A 61-cm-diameter Wheel Accelerates Uniformly About Its Center From 120 Rpm To 280 Rpm In 4.0 S. Determine

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Understanding the dynamics of rotating objects is fundamental in physics and engineering. In this comprehensive guide, we will analyze a scenario involving a wheel that accelerates uniformly about its center. The problem involves calculating various parameters such as angular acceleration, initial and final angular velocities, and the total angular displacement during the acceleration phase. This analysis is crucial for applications ranging from mechanical design to rotational motion analysis in machinery.

Introduction to Rotational Motion

Rotational motion refers to the movement of an object around a center or axis. It shares similarities with linear motion but involves angles, angular velocities, and angular accelerations instead of linear displacements and velocities.

Key Concepts in Rotational Dynamics

- Angular Displacement (θ): The angle through which an object rotates during motion, measured in radians.
- Angular Velocity (ω): The rate at which an object rotates, measured in radians per second (rad/s).
- Angular Acceleration (α): The rate of change of angular velocity, measured in radians per second squared (rad/s^2).
- Moment of Inertia (I): The rotational equivalent of mass, indicating an object's resistance to change in rotation.
- Torque (τ): The measure of the force that causes an object to rotate.

Understanding the Given Problem

The problem states:

> A 61-cm-diameter wheel accelerates uniformly about its center from 120 rpm to 280 rpm in 4.0 seconds. Determine...

While the problem statement is incomplete, based on standard physics problems of this nature, it typically asks to determine one or more of the following:

- The angular acceleration (α)

- The total angular displacement (θ)
- The final or initial angular velocities in rad/s
- The torque required (if mass and moment of inertia are known)
- The total work done or energy involved

For this analysis, we will focus on calculating the angular acceleration, initial and final angular velocities in rad/sec, and the total angular displacement during the acceleration period.

Converting Units and Initial Data

Before performing calculations, it is essential to convert all given quantities into SI units for consistency.

Given Data

- Diameter of the wheel, D = 61 cm = 0.61 meters
- Initial rotational speed, n_1 = 120 rpm
- Final rotational speed, n_2 = 280 rpm
- Time taken for acceleration, t = 4.0 seconds

Calculations of Radius and Angular Velocities

- Radius (r): Since the diameter is given, radius is half of that:

$$r = D/2 = 0.61 / 2 = 0.305 \text{ meters}$$

- Conversion from rpm to rad/sec:

The relation between revolutions per minute (rpm) and radians per second (rad/sec):

$$\omega \text{ (rad/sec)} = \text{(rpm)} \times (2\pi \text{ radians}) / (60 \text{ seconds})$$

Calculating initial and final angular velocities:

1. Initial angular velocity, ω_1:

$$\omega_1 = 120 \times 2\pi / 60 = 120 \times \pi / 30 = 4\pi \text{ rad/sec} \approx 12.566 \text{ rad/sec}$$

2. Final angular velocity, ω_2:

$$\omega_2 = 280 \times 2\pi / 60 = 280 \times \pi / 30 \approx 9.333\times\pi \text{ rad/sec} \approx 29.321 \text{ rad/sec}$$

Calculating Angular Acceleration

Since the wheel accelerates uniformly, the angular acceleration (α) can be found using the kinematic equation:

Angular Kinematic Equation

$$\omega_2 = \omega_1 + \alpha \times t$$

Rearranged to solve for α:

$$\alpha = (\omega_2 - \omega_1) / t$$

Substituting the known values:

$$\alpha = (29.321 - 12.566) / 4.0 \approx 16.755 / 4.0 \approx 4.189 \text{ rad/sec}^2$$

Result:

The wheel's angular acceleration is approximately 4.189 rad/sec^2.

Determining Angular Displacement

The total angular displacement (θ) during the acceleration period can be calculated using another kinematic equation:

Angular Displacement Equation

$$\theta = \omega_1 \times t + (1/2) \times \alpha \times t^2$$

Calculating each term:

- Initial term:

$$\omega_1 \times t = 12.566 \times 4.0 \approx 50.265 \text{ radians}$$

- Acceleration term:

$$(1/2) \times \alpha \times t^2 = 0.5 \times 4.189 \times 16 \approx 0.5 \times 4.189 \times 16 \approx 33.512 \text{ radians}$$

Adding both:

$$\theta \approx 50.265 + 33.512 \approx 83.777 \text{ radians}$$

Interpretation:

The wheel rotates through approximately 83.78 radians during the acceleration phase.

Additional Calculations and Applications

Depending on the context, further calculations may be relevant:

1. Final Linear Velocity at the Rim

The linear velocity (v) of a point on the rim is related to the angular velocity by:

$$v = \omega \times r$$

At the final angular velocity:

$$v_f = \omega_2 \times r \approx 29.321 \times 0.305 \approx 8.94 \text{ m/sec}$$

This indicates the point on the rim moves at approximately 8.94 meters per second at the end of acceleration.

2. Kinetic Energy of the Rotating Wheel

If the moment of inertia (I) is known, the rotational kinetic energy (KE) can be calculated:

$$KE = (1/2) \times I \times \omega^2$$

However, without the mass or moment of inertia, this calculation remains theoretical.

3. Torque Required for the Acceleration

Similarly, torque (τ) needed depends on the moment of inertia:

$$\tau = I \times \alpha$$

Again, specific data on the wheel's mass distribution is needed for precise calculations.

Practical Implications and Engineering Applications

Understanding the dynamics of a accelerating wheel has direct applications in various engineering fields:

- Mechanical Design: Designing components that can withstand the stresses during acceleration.
- Automotive Engineering: Calculating acceleration parameters for wheels and tires.
- Rotational Machinery: Ensuring motors can provide the necessary torque for desired acceleration.
- Safety Analysis: Predicting the stresses and strains during rapid acceleration or deceleration.

Summary of Key Results

Parameter	Calculation	Result
Initial angular velocity, ω_1	120 rpm to rad/sec	approximately 12.566 rad/sec
Final angular velocity, ω_2	280 rpm to rad/sec	approximately 29.321 rad/sec
Angular acceleration, α	$(\omega_2 - \omega_1) / t$	approximately 4.189 rad/sec^2

Total angular displacement, θ	$\omega_1 \times t + \frac{1}{2} \times \alpha \times t^2$	approximately 83.78 radians
Linear velocity at rim (final)	$\omega_2 \times r$	approximately 8.94 m/sec

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Conclusion

Analyzing the uniform acceleration of a wheel involves converting units, applying kinematic equations, and understanding the relationships between angular and linear quantities. The calculations reveal how quickly the wheel accelerates, the total rotation during the acceleration period, and the linear speed of the rim at the end of the process. These insights are fundamental for designing rotating systems, ensuring safety, and optimizing performance in various mechanical and engineering applications.

References and Further Reading

- Classical Mechanics by Herbert Goldstein - An authoritative text on rotational dynamics.
- Physics for Scientists and Engineers by Serway and Jewett - Offers detailed explanations of rotational motion.
- Engineering Mechanics: Dynamics by R.C. Hibbeler - Practical applications of rotational kinematics.
- Online resources such as Khan Academy and HyperPhysics for interactive tutorials.

By mastering these calculations and concepts, engineers and students can effectively analyze and design systems involving rotating bodies, ensuring safety, efficiency, and innovation in their work.

Frequently Asked Questions

What is the initial angular velocity of the wheel in radians per second?

The initial angular velocity $\omega_1 = (120 \text{ rpm} \times 2\pi) / 60 = 12.57$ rad/s.

What is the final angular velocity of the wheel in radians per second?

The final angular velocity $\omega_2 = (280 \text{ rpm} \times 2\pi) / 60 = 29.32$ rad/s.

How do you calculate the angular acceleration of the wheel?

Angular acceleration $\alpha = (\omega_2 - \omega_1) / t = (29.32 - 12.57) / 4.0 = 4.89$ rad/s².

What is the linear acceleration of a point on the rim of the wheel?

Linear acceleration $a = r \times \alpha = (0.61 \text{ m}) \times 4.89 \text{ rad/s}^2 \approx 2.99 \text{ m/s}^2$.

How can the work done to accelerate the wheel be determined?

Work done = $0.5 \times I \times (\omega_2^2 - \omega_1^2)$, where $I = 0.5 \times m \times r^2$; assuming mass m, the work can be calculated once m is known.

Additional Resources

A 61-cm-diameter wheel accelerates uniformly about its center from 120 rpm to 280 rpm in 4.0 seconds. Determine the angular acceleration, the angular displacement during this interval, and the final angular velocity in radians per second. This problem serves as an excellent illustration of rotational kinematics, where understanding the relationships between angular displacement, angular velocity, and angular acceleration is crucial. In this detailed guide, we'll walk through each step carefully, ensuring clarity for those studying rotational motion or preparing for related physics exams.

Understanding the Problem

Before diving into calculations, it's important to interpret what the problem is asking and what data is provided.

Given Data:
- Diameter of the wheel: 61 cm
- Initial rotational speed (ω_0): 120 rpm (revolutions per minute)
- Final rotational speed (ω): 280 rpm
- Time interval (t): 4.0 seconds

What is asked:
1. Determine the angular acceleration (α).
2. Calculate the angular displacement (θ) during this period.
3. Find the final angular velocity in radians per second.

Step 1: Convert Rotational Speeds to Radians per Second

Rotational speeds given in revolutions per minute (rpm) must be converted to radians per second (rad/sec) because SI units are standard for angular quantities.

Conversion factor:
- 1 revolution = 2π radians

- 1 minute = 60 seconds

Conversion process:

$$
\text{Angular velocity} \ (\omega) = \text{rpm} \times \frac{2\pi \ \text{rad}}{1 \ \text{rev}} \times \frac{1 \ \text{min}}{60 \ \text{sec}}
$$

Applying for initial and final speeds:

Initial angular velocity (ω_0):

$$
\omega_0 = 120 \times \frac{2\pi}{60} = 120 \times \frac{\pi}{30} = 4\pi \ \text{rad/sec}
$$

$$
\boxed{\omega_0 \approx 4 \times 3.1416 = 12.566 \ \text{rad/sec}}
$$

Final angular velocity (ω):

$$
\omega = 280 \times \frac{2\pi}{60} = 280 \times \frac{\pi}{30} = \frac{280\pi}{30} = \frac{28\pi}{3} \ \text{rad/sec}
$$

$$
\boxed{\omega \approx \frac{28 \times 3.1416}{3} \approx \frac{87.9648}{3} \approx 29.3216 \ \text{rad/sec}}
$$

Step 2: Find Angular Acceleration (α)

Since the wheel accelerates uniformly, the angular acceleration (α) can be calculated using the kinematic equation:

$$
\omega = \omega_0 + \alpha t
$$

Rearranged for α:

$$
\alpha = \frac{\omega - \omega_0}{t}
$$

Substituting the known values:

$$
\alpha = \frac{29.3216 - 12.566}{4.0} = \frac{16.7556}{4} \approx 4.1889 \ \text{rad/sec}^2
$$

Answer:

$$
\boxed{\text{Angular acceleration } \alpha \approx 4.189 \ \text{rad/sec}^2}
$$

This positive value indicates the wheel speeds up in the same rotational direction throughout.

Step 3: Calculate Angular Displacement (θ)

Angular displacement during the acceleration phase can be found with the equation:

$$
\theta = \omega_0 t + \frac{1}{2} \alpha t^2
$$

Plugging in the known values:

$$
\theta = 12.566 \times 4 + \frac{1}{2} \times 4.189 \times (4)^2
$$

Calculate each term:

- First term:
$$
12.566 \times 4 = 50.264 \ \text{rad}
$$

- Second term:
$$
\frac{1}{2} \times 4.189 \times 16 = 2.0945 \times 16 = 33.512 \ \text{rad}
$$

Adding both:

$$
\theta \approx 50.264 + 33.512 = 83.776 \ \text{rad}
$$

Answer:
$$
\boxed{\text{Angular displacement } \theta \approx 83.78 \ \text{radians}}
$$

This is the total angle (in radians) the wheel covers during the acceleration from 120 rpm to 280 rpm.

Step 4: Final Angular Velocity in Radians per Second

We've already calculated the final angular velocity in radians per second:

$$
\boxed{\omega \approx 29.322 \ \text{rad/sec}}
$$

This confirms the wheel's speed after 4 seconds of acceleration.

Additional Insights: Relating to Linear Speed

While the primary focus is on angular quantities, understanding the linear speed at the rim of the wheel can provide practical context.

Calculate the radius:

$$
r = \frac{\text{diameter}}{2} = \frac{61 \ \text{cm}}{2} = 30.5 \ \text{cm} = 0.305 \ \text{m}
$$

Final linear speed (v):

$$
v = r \times \omega
$$

$$
v = 0.305 \times 29.322 \approx 8.94 \ \text{m/sec}
$$

So, the rim of the wheel reaches nearly 9 m/sec after 4 seconds of acceleration.

Practical Applications and Further Considerations

Understanding the dynamics of a rotating wheel like this has numerous real-world applications:

- Mechanical engineering: Designing flywheels, turbines, and rotating machinery.
- Transportation: Analyzing wheel acceleration in vehicles.
- Sports science: Studying rotational motion in sports equipment.

Additional factors:
- Friction and air resistance may slightly alter actual acceleration.
- The mass distribution affects the moment of inertia, which wasn't explicitly considered here but is vital for more detailed analyses.

Summary of Results

Quantity	Calculation	Result
Initial angular velocity (ω_0)	120 rpm → rad/sec	12.566 rad/sec
Final angular velocity (ω)	280 rpm → rad/sec	29.322 rad/sec
Angular acceleration (α)	$(\omega - \omega_0) / t$	4.189 rad/sec^2
Angular displacement (θ)	$\omega_0 t + ½ \alpha t^2$	83.78 radians
Final linear speed (v)	$r \times \omega$	8.94 m/sec

Final Remarks

This comprehensive analysis demonstrates how to approach a rotational kinematics problem systematically. By converting units, applying the fundamental equations, and interpreting the physical meaning behind the calculations, we gain a deeper understanding of rotational motion. Whether you're a student preparing for exams or an engineer designing rotating systems, mastering these concepts is essential for accurate analysis and effective problem solving.

Remember: always verify your units, double-check your conversions, and interpret your results in the context of the physical system. Happy calculating!

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