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A Line Has A Slope Of 1/2. Which Of The Following Is True About A Line That Is Perpendicular To The Given

Understanding the properties of lines and their slopes is fundamental in coordinate geometry. When analyzing relationships between lines, particularly perpendicular lines, recognizing how slopes interact is crucial. This article explores the characteristics of lines with specific slopes, focusing on the case where a line has a slope of 1/2, and examines what occurs when a line perpendicular to it is considered.

Introduction to Slopes and Perpendicular Lines

What Is a Slope?

The slope of a line measures its steepness and direction. It is typically represented by the letter m in the slope-intercept form of a line: y = mx + b. The slope indicates how much y changes for a unit increase in x.

Mathematically, the slope is calculated as:
$$ m = \frac{\Delta y}{\Delta x} $$
where Δy is the change in the y-coordinate, and Δx is the change in the x-coordinate between two points on the line.

Perpendicular Lines and Their Slopes

Two lines are perpendicular if they intersect at a right angle (90 degrees). A key property of perpendicular lines in the coordinate plane is the relationship between their slopes:

- The slopes are negative reciprocals of each other.
- If one line has slope $ m $, then a line perpendicular to it will have a slope of $ -\frac{1}{m} $.

This relationship is fundamental in solving problems related to perpendicular lines, especially when the slope of the original line is known.

Analyzing the Given Line with Slope 1/2

The Slope of the Given Line

The given line has a slope of:
$$ m_1 = \frac{1}{2} $$
This means for every 2 units increase in x, y increases by 1 unit.

What Does This Imply for Its Perpendicular Line?

Applying the property of perpendicular slopes:
$$ m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{1}{2}} $$
Simplifying:
$$ m_2 = -2 $$

Therefore, a line perpendicular to the original line with slope 1/2 must have a slope of -2.

Key Characteristics of the Perpendicular Line

Slope of the Perpendicular Line

As established, the slope of the perpendicular line is -2. This negative slope indicates the line decreases as x increases, in contrast to the original line, which increases.

Equation of the Perpendicular Line

Knowing the slope is essential, but equations are often more useful in applications. The general form of the equation of a line with slope -2 passing through a point (x_0, y_0) is:
$$ y - y_0 = -2(x - x_0) $$
or, in slope-intercept form:
$$ y = -2x + b $$
where b is the y-intercept, determined by specific points.

Graphical Relationship

On a coordinate plane, the original line with slope 1/2 is less steep and inclined upward, while its perpendicular line with slope -2 is steeper and inclined downward. The two lines intersect at a right angle, confirming their perpendicularity.

Practical Applications and Examples

Example 1: Find the Equation of Perpendicular Line Passing Through a Point

Suppose the original line passes through point (x_1, y_1). To find the equation of the perpendicular line passing through this point:

1. Identify the slope of the perpendicular line:
$$ m_{perp} = -2 $$

2. Use point-slope form:
$$ y - y_1 = -2(x - x_1) $$

3. Simplify to slope-intercept form if needed.

Example: Find the line perpendicular to $ y = \frac{1}{2}x + 3 $ passing through $(4, 5)$.

Solution:
$$
\begin{aligned}
y - 5 &= -2(x - 4) \\
y - 5 &= -2x + 8 \\
y &= -2x + 8 + 5 \\
y &= -2x + 13
\end{aligned}
$$

The perpendicular line has the equation $y = -2x + 13$.

Example 2: Determine if Two Lines are Perpendicular

Given two lines:

- Line A: $y = \frac{1}{2}x + 4$
- Line B: $y = -2x + 1$

Since the slopes are $\frac{1}{2}$ and -2, respectively, and these are negative reciprocals, the lines are perpendicular.

Common Mistakes and Clarifications

Misunderstanding the Slope Relationship

A common mistake is assuming that any negative slope indicates perpendicularity, which is incorrect. The key is the reciprocal, specifically the negative reciprocal.

- Incorrect: Any line with a negative slope is perpendicular.
- Correct: The slopes are negative reciprocals, i.e., $m_2 = -\frac{1}{m_1}$.

Handling Vertical and Horizontal Lines

Special cases occur when:

- The original line is horizontal ($m = 0$). Its perpendicular line is vertical with an undefined slope.
- The original line is vertical (undefined slope). Its perpendicular line is horizontal with slope 0.

In our case, with slope 1/2, the perpendicular slope -2 is well-defined, and no such special case applies.

Summary of Key Points

- The original line has slope $\frac{1}{2}$.
- Any line perpendicular to it must have a slope of -2.
- The slopes are negative reciprocals of each other: $m_{perp} = -\frac{1}{m_{original}}$.
- Equations of perpendicular lines can be written using point-slope form or slope-intercept form, depending on known points.
- Understanding these relationships is crucial in solving geometric problems involving perpendicular lines.

Conclusion

Understanding the relationship between slopes of perpendicular lines is essential in coordinate geometry. When a line has a slope of 1/2, any line perpendicular to it must have a slope of -2. This fundamental property allows you to quickly identify, analyze, and construct perpendicular lines in various mathematical and real-world contexts, such as engineering, architecture, and computer graphics. Mastering this concept enhances problem-solving skills and deepens comprehension of geometric relationships in the coordinate plane.

Frequently Asked Questions

What is the slope of a line perpendicular to a line with a slope of 1/2?

The slope of the perpendicular line is -2.

Why is the slope of a perpendicular line to a line with slope 1/2 equal to -2?

Because perpendicular lines have slopes that are negative reciprocals; the reciprocal of 1/2 is 2, and negating it gives -2.

If a line has a slope of 1/2, which of the following is true about its perpendicular line?

Its perpendicular line has a slope of -2.

Can the slope of a line perpendicular to a line with slope 1/2 be any other value besides -2?

No, the slope must be the negative reciprocal, which is -2.

How do you determine the slope of a line perpendicular to one with slope 1/2?

Take the reciprocal of 1/2, which is 2, then change the sign to negative, resulting in -2.

In coordinate geometry, what is the relationship between slopes of perpendicular lines?

Their slopes are negative reciprocals; multiplying them gives -1.

Additional Resources

Understanding the Perpendicular Line to a Slope of 1/2: A Comprehensive Analysis

When delving into the world of coordinate geometry, the concept of slopes and their relationships is fundamental. Among these, the idea of perpendicular lines holds particular significance, especially in understanding geometric relationships, solving algebraic problems, and visualizing graphs. This detailed exploration aims to clarify what it means for a line to be perpendicular to another with a given slope, specifically a line with a slope of 1/2, and to analyze the properties and implications of such a perpendicular

line.

Fundamentals of Slope and Perpendicularity

Before examining the specific case of a line perpendicular to one with a slope of 1/2, it's essential to review the core concepts:

What is a Slope?

- In coordinate geometry, the slope of a line measures its steepness.
- It is often denoted as m and calculated as the "rise over run" between any two points on the line:

$$
m = \frac{\Delta y}{\Delta x}
$$

- The slope indicates the direction of the line:
- Positive slope: line ascends from left to right.
- Negative slope: line descends from left to right.
- Zero slope: horizontal line.
- Undefined slope: vertical line.

Perpendicular Lines and Their Slopes

- Two lines are perpendicular if they intersect at a 90° (right) angle.
- The slopes of perpendicular lines have a special relationship:

$$
m_1 \times m_2 = -1
$$

where m_1 and m_2 are the slopes of the two lines.
- This relationship holds true provided neither line is vertical or horizontal in a degenerate sense.

Analyzing the Given Line: Slope of 1/2

The line in question has a slope $m = \frac{1}{2}$. This indicates:

- The line rises 1 unit vertically for every 2 units it runs horizontally.
- The line is relatively gentle in inclination, not too steep nor too flat.
- Its equation (if passing through a point (x_0, y_0)) can be written as:

$$
y - y_0 = \frac{1}{2}(x - x_0)
$$

Understanding this baseline helps us analyze the properties of any line perpendicular to it.

Determining the Slope of the Perpendicular Line

Given the relationship between slopes of perpendicular lines:

$$
m_1 \times m_2 = -1
$$

- For $m_1 = \frac{1}{2}$, the slope of the perpendicular line m_2 is:

$$
m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{1}{2}} = -2
$$

Key Point:
The slope of any line perpendicular to a line with slope $\frac{1}{2}$ is -2.

This reciprocal negative relationship signifies that the slopes are negative reciprocals of each other.

Implications of the Perpendicular Slope: Geometric and Algebraic Perspectives

Understanding the perpendicular slope as -2 opens up various insights:

Geometric Interpretation

- The line with slope -2 is steeper than the original line.
- It descends more sharply, with a vertical change of 2 units for every 1 unit horizontally.
- When graphed, the two lines will intersect at right angles, satisfying the orthogonality condition.

Algebraic Representation

- The general form for the perpendicular line passing through a point (x_1, y_1) is:

$$
y - y_1 = -2(x - x_1)
$$

- This equation can be expanded or rearranged into slope-intercept form:

$$
y = -2x + (2x_1 + y_1)
$$

- The specific parameters depend on the point through which the perpendicular line passes.

Multiple Choice Context and Common Misconceptions

Often, problems related to perpendicular lines with specific slopes appear in multiple-choice formats, testing conceptual understanding. Typical options may include:

- A line with slope -2
- A line with slope 2
- A vertical line
- A horizontal line

Correct Choice:
The line perpendicular to one with slope $\frac{1}{2}$ must have a slope -2.

Common misconceptions include:

- Assuming the perpendicular line has the same slope but negative (incorrect; same slope means parallel, not perpendicular).
- Confusing the reciprocal with the negative reciprocal (the reciprocal of $\frac{1}{2}$ is 2, but the negative reciprocal is -2).
- Thinking that a perpendicular line must be vertical or horizontal (not necessarily; unless the original line is horizontal or vertical, its perpendicular will have a specific slope).

Special Cases and Additional Considerations

While the general rule is straightforward, certain cases are worth noting:

Original Line is Horizontal or Vertical

- If the original line is horizontal ($m=0$), the perpendicular line is vertical ($x = c$), which has an undefined slope.
- Conversely, if the original line is vertical (m undefined), the perpendicular line is horizontal ($y = c$), slope 0.

Implications for Equations and Graphs

- The perpendicular line's equation depends on a specific point; without it, we determine only the slope.
- The intersection point can be found by solving the two equations simultaneously if a point is given.

Practical Applications and Real-World Examples

Understanding perpendicular lines with specific slopes has numerous applications:

- Design and Architecture: Ensuring structures or features are at right angles.
- Navigation and Mapping: Calculating perpendicular paths or routes.
- Physics: Analyzing vectors and forces that are orthogonal.
- Computer Graphics: Rendering perpendicular lines or features for visual effects.

Summary and Key Takeaways

To wrap up, here's a concise summary:

- The slope of a line with slope $\frac{1}{2}$ is moderate, rising gently.
- A line perpendicular to it must have a slope of -2, which is the negative reciprocal.
- This relationship is fundamental:
$$
m_{perpendicular} = -\frac{1}{m_{original}}
$$
- The perpendicular line's equation depends on the point it passes through but will always have slope -2.
- Special cases arise when the original line is horizontal or vertical, leading to vertical or horizontal perpendicular lines, respectively.

Final Thoughts: Mastering the Concept of Perpendicular Slopes

Grasping the relationship between slopes in perpendicular lines is essential for a deep understanding of coordinate geometry. Recognizing that the negative reciprocal of a given slope defines its perpendicular counterpart simplifies many problems and enhances geometric intuition.

In the specific case of a line with a slope of $\frac{1}{2}$, the perpendicular line's slope is unequivocally -2. This knowledge allows students, educators, and professionals to solve a broad range of problems involving right angles, optimize geometric designs, and analyze complex systems with precision.

Remember, the core principle hinges on the reciprocal and sign change in slopes: always check whether the line is horizontal or vertical to determine the correct perpendicular slope, and be confident in applying the negative reciprocal rule to find the perpendicular line's slope in algebraic and geometric contexts.

In conclusion, the key fact is:
If a line has a slope of $\frac{1}{2}$, then any line perpendicular to it must have a slope of -2.

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