A Distribution Of Values Is Normal With A Mean Of 138 And A Standard Deviation Of 19.Find The Probability

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Understanding the probability associated with a normal distribution is fundamental in statistics, especially when analyzing data that naturally tends to cluster around a central value. In this article, we will explore how to determine the probability for a normally distributed variable with a given mean and standard deviation, specifically focusing on the scenario where the distribution has a mean of 138 and a standard deviation of 19. We will delve into the concepts of the normal distribution, standardization, and the use of the standard normal table to find the required probability. Additionally, practical examples and step-by-step calculations will be provided to clarify the process.

Understanding the Normal Distribution

What Is a Normal Distribution?

The normal distribution, often referred to as the bell curve, is a continuous probability distribution that is symmetric about its mean. It describes how the values of a variable are distributed, with most observations clustering around the central mean and fewer observations appearing as you move further away.

Key characteristics of a normal distribution include:
- Symmetry about the mean.
- The mean, median, and mode are all equal.
- The distribution is fully described by its mean (μ) and standard deviation (σ).
- The empirical rule (68-95-99.7 rule) applies, stating that:
- Approximately 68% of data falls within one standard deviation of the mean.
- Approximately 95% within two standard deviations.
- Approximately 99.7% within three standard deviations.

Parameters of the Distribution

In our case:
- Mean (μ): 138
- Standard deviation (σ): 19

Knowing these parameters allows us to determine the probability of the variable falling within a certain range.

Standardizing the Normal Variable

The Z-Score Concept

To find probabilities associated with a normal distribution, it is standard practice to convert a raw score into a standard normal variable (Z-score). The Z-score indicates how many standard deviations a data point is from the mean.

The formula for calculating Z-score:

$$Z = \frac{X - \mu}{\sigma}$$

Where:
- X is the value of interest.
- μ is the mean.
- σ is the standard deviation.

This transformation allows us to use the standard normal distribution table (Z-table) to find probabilities.

Application to Our Scenario

Suppose we are interested in the probability that a value, X, is at least some number, say 1000, or within a certain range. We would:
1. Calculate the Z-score for X.
2. Find the corresponding probability from the Z-table.
3. Interpret the probability in context.

Note: Since 1000 is significantly larger than the mean (138), we will see that the probability of X being at least 1000 is extremely low.

Calculating the Probability for $X \geq 1000$

Step 1: Calculate the Z-Score for X = 1000

Using the Z-score formula:

$$ Z = \frac{1000 - 138}{19} = \frac{862}{19} \approx 45.37 $$

This Z-score is extraordinarily high, indicating that 1000 is 45.37 standard deviations above the mean.

Step 2: Find Probability Corresponding to Z = 45.37

From the Z-table or standard normal distribution calculator:
- For $Z = 45.37$, the probability $P(Z \leq 45.37)$ is effectively 1.
- Since we're interested in $P(X \geq 1000)$, this corresponds to $P(Z \geq 45.37)$.

Therefore:

$$ P(X \geq 1000) = P(Z \geq 45.37) $$

Given the extreme Z-score, the probability is practically zero, meaning it is virtually impossible for the variable to reach 1000 or more.

Conclusion for X ≥ 1000

- Probability ≈ 0.0000 (practically zero).

This illustrates that in a normal distribution with the given parameters, values as high as 1000 are astronomically unlikely.

Calculating the Probability for X ≤ a Certain Value

Suppose we wanted to find the probability that the variable is less than or equal to a specific value, say 150.

Step 1: Calculate Z-Score for X = 150

$$ Z = \frac{150 - 138}{19} = \frac{12}{19} \approx 0.63 $$

Step 2: Find the Probability P(X ≤ 150)

Using the Z-table:
- $P(Z \leq 0.63) \approx 0.7357$

Interpretation:
- About 73.57% of the data falls below 150.

Finding Probabilities for Other Ranges

The process for other ranges involves similar steps:
1. Calculate the Z-scores for the boundary values.
2. Use the Z-table or calculator to find the corresponding probabilities.
3. Subtract or add probabilities as needed to find the probability within an interval.

Examples:
- Between 120 and 150: $P(120 \leq X \leq 150)$
- Less than 100: $P(X \leq 100)$
- Greater than 200: $P(X \geq 200)$

Example: Probability that X is between 120 and 150

- Z for 120: $Z = \frac{120 - 138}{19} = -0.95$
- Z for 150: $Z = 0.63$

Using Z-table:
- $P(Z \leq 0.63) \approx 0.7357$
- $P(Z \leq -0.95) \approx 0.1714$

Thus:
$$P(120 \leq X \leq 150) = 0.7357 - 0.1714 = 0.5643$$

Approximately 56.43% of the data falls within this interval.

Practical Applications of These Calculations

Understanding how to compute these probabilities is crucial in various fields such as:
- Quality control: Determining the likelihood of products falling outside specifications.
- Finance: Assessing the probability of returns exceeding or falling below certain thresholds.
- Education: Analyzing test score distributions.
- Health sciences: Estimating the probability of measurements being within normal ranges.

Real-world Example: Medical Testing

Suppose a blood test measurement is normally distributed with the same parameters ($\mu=138$, $\sigma=19$). If a patient's reading is 150, what's the probability that a randomly selected individual has a reading less than or equal to this value? Using the above calculations, the probability is approximately 73.57%. Therefore, the patient's result is within the common range for the population.

Summary and Key Takeaways

- The normal distribution is symmetric and defined by its mean and standard deviation.
- Standardization via Z-scores allows us to use standard normal tables to find probabilities.
- Values far from the mean (like 1000 in this case) have negligible probabilities.
- Calculating probabilities involves converting raw scores to Z-scores and referencing the Z-table.
- These techniques are widely applicable across various disciplines for statistical analysis and decision-making.

Conclusion

Determining the probability associated with a normal distribution involves understanding its parameters, standardizing values, and consulting the standard normal distribution table. In our specific scenario, with a mean of 138 and a standard deviation of 19, the probability of observing a value greater than or equal to 1000 is effectively zero, emphasizing the rarity of such an outlier. Conversely, for values closer to the mean, probabilities are more substantial and can be precisely calculated. Mastering these methods equips practitioners and students alike with essential tools for statistical reasoning and data analysis across many fields.

Note: For precise calculations, especially for probabilities involving Z-scores beyond the typical range, advanced statistical software or calculators may be used to obtain extremely accurate results.

Frequently Asked Questions

What is the probability that a value from the normal distribution with mean 138 and standard deviation 19 is less than 120?

First, find the z-score: (120 - 138) / 19 ≈ -0.95. Using standard normal tables or a calculator, P(Z < -0.95) ≈ 0.1711. Therefore, the probability is approximately 17.11%.

How do you calculate the probability that a value exceeds 160 in this distribution?

Calculate the z-score: (160 - 138) / 19 ≈ 1.16. Using a Z-table or calculator, P(Z > 1.16) ≈ 0.1230. So, the probability is approximately 12.30%.

What is the probability that a value falls between 125 and 150 in this distribution?

Calculate z-scores: (125 - 138) / 19 ≈ -0.68 and (150 - 138) / 19 ≈ 0.63. Using Z-tables: P(Z < 0.63) ≈ 0.7357, P(Z < -0.68) ≈ 0.2483. The probability between 125 and 150 is 0.7357 - 0.2483 ≈ 0.4874, or 48.74%.

What is the probability that a value is exactly 138 in this continuous distribution?

In a continuous distribution, the probability of exactly any single value is zero. Therefore, P(X = 138) = 0.

Find the value below which 90% of the data falls in this distribution.

Find the z-score corresponding to the 90th percentile: z ≈ 1.28. Then, value = mean + z standard deviation = 138 + 1.28 19 ≈ 138 + 24.32 ≈ 162.32. So, approximately 90% of values are below 162.32.

What is the probability that a randomly selected value is more than 155?

Calculate z-score: (155 - 138) / 19 ≈ 0.89. Using Z-tables: P(Z > 0.89) ≈ 0.1867. Therefore, about 18.67% of values exceed 155.

If a value is 138, what is its percentile rank in this distribution?

Since 138 is the mean, its z-score is 0. The percentile rank corresponds to P(Z < 0) = 0.5, so 138 is at the 50th percentile.

How would you find the probability that a value is between 125 and 155?

Calculate z-scores: (125 - 138) / 19 ≈ -0.68 and (155 - 138) / 19 ≈ 0.89. Using Z-tables: $P(Z < 0.89) \approx 0.8133$ and $P(Z < -0.68) \approx 0.2483$. The probability is 0.8133 - 0.2483 ≈ 0.565, or 56.5%.

Additional Resources

A Distribution Of Values Is Normal With A Mean Of 138 And A Standard Deviation Of 19. Find The Probability

Understanding the nuances of probability within a normal distribution is a foundational aspect of statistics, essential for fields ranging from medicine and engineering to economics and social sciences. When a data set follows a normal distribution with a specified mean and standard deviation, it enables us to make predictions about the likelihood of observing certain values. In this article, we will explore how to interpret and calculate these probabilities, focusing on a specific example: a distribution with a mean of 138 and a standard deviation of 19.

Foundations of Normal Distribution

Before delving into probability calculations, it is crucial to understand what a normal distribution entails.

What Is a Normal Distribution?

A normal distribution, often called a bell curve, is a probability distribution that is symmetric about its mean. It describes how the values of a variable are distributed, with most observations clustering around the mean and fewer observations appearing as you move away toward the extremes.

Key characteristics include:
- Symmetry about the mean
- The mean, median, and mode are equal
- The total area under the curve equals 1 (representing total probability)
- The shape is determined by the mean (μ) and standard deviation (σ)

Parameters: Mean and Standard Deviation

- Mean (μ): The average of all data points, which in our case is 138.
- Standard Deviation (σ): Measures the spread of the data around the mean, here 19. It indicates how tightly or loosely data points cluster.

Standardizing Values: Z-scores

Calculating probabilities involves understanding where a specific value falls within the distribution. Since normal distributions are characterized by their parameters, transforming values into a common scale simplifies calculations.

What is a Z-score?

A Z-score indicates how many standard deviations a particular value (X) is from the mean. It is calculated as:

$$Z = \frac{X - \mu}{\sigma}$$

Where:
- X is the value of interest
- $\mu = 138$ is the mean
- $\sigma = 19$ is the standard deviation

For example, if you want to find the probability that a value exceeds 160, you first find the Z-score for 160.

Calculating Probabilities Using the Standard Normal Distribution

Once the Z-score is determined, the next step is to find the probability associated with that Z-score using the standard normal distribution table or statistical software.

Using Z-tables

Z-tables provide the area (probability) to the left of a given Z-score on the standard normal curve.

Example:

Find the probability that a value is less than 160.

1. Calculate Z-score:

$$ Z = \frac{160 - 138}{19} = \frac{22}{19} \approx 1.16 $$

2. Look up Z = 1.16 in the Z-table.

Typically, Z = 1.16 corresponds to an area of approximately 0.8770, meaning:

$$ P(X < 160) \approx 0.8770 $$

This indicates an approximately 87.70% chance that a randomly selected value from the distribution is less than 160.

Using Statistical Software or Calculators

Modern tools like Excel, R, Python, or online calculators can directly compute probabilities:

- Excel: `=NORM.DIST(160, 138, 19, TRUE)`
- Python (SciPy): `scipy.stats.norm.cdf(160, loc=138, scale=19)`

These methods streamline probability calculations, especially for more complex queries.

Practical Examples of Probability Calculations

Let's explore different probability scenarios based on the distribution parameters.

1. Probability of a Value Less Than a Certain Number

Suppose we want to find the probability that a value is less than 120.

- Calculate Z-score:

$$Z = \frac{120 - 138}{19} = \frac{-18}{19} \approx -0.95$$

- Look up Z = -0.95 in Z-table or use software, which gives approximately 0.1719.

- Interpretation: Around 17.19% of the values are less than 120.

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2. Probability of a Value Greater Than a Certain Number

Find the probability that a value exceeds 160.

- Z-score:

$$Z = \frac{160 - 138}{19} \approx 1.16$$

- Probability to the left of Z = 1.16 is 0.8770, so:

$$P(X > 160) = 1 - 0.8770 = 0.1230$$

- Interpretation: Approximately 12.30% of the values are greater than 160.

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3. Probability of a Value Between Two Numbers

Calculate the probability that a value falls between 120 and 160.

- Z-scores:

$$Z_{120} = -0.95$$
$$Z_{160} = 1.16$$

- Probabilities:

$$P(X < 160) \approx 0.8770$$
$$P(X < 120) \approx 0.1719$$

- Therefore:

$$P(120 < X < 160) = 0.8770 - 0.1719 = 0.7051$$

- Interpretation: About 70.51% of the data lies between 120 and 160.

Application: Real-World Implications of These Probabilities

Understanding these probabilities allows practitioners to make informed decisions. For example:

- Quality Control: If measurements follow this distribution, a manufacturer can estimate the likelihood of products falling outside acceptable ranges.
- Education Testing: Educators can determine the percentage of students scoring below or above specific thresholds.
- Healthcare: Medical professionals can assess the probability of certain test results falling within or outside normal ranges.

Limitations and Assumptions

While the normal distribution provides a powerful model, several assumptions and limitations should be acknowledged:

- Normality Assumption: The data must be approximately normally distributed. Deviations can lead to inaccurate probability estimates.
- Outliers: Extreme values can distort the distribution.
- Sample Size: Small samples may not accurately reflect the true distribution parameters.
- Contextual Factors: External factors may influence the data, making the normal model less appropriate.

Conclusion: From Distribution to Decision-Making

Knowing how to calculate probabilities within a normal distribution is vital for interpreting data and making data-driven decisions. In the case of a distribution with a mean of 138 and a standard deviation of 19, understanding how to convert raw values to Z-scores and then consult tables or software allows for efficient probability estimation.

Whether you're estimating the likelihood of a test score falling below a certain threshold, predicting the chance of extreme measurements, or assessing the spread of data, mastery of these techniques empowers statisticians, analysts, and decision-makers alike.

This process underscores the importance of statistical literacy in navigating the complexities of real-world data, transforming raw numbers into meaningful insights and actionable strategies.

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