, and neglecting other forces or distributed mass effects unless specified.

How does changing the position of the 8.4-kg mass affect the system's moment of inertia?

Moving the 8.4-kg mass farther from the axis increases the total moment of inertia, while moving it closer decreases the moment of inertia, following the r^2 relationship.

Is it possible to place the 8.4-kg mass at multiple locations to achieve the same moment of inertia? If so, how?

Yes, because the moment of inertia depends on the square of the distance, multiple positions at different distances can produce the same value. For example, placing the mass at distances r and r' where $m r^2 = m r'^2$ will yield identical moments of inertia.

Additional Resources

A 4.2-kg mass is placed at (3.0, 4.0) m. Where can an 8.4-kg mass be placed so that the moment of inertia? This intriguing question touches on fundamental principles of rotational dynamics and the calculation of moments of inertia—a key concept in physics that describes an object's resistance to changes in its rotation. Understanding how to determine the placement of masses to achieve a specific moment of inertia is essential in fields ranging from mechanical engineering to astrophysics. In this comprehensive guide, we will explore the principles involved, step through the problem-solving process, and discuss how to find the position where an 8.4-kg mass can be placed to satisfy a particular moment of inertia.

Understanding the Moment of Inertia

Before diving into the problem itself, it's important to grasp what the moment of inertia (denoted as I) represents. It quantifies how much an object resists rotational acceleration about a specific axis. Unlike mass, which measures an object's resistance to linear acceleration, the moment of inertia depends on the mass distribution relative to the axis of rotation.

Key Concepts:

- Moment of Inertia (I): For a collection of point masses, it is given by the sum of each mass times the square of its distance from the axis of rotation:

$I = \sum m_i r_i^2$

- Axis of Rotation: The line about which the object rotates. The calculation of I depends on this axis.
- Mass Distribution: The placement of masses relative to the axis affects the total I.

The Given Data and the Problem Statement

Let's restate the problem with clarity:

- A 4.2-kg mass is fixed at the point (3.0, 4.0) meters.
- The question: Where can an 8.4-kg mass be placed so that the total moment of inertia about a specific axis? (Note: The axis is typically assumed to be the origin unless specified otherwise.)

While the axis of rotation isn't explicitly specified, the standard assumption in many problems is that the axis passes through the origin (0, 0). If the problem states otherwise, adjustments can be made accordingly.

Step-by-Step Approach to Solving the Problem

1. Clarify the Axis of Rotation

The first step is to identify the axis about which the moment of inertia is to be calculated. For this problem, we'll assume:

- Rotation axis: The z-axis passing through the origin, perpendicular to the xy-plane.

This is a common assumption in two-dimensional problems involving planar masses.

2. Understand the Moment of Inertia for Point Masses

For a point mass m located at position (x, y) relative to the axis, the moment of inertia is:

$$ I = m r^2 $$

where r is the perpendicular distance from the axis to the mass.

In the case of rotation about the z-axis through the origin:

$$ r = \sqrt{x^2 + y^2} $$

Thus, for each mass:

$$ I = m (x^2 + y^2) $$

3. Calculate the Moment of Inertia of the First Mass

Given:

- Mass m_1 = 4.2 kg
- Coordinates (x_1, y_1) = (3.0, 4.0) m

Compute:

$$ r_1 = \sqrt{3.0^2 + 4.0^2} = \sqrt{9 + 16} = \sqrt{25} = 5\, \text{m} $$

Therefore:

$$ I_1 = 4.2 \times 5^2 = 4.2 \times 25 = 105\, \text{kg·m}^2 $$

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4. Determine the Required Total Moment of Inertia

Suppose the problem specifies a target total moment of inertia I_{total} (say, for example, to balance a certain rotational system). If the target I_{total} is given, the total moment of inertia with the two masses should satisfy:

$$ I_{total} = I_1 + I_2 $$

where:

- I_2 is the moment of inertia of the 8.4-kg mass at a new position (x, y).

Rearranged:

$$ I_2 = I_{total} - I_1 $$

If the problem does not specify a target I_{total}, then we need to find the position of the 8.4-kg mass such that the total I satisfies a certain condition, perhaps equal to a known value or balancing a system.

Note: For this guide, we'll assume the goal is to find the position of the 8.4-kg mass such that the total moment of inertia equals a specified value (say, I_{target}). For illustration, suppose:

$$ I_{target} = 300\, \text{kg·m}^2 $$

Then:

$$ I_2 = 300 - 105 = 195\, \text{kg·m}^2 $$

---

5. Find the Position of the 8.4-kg Mass

Using:

$$ I_2 = m_2 r_2^2 $$

$$ r_2^2 = \frac{I_2}{m_2} $$

$$ r_2^2 = \frac{195}{8.4} \approx 23.214 $$

$$ r_2 = \sqrt{23.214} \approx 4.82\, \text{m} $$

This means the 8.4-kg mass must be located approximately 4.82 meters from the axis (origin).

6. Determine Possible Coordinates for the 8.4-kg Mass

Since the distance r from the origin is fixed at approximately 4.82 m, the mass can be placed anywhere on the circle:

$$ x^2 + y^2 = r_2^2 \approx 23.214 $$

Possible placements include:

- Along the x-axis: (±4.82, 0)
- Along the y-axis: (0, ±4.82)
- Or at any point satisfying the circle equation.

Extending the Analysis: Considering Different Objectives

The above calculations assume a specific target I_total. Depending on the context, the goal might be:

- To minimize or maximize the moment of inertia.
- To balance the system symmetrically.
- To place the mass optimally for stability or rotational dynamics.

In each case, the approach involves similar steps but with different target values or constraints.

Visualizing the Solution

Graphical representation helps solidify understanding:

- Plot the fixed mass at (3.0, 4.0).
- Draw a circle centered at the origin with radius ~4.82 m.
- Any point on this circle represents a potential position for the 8.4-kg mass.

This visualization allows for intuitive grasping of the problem—seeing the geometric locus of possible positions.

Practical Considerations

Real-World Applications

- Mechanical systems: Designing rotating machinery with precise mass distributions.
- Robotics: Positioning components to achieve desired rotational inertia.
- Spacecraft: Distributing mass for stability and maneuverability.

Limitations and Assumptions

- The analysis assumes point masses and ignores size or shape.
- Rotation axis is assumed to be perpendicular to the xy-plane at the origin.
- External factors like gravity, friction, or constraints are not considered.

Summary and Final Thoughts

Determining where to place a mass to achieve a specific moment of inertia involves understanding the relationship between mass distribution and rotational resistance. By calculating the distances from the axis and applying the formula $I = m r^2$, one can find the required placement. This process combines geometric reasoning with algebraic calculations, illustrating the beautiful interplay between physics and mathematics.

In our example, placing the 8.4-kg mass approximately 4.82 meters from the origin ensures the desired total moment of inertia, assuming the target value is 300 kg·m². The flexibility of placing the mass anywhere on the circle of radius 4.82 meters offers multiple solutions, emphasizing the importance of context and additional constraints in real-world problems.

Mastering the calculation of moments of inertia and mass placement strategies is essential for engineers, physicists, and designers aiming to optimize systems for stability, efficiency, and performance.

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