A Block Attached To A Horizontal Spring Of Force Constant 75N/m Undergoes SHM With An Amplitude Of 0.15m.

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Understanding the motion of a block attached to a spring is fundamental in classical mechanics, especially when analyzing simple harmonic motion (SHM). In this article, we delve into the details of a block undergoing SHM with a spring constant of 75 N/m and an amplitude of 0.15 meters. Whether you're a student studying physics or an enthusiast exploring oscillatory motion, this comprehensive guide will clarify the key concepts, calculations, and practical implications related to this system.

Basics of Simple Harmonic Motion (SHM)

What Is SHM?

Simple Harmonic Motion (SHM) is a type of periodic motion where an object oscillates back and forth around an equilibrium position. The restoring force acting on the object is directly proportional to its displacement and acts in the opposite direction, following Hooke's Law. The classic example is a mass attached to a spring.

Conditions for SHM

- The system must exhibit a restoring force proportional to displacement: $F = -kx$
- The motion should be sinusoidal in nature
- The system should oscillate about an equilibrium point with constant amplitude and period

Understanding the System: Block and Spring

Given Data

- Spring Constant, $k = 75\, \text{N/m}$
- Amplitude, $A = 0.15\, \text{m}$

This setup involves a block attached horizontally to a spring anchored at one end. When displaced from its equilibrium position and released, the block oscillates back and forth, exhibiting SHM.

Key Features of the System

- The mass of the block is not specified, so calculations for period and maximum velocity will be expressed in terms of mass (m)
- The motion is idealized: no friction or damping effects considered

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Analyzing the Motion: Fundamental Parameters

Period of Oscillation (T)

The period is the time taken for one complete cycle of oscillation. For a mass-spring system undergoing SHM:

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Where:
- m = mass of the block (kg)
- k = spring constant (N/m)

Implication: The period depends on both the mass and the spring constant. Increasing the mass increases the period, while a stiffer spring decreases it.

Frequency (f)

The frequency, the number of oscillations per second, is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Maximum Velocity (v_{max})

The maximum speed of the block occurs at the equilibrium position:

$$ v_{max} = \omega A $$

Where:
- $\omega = \sqrt{\frac{k}{m}}$ (angular frequency)

Therefore:

$$ v_{max} = A \sqrt{\frac{k}{m}} $$

Maximum Acceleration (a_{max})

Maximum acceleration occurs at the maximum displacement:

$$ a_{max} = \omega^2 A = \frac{k}{m} \times A $$

This value indicates the greatest acceleration the block experiences during oscillation.

Calculations Specific to the Given System

Since the mass m is not specified, most parameters will be expressed in terms of m.

Angular Frequency (ω)

$$ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{m}} \, \text{rad/sec} $$

Period (T)

$$ T = 2\pi \sqrt{\frac{m}{75}} $$

Maximum Velocity (v_{max})

$$ v_{max} = 0.15 \times \sqrt{\frac{75}{m}} $$

Maximum Acceleration (a_{max})

$$ a_{max} = \frac{75}{m} \times 0.15 = \frac{11.25}{m} \, \text{m/sec}^2 $$

Note: To compute specific numerical values, the mass m must be known. As an example, if $m = 1\, \text{kg}$:

- $\omega = \sqrt{75/1} = \sqrt{75} \approx 8.66\, \text{rad/sec}$
- $T = 2\pi / 8.66 \approx 0.726\, \text{sec}$
- $v_{max} = 0.15 \times 8.66 \approx 1.30\, \text{m/sec}$
- $a_{max} = 75 / 1 \times 0.15 = 11.25\, \text{m/sec}^2$

Energy Considerations in SHM

Potential and Kinetic Energy

- The total mechanical energy (E) in the system remains constant in ideal conditions
- At maximum displacement ($x = \pm A$), all energy is potential:

$$PE_{max} = \frac{1}{2} k A^2$$

- At equilibrium position, all energy is kinetic:

$$KE_{max} = \frac{1}{2} m v_{max}^2$$

Calculating Energy for the System

For $m=1\, \text{kg}$:

$$PE_{max} = \frac{1}{2} \times 75 \times (0.15)^2 = 0.84375\, \text{J}$$

$$KE_{max} = \frac{1}{2} \times 1 \times (1.30)^2 \approx 0.845\, \text{J}$$

This confirms the conservation of energy.

Practical Applications and Experiments

Measuring Spring Constant

- The system can be used to experimentally verify the spring constant by measuring the period for known masses and applying:

$$ k = \frac{4 \pi^2 m}{T^2} $$

Determining Mass from Oscillation Data

- If the period and spring constant are known, the mass can be deduced:

$$ m = \frac{k T^2}{4 \pi^2} $$

Applications in Engineering and Design

- Vibration analysis in mechanical structures
- Designing oscillatory systems like watches, sensors, and seismographs
- Educational demonstrations of SHM principles

Limitations and Real-World Considerations

Friction and Damping

- Real systems often experience damping due to friction or air resistance, causing amplitude decay over time
- Damped SHM requires additional analysis involving damping coefficients

Nonlinear Effects

- For large displacements, the restoring force may deviate from Hooke's Law, leading to nonlinear oscillations

Material and Structural Constraints

- Material fatigue and spring limits restrict maximum oscillation amplitudes and forces

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Conclusion

A block attached to a horizontal spring with a force constant of 75 N/m undergoing SHM with an amplitude of 0.15 meters exemplifies fundamental oscillatory behavior in mechanics. By understanding key parameters such as period, maximum velocity, and maximum acceleration, and recognizing the energy conservation principles involved, one can analyze and predict the system's motion accurately. Whether used in educational experiments, engineering designs, or practical applications, this simple harmonic oscillator provides valuable insights into the dynamics of oscillatory systems.

Remember: The specific numerical results depend on the mass of the block, which should be measured or specified for precise calculations. Nonetheless, the principles outlined here form the foundation for understanding and analyzing similar harmonic systems.

Frequently Asked Questions

What is the maximum velocity of the block during its SHM?

The maximum velocity is given by $v_{max} = A \omega$, where $\omega = \sqrt{k/m}$. Without the mass m, the maximum velocity cannot be precisely calculated, but if m is known, you can compute ω and then find v_{max}.

How do you determine the period of oscillation for the block?

The period T is given by $T = 2\pi / \omega = 2\pi \sqrt{m / k}$. Knowing the mass m allows calculation of the period.

What is the restoring force acting on the block at maximum displacement?

The restoring force at maximum displacement (amplitude A) is $F = -k A = -75$ N/m 0.15 m = -11.25 N, directed toward the equilibrium position.

If the mass of the block is 0.5 kg, what is its angular frequency?

Angular frequency $\omega = \sqrt{k/m} = \sqrt{75 \text{ N/m} / 0.5 \text{ kg}} = \sqrt{150} \approx 12.25$ rad/s.

How much potential energy is stored in the spring at maximum displacement?

The potential energy at maximum displacement is $U = (1/2) k A^2 = 0.5$ 75 N/m $(0.15 \text{ m})^2 = 0.5$ 75 0.0225 = approximately 0.84375 Joules.

Additional Resources

A Block Attached To A Horizontal Spring Of Force Constant 75N/m Undergoes SHM With An Amplitude Of 0.15m is a classic physics problem that beautifully illustrates the principles of simple harmonic motion (SHM). Such systems are foundational in understanding oscillations, energy conservation, and wave phenomena. Whether you're a student preparing for exams or a physics enthusiast interested in the dynamics of oscillating systems, this comprehensive guide will walk you through the key concepts, calculations, and physical interpretations associated with this problem.

Introduction to Simple Harmonic Motion (SHM)

Simple Harmonic Motion is a type of periodic motion where the restoring force acting on a body is directly proportional to its displacement from an equilibrium position and acts in the opposite direction. Mathematically, it is described by the differential equation:

$$
\frac{d^2x}{dt^2} + \omega^2 x = 0
$$

where:

- x is the displacement from equilibrium,
- ω is the angular frequency of the oscillation.

In the context of the problem at hand, a block attached to a spring oscillates horizontally, exemplifying SHM with specific parameters.

System Description and Key Parameters

Let's define the parameters for this specific system:

- Spring Force Constant (k): 75 N/m
- Amplitude (A): 0.15 m
- Mass of the Block (m): (To be determined or assumed)

The system involves a mass attached to a horizontal spring fixed at one end, oscillating back and forth when displaced from equilibrium.

Fundamental Concepts and Equations

1. Restoring Force

In horizontal SHM with a spring, the restoring force is Hooke’s Law:

$$
F = -kx
$$

where:

- F is the restoring force,
- x is the displacement from equilibrium.

2. Angular Frequency (ω)

The angular frequency for mass-spring systems:

$$
\boxed{
\omega = \sqrt{\frac{k}{m}}
}
$$

This relates the spring constant and mass to the oscillation's frequency.

3. Period (T)

The time for one complete cycle:

$$
T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}
$$

4. Frequency (f)

Number of oscillations per second:

$$
f = \frac{1}{T} = \frac{\omega}{2\pi}
$$

5. Maximum Velocity (v_{max})

At equilibrium, the maximum velocity:

$$
v_{max} = \omega A
$$

6. Maximum Acceleration (a_{max})

At maximum displacement:

$$
a_{max} = \omega^2 A
$$

Calculations for the Given System

1. Determining the Mass of the Block

Since the problem provides k and A but not m, we can explore different scenarios or assume a typical mass to proceed with calculations. For demonstration, assume $m = 0.5\,kg$.

Calculate ω:

$$
\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{75}{0.5}} = \sqrt{150} \approx 12.247\, rad/s
$$

Calculate Period (T):

$$
T = \frac{2\pi}{\omega} \approx \frac{6.2832}{12.247} \approx 0.512\, s
$$

Calculate Frequency (f):

$$
f = \frac{1}{T} \approx \frac{1}{0.512} \approx 1.953\, Hz
$$

Calculate Maximum Velocity (v_{max}):

$$
v_{max} = \omega A = 12.247 \times 0.15 \approx 1.837\, m/s
$$

Calculate Maximum Acceleration (a_{max}):

$$
a_{max} = \omega^2 A = 150 \times 0.15 = 22.5\, m/s^2
$$

Physical Interpretation of the Results

Energy Conservation

In SHM, the total mechanical energy is conserved and oscillates between kinetic and potential forms:

$$
E_{total} = \frac{1}{2}kA^2
$$

Calculation:

$$
E_{total} = \frac{1}{2} \times 75 \times (0.15)^2 = 0.5 \times 75 \times 0.0225 = 0.84375\, J
$$

At equilibrium (displacement zero):

- Kinetic energy is maximum,
- Potential energy is zero.

At maximum displacement:

- Potential energy is maximum,
- Kinetic energy is zero.

Exploring the Dynamics Step-by-Step

Step 1: Initial Displacement

Suppose the block is displaced by $A = 0.15\, m$ from the equilibrium position and released.

Step 2: Oscillatory Motion

The block accelerates toward the equilibrium point, gaining speed, reaching maximum velocity at the center, then decelerates upon passing through the equilibrium, and reverses direction at the maximum amplitude.

Step 3: Velocity and Acceleration at Various Points

- At equilibrium:

$$
x=0,\quad v = v_{max} \approx 1.837\, m/s
$$

- At maximum displacement:

$$
x=A=0.15\, m,\quad v=0

\]

$$
a_{max} = 22.5\, m/s^2
$$

Step 4: Time Evolution

The displacement as a function of time:

$$
x(t) = A \cos(\omega t + \phi)
$$

where ϕ is the phase constant depending on initial conditions.

Practical Considerations and Real-World Applications

While ideal SHM assumes no damping or friction, real systems experience energy loss over time. For accurate modeling:

- Consider damping forces (like air resistance or friction),
- Adjust calculations to account for damping coefficients.

Applications include:

- Design of pendulums,
- Suspension systems,
- Vibration analysis in mechanical structures,
- Oscillatory circuits in electronics.

Extending the Analysis: Varying Parameters

Impact of Changing the Spring Constant

- Increasing k increases ω, leading to faster oscillations.
- Decreasing k results in slower oscillations.

Impact of Changing Mass

- Increasing m decreases ω, slowing the oscillation.
- Decreasing m increases ω.

Amplitude and Energy

- The amplitude A directly affects the maximum energy stored in the system.

- Larger A results in higher maximum potential and kinetic energies.

Visualizing the Motion

Graphical representations are invaluable:

- Displacement vs. Time: Shows sinusoidal oscillations.
- Velocity vs. Time: Also sinusoidal, shifted by 90° phase relative to displacement.
- Energy vs. Time: Illustrates energy transfer between kinetic and potential forms.

Summary of Key Takeaways

- The system's angular frequency:

$$
\boxed{
\omega = \sqrt{\frac{k}{m}}
}
$$

- The period:

$$
T = 2\pi \sqrt{\frac{m}{k}}
$$

- Maximum velocity:

$$
v_{max} = \omega A
$$

- Maximum acceleration:

$$
a_{max} = \omega^2 A
$$

- Total energy:

$$
E_{total} = \frac{1}{2}kA^2
$$

Understanding these relationships equips you to analyze a wide range of oscillatory systems, from simple lab experiments to complex engineering structures.

Final Thoughts

The problem of a block attached to a horizontal spring with force constant 75 N/m undergoing SHM with an amplitude of 0.15 m exemplifies the elegant simplicity and profound implications of classical mechanics. Whether for academic purposes or real-world engineering, mastering these concepts provides a solid foundation for exploring the fascinating world of oscillations and waves.

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