A.Find The Equation Of Plane Through The Point (4,3,4) And Parallel To The Xy-plane. B. Find The Equation

Understanding the Problem: Finding the Equation of a Plane Through a Point and Parallel to a Given Plane

A.Find The Equation Of Plane Through The Point (4,3,4) And Parallel To The Xy-plane. B. Find The Equation introduces a common problem in three-dimensional geometry: determining the equation of a plane that passes through a specific point and runs parallel to a known plane. This type of problem helps students and mathematicians understand the fundamental concepts of planes in 3D space, including the role of normal vectors and the importance of parallelism.

In this article, we will explore step-by-step how to approach such problems, focusing on the geometric intuition, the algebraic formulation, and practical methods for deriving the equations of planes under these conditions. Whether you're preparing for exams, solving real-world engineering problems, or just deepening your understanding of 3D geometry, mastering these techniques is essential.

Basics of Plane Equations in 3D Space

Before diving into the specific problem, it's important to review the fundamental concepts related to planes in three-dimensional space.

Standard Equation of a Plane

A plane in 3D space can be expressed using the general equation:

$$ ax + by + cz + d = 0 $$

where:

- (a, b, c) is the normal vector perpendicular to the plane.
- d is a scalar constant.

The normal vector plays a crucial role because it defines the orientation of the plane.

Normal Vectors and Plane Orientation

- The normal vector $\vec{n} = (a, b, c)$ determines the tilt and orientation.
- If two planes are parallel, their normal vectors are scalar multiples (i.e., they are parallel or identical).

Planes Parallel to the XY-Plane

The XY-plane itself has the equation:

$$z = 0$$

- Its normal vector is $(0, 0, 1)$.

A plane parallel to the XY-plane will also have a normal vector parallel to $(0, 0, 1)$, which indicates that the plane's equation will have no x or y terms affecting the normal vector's direction.

Part A: Finding the Equation of a Plane Through a Point and Parallel to the XY-plane

Step 1: Recognize the Geometric Condition

Given that the plane is parallel to the XY-plane:

- The normal vector of the desired plane must be parallel to the normal vector of the XY-plane.
- The normal vector of the XY-plane is $(0, 0, 1)$.

Therefore, the plane's normal vector $\vec{n}$ can be expressed as:

$$\vec{n} = (0, 0, c)$$

where $c \neq 0$. Usually, for simplicity, c can be taken as 1, since scalar multiples do not change the plane's orientation.

Step 2: Use the Point to Find the Equation

The plane passes through the point $(4, 3, 4)$. Substituting into the general plane equation:

$$0 \cdot x + 0 \cdot y + 1 \cdot z + d = 0$$

which simplifies to:

$$ z + d = 0 $$

Applying the point $(4, 3, 4)$:

$$ 4 + d = 0 \Rightarrow d = -4 $$

Step 3: Write the Final Equation

Thus, the equation of the plane is:

$$ z - 4 = 0 \quad \text{or simply} \quad z = 4 $$

This plane is parallel to the XY-plane and passes through the point $(4, 3, 4)$.

Part B: Finding the Equation of a Plane with Different Conditions

Since the original prompt mentions "B. Find The Equation," it suggests another plane or condition to be considered. Depending on the context, this could involve:

- A plane passing through different points.
- A plane parallel to a different plane (e.g., XZ-plane or YZ-plane).
- A plane with a specific normal vector or orientation.

Let's explore some common scenarios.

Scenario 1: Plane Parallel to the XZ-Plane

The XZ-plane has the equation:

$$ y = 0 $$

Its normal vector is $(0, 1, 0)$.

- For a plane parallel to the XZ-plane, the normal vector must be parallel to $(0, 1, 0)$.

Given the point $(4, 3, 4)$, the plane's equation will be:

$$ 0 \cdot x + 1 \cdot y + 0 \cdot z + d = 0 \Rightarrow y + d = 0 $$

Substituting the point:

$$3 + d = 0 \Rightarrow d = -3$$

Final equation:

$$y - 3 = 0 \quad \text{or} \quad y = 3$$

This plane passes through the point $(4, 3, 4)$ and is parallel to the XZ-plane.

Scenario 2: Plane Parallel to the YZ-Plane

The YZ-plane has the equation:

$$x = 0$$

Normal vector: $(1, 0, 0)$.

- For a plane parallel to the YZ-plane:

$$x + d = 0$$

Substituting point $(4, 3, 4)$:

$$4 + d = 0 \Rightarrow d = -4$$

Equation:

$$x - 4 = 0 \quad \text{or} \quad x = 4$$

Additional Tips for Solving Plane Equations

1. Identifying the Normal Vector

- Always analyze the plane's orientation relative to known planes or axes.
- Parallel planes share the same normal vector or scalar multiples.

2. Using Point-Normal Form

- The point-normal form of a plane:

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$

where:

- $\vec{n}$ is the normal vector.
- $\vec{r} = (x, y, z)$ is a general point on the plane.
- $\vec{r}_0$ is a point on the plane.

This form makes it straightforward to write the plane's equation once the normal vector and a point are known.

3. Confirming Parallelism

- Two planes are parallel if their normal vectors are scalar multiples.
- To check if a plane is parallel to a known plane, compare their normal vectors.

Conclusion: Mastering the Art of Plane Equations

Understanding how to find the equation of a plane through a specific point and parallel to a known plane is fundamental in 3D geometry. The key steps involve identifying the normal vector based on the plane's orientation, substituting the point into the general equation, and simplifying to find the specific plane equation.

In the case of the plane passing through $(4, 3, 4)$ and parallel to the XY-plane, the solution is straightforward: the plane's equation is $z = 4$. For other orientations, such as parallel to the XZ-plane or YZ-plane, similar methods apply, focusing on the normal vector and point substitution.

Mastering these techniques enhances spatial reasoning and problem-solving skills in mathematics, engineering, computer graphics, and related fields. Whether dealing with simple cases or complex arrangements, understanding the relationship between points, normals, and planes is essential for accurate and efficient solutions.

Keywords: plane equation, parallel planes, point-normal form, 3D geometry, XY-plane, XZ-plane, YZ-plane, normal vector, algebraic geometry, spatial reasoning

Frequently Asked Questions

What is the general form of the equation of a plane parallel to the XY-plane passing through the point (4, 3, 4)?

Since the plane is parallel to the XY-plane, its equation is of the form $z = k$. Given the point $(4, 3, 4)$, the equation is $z = 4$.

How do you find the equation of a plane passing through a specific point and parallel to the XY-plane?

A plane parallel to the XY-plane has a constant z-value, so its equation is $z = z_0$, where z_0 is the z-coordinate of the given point. For the point (4, 3, 4), the equation is $z = 4$.

Is the equation of the plane through (4, 3, 4) parallel to the XY-plane unique?

Yes. Since the plane is parallel to the XY-plane, its equation is uniquely determined by the z-coordinate of the given point, resulting in $z = 4$.

What is the significance of the point (4, 3, 4) in determining the plane's equation?

The point provides the specific z-value ($z=4$) that the plane must pass through, which, combined with the parallelism to XY-plane, defines the plane's equation as $z = 4$.

Can the plane parallel to XY-plane through (4, 3, 4) be expressed in a different form?

No. Since the plane is parallel to XY-plane, its equation is simply $z = 4$, which is the most straightforward form.

What is the next step after finding the plane's equation if asked for a different condition?

If a different condition is specified, such as passing through another point or being parallel to another plane, you would incorporate that information into the equation accordingly.

How does the concept of parallel planes help in determining the equation in this problem?

Parallel planes share the same normal vector. Since the XY-plane is normal to the z-axis, any plane parallel to it must have an equation with a constant z-value, simplifying the problem to z = constant.

What is the final answer to the problem 'Find the equation of the plane through (4, 3, 4) and parallel to the XY-plane'?

The equation of the plane is $z = 4$.

Additional Resources

A. Find The Equation Of Plane Through The Point (4,3,4) And Parallel To The XY-Plane. B. Find The Equation

When tackling problems related to the equation of a plane through a specific point and parallel to a given plane, it is essential to understand the fundamental concepts of planes in three-dimensional space. These problems often involve identifying the normal vector to the plane, applying the point-normal form of a plane's equation, and understanding the geometric implications of parallelism. This guide aims to provide a comprehensive, step-by-step approach to solving such problems, focusing on the specific case of finding the equation of a plane passing through the point (4,3,4) and parallel to the XY-plane, followed by a similar analysis for the second part of the problem.

Understanding the Geometry of Planes in 3D Space

Before diving into the solution, let's review some critical concepts:

The Equation of a Plane

A plane in three-dimensional space can be represented by the general equation:

$$Ax + By + Cz + D = 0$$

where:

- (A, B, C) is a normal vector to the plane.
- D is a scalar constant.

Normal Vector to a Plane

The normal vector, often denoted as $\vec{n} = (A, B, C)$, is perpendicular to the plane. It plays a central role in deriving the plane's equation once a point on the plane is known.

Parallel Planes

Two planes are parallel if their normal vectors are scalar multiples of each other. Equivalently, they share the same direction of normal vector, meaning the coefficients (A, B, C) are proportional.

Part A: Finding the Equation of the Plane Through (4,3,4) and Parallel to the XY-plane

Step 1: Recognize the Characteristics of the XY-plane

The XY-plane is described by the equation:

$$z = 0$$

or equivalently:

$$ 0x + 0y + 1z + 0 = 0 $$

which indicates that its normal vector is:

$$ \vec{n}_{XY} = (0, 0, 1) $$

Step 2: Determine the Normal Vector of the Desired Plane

Since the plane we're seeking is parallel to the XY-plane, it must have the same normal vector as the XY-plane:

$$ \vec{n} = (0, 0, 1) $$

Step 3: Use the Point-Normal Form

The point-normal form of a plane's equation is:

$$ A(x - x_0) + B(y - y_0) + C(z - z_0) = 0 $$

where (x_0, y_0, z_0) is a point on the plane, and (A, B, C) is the normal vector.

Given:

- Point $(4, 3, 4)$
- Normal vector $(0, 0, 1)$

Substituting into the point-normal form:

$$ 0(x - 4) + 0(y - 3) + 1(z - 4) = 0 $$

which simplifies to:

$$ z - 4 = 0 $$

Final Equation for Part A:

$$ z = 4 $$

This is the equation of the plane passing through the point $(4, 3, 4)$ and parallel to the XY-plane. Geometrically, it's a plane parallel to XY-plane at a height $z = 4$.

Part B: Find the Equation of a Plane Through a Given Point and Parallel to a Different Plane

Suppose the second part of the problem asks us to find the equation of a plane passing through the point $(4, 3, 4)$ and parallel to another plane, say, the YZ-plane.

Step 1: Recognize the YZ-plane Equation

The YZ-plane is described by:

$$ x = 0 $$

with the normal vector:

$$ \vec{n}_{YZ} = (1, 0, 0) $$

Step 2: Determine the Normal Vector of the New Plane

Since the new plane must be parallel to the YZ-plane, it must share the same normal vector:

$$ \vec{n} = (1, 0, 0) $$

Step 3: Apply Point-Normal Form

Using the point $(4, 3, 4)$:

$$ 1(x - 4) + 0(y - 3) + 0(z - 4) = 0 $$

which simplifies to:

$$ x - 4 = 0 $$

Final Equation for Part B:

$$ x = 4 $$

This plane passes through $(4, 3, 4)$ and is parallel to the YZ-plane.

Extending the Concept: General Approach to Plane Problems

While the specific examples above involve planes parallel to coordinate planes, the general approach to similar problems involves:

1. Identifying the Given Conditions
- Determine the plane's orientation (parallel to which plane or plane's normal vector).
- Note the point through which the plane passes.

2. Find the Normal Vector
- For plane parallel to a known plane, use the same normal vector.
- For planes with specified orientation, derive the normal vector accordingly.

3. Write the Equation in Point-Normal Form
- Use the known point and the normal vector to write the equation.

4. Simplify and Express the Equation
- Convert into the standard form, if needed.

Practical Tips for Solving Plane Equations

- Visualization: Visualize the plane relative to coordinate axes and the given constraints.
- Normal Vectors: Always confirm the normal vector when dealing with parallel planes.
- Special Planes: Recognize common planes like XY, YZ, ZX, and their equations to speed up problem-solving.
- Coordinate Substitutions: Use the point-normal form directly to avoid unnecessary algebra.

Summary

- To find the equation of a plane through a point and parallel to a given plane, identify the normal vector of the given plane.
- Use the point-normal form to derive the plane's equation.
- For the specific problem of a plane through \((4,3,4) \) parallel to the XY-plane, the answer is simply \(z=4 \).
- For other orientations, follow the same method, adjusting the normal vector accordingly.

By mastering these fundamental steps, you can confidently approach a wide variety of plane equations in three-dimensional geometry, whether for academic exercises, engineering applications, or advanced mathematical analysis.

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