A Damped Oscillator With A Period Of 30 S Shows A Reduction Of 23% In Amplitude After 1.0 Min.1)Calculate

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Introduction

The phenomenon of damping in oscillatory systems is fundamental in physics, engineering, and various applied sciences. It describes how oscillations gradually decrease in amplitude over time due to resistive forces such as friction, air resistance, or other dissipative effects. Understanding damping is crucial in designing systems like suspension bridges, mechanical watches, electronic circuits, and more.

In this article, we analyze a specific case: a damped oscillator with a period of 30 seconds exhibits a 23% reduction in amplitude after 1.0 minute. This scenario provides an excellent opportunity to explore the mathematical relationships governing damping, such as the damping coefficient, logarithmic decrement, and the quality factor of the system. Through detailed calculations, we will determine essential parameters characterizing the damping behavior of this oscillator.

Understanding the Basic Concepts

What is a Damped Oscillator?

A damped oscillator is a system that oscillates with a decreasing amplitude over time due to energy dissipation. The classic example is a mass attached to a spring with a damping force proportional to velocity. The key components of a damped harmonic oscillator include:

- Restoring force: Typically proportional to displacement, characterized by the spring constant k.
- Mass: Denoted as m, influencing the natural frequency.
- Damping force: Opposes motion, proportional to velocity, characterized by damping coefficient b.

The Differential Equation

The motion of a damped harmonic oscillator is described by:

$$
m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0
$$

Where:
- $x(t)$: displacement as a function of time.
- m: mass.

- b: damping coefficient.
- k: spring constant.

The solution depends on the damping ratio, leading to underdamped, critically damped, or overdamped behavior.

Key Parameters

- Natural (undamped) angular frequency:

$$
\omega_0 = \sqrt{\frac{k}{m}}
$$

- Damped angular frequency:

$$
\omega_d = \omega_0 \sqrt{1 - \zeta^2}
$$

- Damping ratio:

$$
\zeta = \frac{b}{2 \sqrt{mk}}
$$

- Period of damped oscillation:

$$
T_d = \frac{2\pi}{\omega_d}
$$

Calculating the Damping Parameters

Given Data:
- Period of oscillation: $T_d = 30\, \text{s}$
- Time interval: $t = 1.0\, \text{min} = 60\, \text{s}$
- Amplitude reduction: 23% after 1 minute.

Step 1: Determine the Damped Angular Frequency

From the period:

$$
\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{30} = \frac{\pi}{15} \approx 0.2094\, \text{rad/sec}
$$

Step 2: Understand Amplitude Decay in a Damped Oscillator

The amplitude $A(t)$ of a damped oscillator decays exponentially:

$$
A(t) = A_0 e^{-\beta t}
$$

Where:
- A_0: initial amplitude.
- β: damping decay constant, related to damping coefficient b, and the damping ratio ζ.

The logarithmic decrement δ quantifies the rate of amplitude decay per cycle:

$$
\delta = \frac{1}{n} \ln \frac{A(t)}{A(t + n T_d)} \quad \text{for } n \text{ cycles}
$$

Alternatively, for a single cycle:

$$
\delta = \ln \frac{A_0}{A_1}
$$

Step 3: Calculate the Logarithmic Decrement

Given that the amplitude reduces by 23% after 60 seconds:

$$
\frac{A(t)}{A_0} = 1 - 0.23 = 0.77
$$

Using the exponential decay relation:

$$
A(t) = A_0 e^{-\beta t}
$$

Thus:

$$
0.77 = e^{-\beta \times 60}
$$

Taking natural logarithm:

$$
\ln 0.77 = -\beta \times 60
$$

$$
\beta = - \frac{\ln 0.77}{60}
$$

\]

Calculating:

\[
\ln 0.77 \approx -0.2614
\]

\[
\beta = \frac{0.2614}{60} \approx 0.00436\, \text{sec}^{-1}
\]

Calculating the Damping Coefficient \(b\)

The damping coefficient relates to \(\beta\) through:

\[
\beta = \frac{b}{2m}
\]

Rearranged:

\[
b = 2m \beta
\]

Without the mass \(m\), we can only express \(b\) in terms of \(m\). However, the damping ratio and energy dissipation can be analyzed further once \(m\) is known or assumed.

Determining the Damping Ratio \(\zeta\)

The damping ratio \(\zeta\) is related to the damping coefficient \(b\) and the natural frequency \(\omega_0\):

\[
\zeta = \frac{b}{2 \sqrt{mk}} = \frac{\beta}{\omega_0}
\]

Given that:

\[
\omega_d = \omega_0 \sqrt{1 - \zeta^2}
\]

And knowing \(\omega_d\) from the period:

\[

$$
\omega_d = 0.2094\, \text{rad/sec}
$$

We need to find ζ.

Since:

$$
\zeta = \frac{\beta}{\omega_0}
$$

And:

$$
\omega_0 = \frac{\omega_d}{\sqrt{1 - \zeta^2}}
$$

Rearranged:

$$
\omega_0 = \frac{\omega_d}{\sqrt{1 - \zeta^2}}
$$

But without ω_0, direct calculation is complex. Alternatively, for small damping ($\zeta \ll 1$), $\omega_d \approx \omega_0$, so:

$$
\omega_0 \approx 0.2094\, \text{rad/sec}
$$

Then:

$$
\zeta \approx \frac{\beta}{\omega_0} = \frac{0.00436}{0.2094} \approx 0.0208
$$

This indicates a very lightly damped system.

Calculating the Quality Factor Q

The quality factor Q measures the damping's effect on the oscillation:

$$
Q = \frac{1}{2\zeta}
$$

Using $\zeta \approx 0.0208$:

$$

$$
Q \approx \frac{1}{2 \times 0.0208} \approx 24
$$

A high Q indicates low damping and sustained oscillations over many cycles.

Summary of Key Results

Parameter	Value	Explanation
Damped angular frequency ω_d	0.2094 rad/sec	Derived from period $T_d = 30\,\text{s}$
Damping decay constant β	0.00436 sec^{-1}	From amplitude reduction over 60 s
Damping ratio ζ	~0.0208	Approximate, assuming $\omega_0 \approx \omega_d$
Quality factor Q	~24	Indicates lightly damped oscillations

Practical Applications and Implications

Understanding damping parameters is vital in various fields:

- Mechanical systems: Ensuring structures like bridges and buildings can withstand oscillations caused by wind or traffic.
- Electronics: Designing circuits with controlled damping to prevent oscillations or overshoot.
- Seismology: Analyzing damping in seismic waves and building responses.
- Automotive engineering: Designing suspension systems that balance comfort and stability.

In this scenario, recognizing that the damping is very slight (low ζ, high Q) helps engineers predict how long oscillations will persist and how quickly energy dissipates.

Conclusion

Analyzing a damped oscillator with a period of 30 seconds that exhibits a 23% amplitude reduction after 1 minute reveals key insights into the damping characteristics of the system. The decay constant β is approximately 0.00436 sec^{-1}, corresponding to a damping ratio of about 0.0208, indicating a lightly damped system with a high quality factor (~24).

These calculations highlight the importance of damping parameters in predicting oscillatory behavior, designing resilient structures, and optimizing system performance. By understanding how damping influences oscillations, engineers and scientists can better control and utilize these phenomena across

Frequently Asked Questions

What is the initial amplitude of the damped oscillator if the amplitude reduces by 23% after 1 minute?

To find the initial amplitude, we use the decay formula: $A = A_0 e^{(-bt)}$. Given that $A = 0.77 A_0$ after 60 seconds, $0.77 = e^{(-b 60)}$. Solving for b gives $b = -\ln(0.77)/60$. The initial amplitude A_0 remains as the starting value, which can be any arbitrary unit since it cancels out in the ratio.

How can we determine the damping coefficient (b) of the oscillator from the given data?

Using the amplitude decay formula, $0.77 = e^{(-b 60)}$. Taking natural logs, $b = -\ln(0.77)/60 \approx 0.0037$ s^{-1}.

What is the relationship between the damping coefficient and the damping ratio in this context?

The damping ratio (ζ) relates to the damping coefficient (b), the mass (m), and the undamped natural frequency (ω_0) as $\zeta = b / (m \omega_0)$. Knowing b and the period allows calculation of ω_0, which helps determine ζ, indicating whether the oscillator is underdamped, critically damped, or overdamped.

How does the period of 30 seconds relate to the undamped natural frequency of the oscillator?

The period T relates to the undamped natural frequency ω_0 by $T = 2\pi / \omega_0$. Therefore, $\omega_0 = 2\pi / T = 2\pi / 30 \approx 0.2094$ rad/sec.

Can we estimate the energy loss in the oscillator after 1 minute based on the amplitude reduction?

Yes. Since potential energy is proportional to the square of amplitude, the energy after 1 minute is approximately $(0.77)^2 \approx 0.593$, or about a 40.7% decrease in energy compared to the initial state.

What is the significance of the damping factor in predicting long-term behavior of the oscillator?

The damping factor determines how quickly the oscillator's amplitude diminishes over time. A higher damping coefficient leads to faster energy loss and quicker cessation of oscillations, indicating whether the system is underdamped, critically damped, or overdamped.

How would the amplitude change if the damping coefficient were doubled, assuming all other parameters remain constant?

Doubling the damping coefficient b would increase the rate of exponential decay, resulting in a more rapid reduction in amplitude over the same period, thus leading to a larger percentage decrease in amplitude after 1 minute.

Additional Resources

Damped Oscillator: An In-Depth Analysis of Amplitude Decay in a 30-Second Period System

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Introduction: Understanding the Damped Oscillator Phenomenon

In the realm of physics and engineering, oscillatory systems are fundamental to numerous applications—from seismic activity analysis and electronic circuits to mechanical systems like suspension bridges and clocks. Among these, the damped oscillator stands out for its ability to model realistic systems where energy loss occurs over time due to friction, air resistance, or other dissipative forces.

This article explores a specific scenario involving a damped oscillator with a period of 30 seconds that exhibits a significant reduction in amplitude—23%—over a span of 1 minute (60 seconds). Our goal is to analyze, calculate, and interpret the underlying parameters governing this decay, providing a comprehensive understanding of the system's damping characteristics.

The Basics of Damped Oscillation

A damped oscillator is characterized by its gradually decreasing amplitude as it oscillates around an equilibrium position. The fundamental differential equation governing such systems is:

$$
m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0
$$

Where:
- m is the mass of the oscillating object,
- c is the damping coefficient,
- k is the spring or restoring force constant,
- $x(t)$ is displacement as a function of time.

The solution to this equation, for underdamped systems (where damping is not strong enough to prevent oscillation), is:

$$
x(t) = A_0 e^{-\zeta \omega_0 t} \cos(\omega_d t + \phi)
$$

Where:
- A_0 is the initial amplitude,
- ζ is the damping ratio,
- ω_0 is the natural angular frequency,
- $\omega_d = \omega_0 \sqrt{1 - \zeta^2}$ is the damped angular frequency,
- ϕ is the phase angle.

The key insight here is that the exponential term $e^{-\zeta \omega_0 t}$ governs the decay of amplitude over time.

Key Parameters of Our System

Period and Natural Frequency
Given:
- Period T = 30 seconds

The natural angular frequency ω_0 is related to the period as:

$$
\omega_0 = \frac{2\pi}{T}
$$

Calculating:

$$
\omega_0 = \frac{2\pi}{30} = \frac{\pi}{15} \approx 0.2094 \text{ rad/sec}
$$

This frequency describes how rapidly the system would oscillate in the absence of damping.

Quantifying Damping Through Amplitude Reduction

The Observed Reduction
- After 1 minute (60 seconds), the amplitude decreases by 23%.

Expressed mathematically:

$$
A(t) = A_0 e^{-\zeta \omega_0 t}
$$

At $t = 60$ seconds:

$$
A(60) = A_0 \times (1 - 0.23) = 0.77 A_0
$$

Applying the exponential decay model:

$$
0.77 A_0 = A_0 e^{-\zeta \omega_0 \times 60}
$$

Dividing both sides by A_0:

$$
0.77 = e^{-\zeta \omega_0 \times 60}
$$

Taking the natural logarithm:

$$
\ln(0.77) = -\zeta \omega_0 \times 60
$$

Calculating:

$$
\ln(0.77) \approx -0.2614
$$

Now, solving for ζ:

$$
\zeta = \frac{-\ln(0.77)}{\omega_0 \times 60}
$$

Substituting $\omega_0 \approx 0.2094$:

$$
\zeta = \frac{0.2614}{0.2094 \times 60} \approx \frac{0.2614}{12.564} \approx 0.0208
$$

Result:

$$
\boxed{
\text{Damping ratio } \zeta \approx 0.0208
}
$$

This small damping ratio signifies a lightly damped system, where oscillations persist over many cycles with gradually decreasing amplitude.

Calculating the Damped Frequency

The damped angular frequency ω_d is:

$$
\omega_d = \omega_0 \sqrt{1 - \zeta^2}
$$

Substituting the known values:

$$
\omega_d = 0.2094 \times \sqrt{1 - (0.0208)^2} \approx 0.2094 \times \sqrt{1 - 0.000433} \approx 0.2094 \times 0.99978 \approx 0.2093 \text{ rad/sec}
$$

The damped period T_d then is:

$$
T_d = \frac{2\pi}{\omega_d} \approx \frac{6.2832}{0.2093} \approx 30.0 \text{ seconds}
$$

Thus, damping has a negligible effect on the period in this lightly damped system, which aligns with expectations.

Interpreting the Results

The damping ratio $\zeta \approx 0.0208$ indicates that the oscillator exhibits very slight damping. Over the course of 60 seconds, the amplitude drops by 23%, which is consistent with a lightly damped oscillatory system.

This analysis reveals:
- The system's natural frequency: approximately 0.2094 rad/sec, correlating to a 30-second period.
- The damping ratio: a very low value, confirming minimal energy dissipation per cycle.
- The amplitude decay: predictable through exponential decay, enabling precise modeling of how the system behaves over time.

Practical Significance and Applications

Understanding damping in oscillatory systems is critical for numerous engineering domains:
- Structural engineering: Ensuring buildings and bridges can withstand vibrations without excessive damping that could compromise stability.
- Mechanical systems: Designing suspensions or shock absorbers with desired damping characteristics.
- Electronics: Tuning circuits to avoid unwanted oscillations or to control signal decay.
- Metrology: Accurate timekeeping devices like pendulum clocks require a balance between damping and oscillation longevity.

The ability to quantify damping through simple measurements—like amplitude decay over a known time—empowers engineers and scientists to optimize system performance, predict longevity, and prevent failures.

Summary of Key Findings

Parameter	Value	Explanation

Period T	30 seconds	Time for one complete oscillation
Natural frequency ω_0	~0.2094 rad/sec	Frequency without damping
Amplitude reduction over 60s	23%	Decay due to damping
Damping ratio ζ	~0.0208	Measure of damping strength
Damped frequency ω_d	~0.2093 rad/sec	Slightly less than ω_0
Damped period T_d	~30 seconds	Nearly identical to natural period

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Final Thoughts: The Elegance of Damped Oscillatory Systems

This exploration underscores how even minimal damping can influence system behavior over time. The precise calculation of damping parameters from simple amplitude decay data exemplifies the power of classical mechanics in predicting real-world phenomena.

For professionals and enthusiasts alike, understanding these concepts not only enriches theoretical knowledge but also informs practical design and troubleshooting. Whether constructing resilient structures or fine-tuning electronic circuits, mastering the principles of damped oscillators ensures systems perform reliably, safely, and efficiently over their operational lifespan.

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