A Non-relativistic Particle Of Mass M With Kinetic Energy 0.01 Ev Has Wavelength Of 0.1 Nm. If The Energy

A Non-relativistic Particle Of Mass M With Kinetic Energy 0.01 Ev Has Wavelength Of 0.1 Nm. If The Energy

Understanding the behavior of particles at the quantum level is fundamental to many areas of physics and materials science. When dealing with particles such as electrons, protons, or other subatomic entities, their wave-like nature becomes significant, especially at small scales. In this article, we explore a specific scenario involving a non-relativistic particle with a given kinetic energy and wavelength, and analyze how to determine its energy, mass, and related properties.

Introduction to Particle-Wave Duality

The principle of wave-particle duality states that particles can exhibit both particle-like and wave-like behavior, depending on the circumstances. This duality is central to quantum mechanics and is exemplified by the de Broglie hypothesis, which relates a particle's wavelength to its momentum.

De Broglie Wavelength and Momentum

The de Broglie wavelength (\(\lambda\)) of a particle is given by the relation:

De Broglie Relation

\[
\lambda = \frac{h}{p}
\]

Where:
- \(h\) is Planck's constant (\(6.626 \times 10^{-34}\; \text{Js}\))
- \(p\) is the momentum of the particle

Since the particle is non-relativistic, its momentum can be expressed as:

Momentum in Non-Relativistic Case

\[
p = \sqrt{2 M KE}

$$
$$

Where:
- M is the mass of the particle
- KE is its kinetic energy

Given Data and Objective

From the problem statement, we have:

- Kinetic energy, $KE = 0.01\, \text{eV}$
- Wavelength, $\lambda = 0.1\, \text{nm} = 1 \times 10^{-10}\, \text{m}$

Our goal is to determine the energy of the particle, verify the consistency of the data, and find the mass M.

Converting Energy Units

To proceed, we need to convert the kinetic energy from electronvolts (eV) to joules (J):

Conversion Factor

$$
1\, \text{eV} = 1.602 \times 10^{-19}\, \text{J}
$$

Thus,

$$
KE = 0.01\, \text{eV} = 0.01 \times 1.602 \times 10^{-19} = 1.602 \times 10^{-21}\, \text{J}
$$

Calculating Momentum

Using the de Broglie relation:

$$
p = \frac{h}{\lambda}
$$

Substituting known values:

$$
p = \frac{6.626 \times 10^{-34}}{1 \times 10^{-10}} = 6.626 \times 10^{-24}\, \text{kg·m/s}
$$

Calculating the Particle's Mass

Since the particle is non-relativistic, its kinetic energy relates to momentum as:

$$
KE = \frac{p^2}{2 M}
$$

Rearranged to solve for M:

$$
M = \frac{p^2}{2 KE}
$$

Plugging in the known values:

$$
M = \frac{(6.626 \times 10^{-24})^2}{2 \times 1.602 \times 10^{-21}}
$$

Calculating numerator:

$$
(6.626 \times 10^{-24})^2 = 4.392 \times 10^{-47}
$$

Calculating denominator:

$$
2 \times 1.602 \times 10^{-21} = 3.204 \times 10^{-21}
$$

Thus,

$$
M = \frac{4.392 \times 10^{-47}}{3.204 \times 10^{-21}} \approx 1.37 \times 10^{-26}\, \text{kg}
$$

This mass is comparable to the mass of a proton ($1.67 \times 10^{-27}\, \text{kg}$), suggesting the particle could be a proton or similar subatomic particle.

Verifying the Consistency of Data

To verify the consistency, we can check if the calculated momentum from the energy matches the de Broglie wavelength:

$$
p = \sqrt{2 M KE}
$$

Calculating:

$$
p = \sqrt{2 \times 1.37 \times 10^{-26} \times 1.602 \times 10^{-21}}
$$

$$
p = \sqrt{2 \times 1.37 \times 10^{-26} \times 1.602 \times 10^{-21}} \approx \sqrt{4.392 \times 10^{-47}} \approx 6.626 \times 10^{-24}\, \text{kg·m/s}
$$

which matches our earlier calculation based on the de Broglie relation. This confirms the internal consistency of the data.

Determining the Total Energy of the Particle

In non-relativistic mechanics, the total energy E of a particle is the sum of its kinetic and potential energies. Assuming the particle is free (no potential energy), the total energy equals the kinetic energy:

$$
E = KE
$$

Expressed in electronvolts:

$$
E = 0.01\, \text{eV}
$$

Therefore, the total energy of the particle is 0.01 eV.

Implications and Applications

Understanding the properties of such particles is essential in various fields:

1. Quantum Mechanics and Wave Behavior

- The de Broglie wavelength indicates the wave nature of particles, crucial for understanding phenomena like electron diffraction.

2. Electron Microscopy

- Knowledge of particle wavelengths helps optimize imaging resolution.

3. Material Science and Nanotechnology

- Particle behavior at atomic scales influences the design of nanomaterials.

4. Particle Physics

- Determining particle masses and energies aids in identifying subatomic particles in experiments.

Summary of Key Calculations

- Mass of particle: approximately $1.37 \times 10^{-26}\, \text{kg}$
- Particle's kinetic energy: 0.01 eV or $1.602 \times 10^{-21}\, \text{J}$
- Wavelength: 0.1 nm or $1 \times 10^{-10}\, \text{m}$
- Momentum: approximately $6.626 \times 10^{-24}\, \text{kg·m/s}$

Conclusion

In this analysis, we examined a non-relativistic particle with a known kinetic energy and wavelength, estimated its mass, and confirmed the consistency of the data through fundamental quantum relations. The particle's mass closely resembles that of a proton, and its wavelength indicates significant wave-like behavior at nanometer scales. This understanding is foundational in quantum physics and has practical implications across scientific disciplines.

Further Reading

- Quantum Mechanics: Principles and Applications
- De Broglie Wavelength and Its Significance
- Non-Relativistic Kinetic Energy and Momentum Relations
- Applications of Wave-Particle Duality in Modern Technology

Note: The detailed calculations provided serve as a guide for understanding how to analyze particle properties based on energy and wavelength data. Always ensure that units are consistent and conversions are correctly performed for accurate results.

Frequently Asked Questions

What is the kinetic energy of a non-relativistic particle with mass M and wavelength 0.1 nm?

The kinetic energy is given as 0.01 eV as per the problem statement.

How is the wavelength related to the momentum of a non-relativistic particle?

The wavelength λ is related to momentum p by de Broglie's equation: $\lambda = h / p$, where h is Planck's constant.

What is the value of Planck's constant used in calculations?

Planck's constant h is approximately 6.626×10^{-34} Js.

How do you convert the given wavelength from nanometers to meters?

Since 1 nm = 1×10^{-9} m, 0.1 nm = 1×10^{-10} m.

How can you find the momentum of the particle using its wavelength?

Using de Broglie's relation: $p = h / \lambda$. Substituting λ = 0.1 nm, $p \approx 6.626 \times 10^{-34}$ Js / 1×10^{-10} m = 6.626×10^{-24} kg·m/s.

How do you convert the kinetic energy from eV to joules?

1 eV = 1.602×10^{-19} J; thus, 0.01 eV = 1.602×10^{-21} J.

What formula relates kinetic energy, mass, and velocity for a non-relativistic particle?

Kinetic energy $KE = (1/2) M v^2$, where M is mass and v is velocity.

How can you find the mass M of the particle from the given data?

Using $KE = p^2 / (2 M)$, rearranged as $M = p^2 / (2 KE)$. Substituting $p \approx 6.626 \times 10^{-24}$ kg·m/s and $KE = 1.602 \times 10^{-21}$ J.

What is the approximate mass of the particle based on the provided data?

Calculating M: $M \approx (6.626 \times 10^{-24})^2 / (2 \times 1.602 \times 10^{-21}) \approx 1.37 \times 10^{-26}$ kg.

Why is the particle considered non-relativistic in this context?

Because its kinetic energy (0.01 eV) is much less than its rest energy (mc^2), indicating speeds much less than the speed of light, thus non-relativistic assumptions are valid.

Additional Resources

Quantum Mechanics and Particle Behavior: An In-Depth Analysis of a Non-Relativistic Particle

---

Introduction

In the realm of quantum mechanics, understanding the behavior of particles at microscopic scales is fundamental. Whether you're a researcher, student, or enthusiast, grasping the relationships between a particle's energy, wavelength, and mass offers critical insights into phenomena observed at the atomic and subatomic levels. Today, we explore a fascinating scenario: a non-relativistic particle of mass M possessing a kinetic energy of 0.01 eV and exhibiting a wavelength of 0.1 nm. This case study provides a window into the intricate dance between energy, momentum, and wave properties that define quantum particles.

Setting the Stage: The Physics of Non-Relativistic Particles

Before delving into calculations, it’s essential to clarify what "non-relativistic" signifies. In physics, a particle is considered non-relativistic when its velocity is much less than the speed of light (~3 x 10^8 m/s). Under these conditions, classical Newtonian mechanics and non-relativistic quantum mechanics adequately describe the particle's behavior, simplifying the analysis of energy and momentum relationships.

The key parameters in our scenario are:

- Kinetic Energy (K): 0.01 eV
- Wavelength (λ): 0.1 nm (nanometers)
- Mass (M): Unknown, to be inferred

Understanding the Core Concepts

Quantum Wavelength and de Broglie Hypothesis

The wave-particle duality, central to quantum physics, posits that particles exhibit wave-like properties characterized by a wavelength λ. Louis de Broglie famously formulated this relationship:

$$
\lambda = \frac{h}{p}
$$

where:

- h: Planck’s constant (~6.626 x 10^-34 Js)
- p: Momentum of the particle

This relation implies that measuring the wavelength gives direct insight into the particle's momentum, which in turn relates to its kinetic energy.

Relationship Between Kinetic Energy and Momentum

For a non-relativistic particle, the kinetic energy (K) is related to momentum (p) by:

$$
K = \frac{p^2}{2M}
$$

Rearranged to find p:

$$
p = \sqrt{2 M K}
$$

This allows us to determine the particle’s mass once we compute its momentum from the known wavelength or energy.

Step-by-Step Analysis

1. Calculating the Momentum (p)

Using the de Broglie relation:

\[
p = \frac{h}{\lambda}
\]

Given:

- \(h = 6.626 \times 10^{-34}\) Js
- \(\lambda = 0.1\, \text{nm} = 0.1 \times 10^{-9}\, \text{m} = 1 \times 10^{-10}\, \text{m}\)

Plugging in:

\[
p = \frac{6.626 \times 10^{-34}}{1 \times 10^{-10}} = 6.626 \times 10^{-24} \text{kg·m/s}
\]

This is the particle's momentum.

---

2. Confirming the Kinetic Energy from Momentum

Convert the kinetic energy from electronvolts to joules:

\[
1\, \text{eV} = 1.602 \times 10^{-19}\, \text{J}
\]
\[
K = 0.01\, \text{eV} = 0.01 \times 1.602 \times 10^{-19} = 1.602 \times 10^{-21}\, \text{J}
\]

Now, from the momentum:

\[
p = \sqrt{2 M K}
\]

Solve for M:

\[
M = \frac{p^2}{2K}
\]

Plugging in the known values:

$$
M = \frac{(6.626 \times 10^{-24})^2}{2 \times 1.602 \times 10^{-21}}
$$

Calculating numerator:

$$
(6.626 \times 10^{-24})^2 = 4.39 \times 10^{-47}
$$

Calculating denominator:

$$
2 \times 1.602 \times 10^{-21} = 3.204 \times 10^{-21}
$$

Thus,

$$
M = \frac{4.39 \times 10^{-47}}{3.204 \times 10^{-21}} \approx 1.37 \times 10^{-26} \text{kg}
$$

Interpreting the Results

The inferred mass $M \approx 1.37 \times 10^{-26} \text{kg}$ is remarkably close to the mass of a proton (~1.67×10^{-27} kg), suggesting that the particle could be a proton or a similar subatomic particle. The slight difference might be due to approximation or experimental uncertainties, but this analysis strongly indicates a subatomic particle at rest or moving slowly enough to be non-relativistic.

Additional Insights and Implications

1. Velocity Estimation

Using momentum and mass:

$$
v = \frac{p}{M}
$$

$$
v = \frac{6.626 \times 10^{-24}}{1.37 \times 10^{-26}} \approx 48.4\, \text{m/s}
$$

This velocity (~48 m/s) is well below the speed of light, reaffirming the non-relativistic assumption.

2. De Broglie Wavelength Validity

The given wavelength of 0.1 nm aligns with typical atomic-scale phenomena. For context:

- Atomic radii are about 0.05 to 0.2 nm.
- Electron wavelengths in atomic orbitals are often within this scale.

This suggests the particle could be an electron or proton involved in atomic or molecular processes.

3. Energy-Wavelength Relationship in Practice

The close match between the calculated and provided parameters showcases the power of quantum mechanics in predicting particle behavior. It also demonstrates how experimental measurements of wavelength and energy can be used to infer fundamental properties like mass and velocity, critical for designing experiments or interpreting data in nanotechnology, materials science, and particle physics.

Broader Context: Applications and Significance

Understanding the interplay of energy and wavelength in particles like this is vital across multiple disciplines:

- Nanotechnology: Electron microscopy relies on electron wavelengths to resolve atomic structures.
- Material Science: Quantum tunneling and electron behavior influence semiconductor design.
- Fundamental Physics: Insights into particle masses and energies help probe the Standard Model and beyond.
- Medical Imaging: Techniques like PET scans depend on particle energies and wavelengths.

Final Thoughts

This exploration underscores the elegance of quantum mechanics: from a simple set of parameters—energy and wavelength—we can deduce the mass, velocity, and even infer potential identities of particles. The non-relativistic assumptions hold true here, simplifying the calculations and confirming that the particle's behavior aligns with well-understood quantum principles. Whether you're analyzing fundamental particles or designing cutting-edge nanodevices, these relationships form the backbone of modern physics, bridging the microscopic world with macroscopic observations.

Summary Checklist

- Given Data:
- Kinetic energy \(K = 0.01\, \text{eV} \)

- Wavelength $\lambda = 0.1\, \text{nm}$

- Derived Quantities:
- Momentum $p \approx 6.626 \times 10^{-24}\, \text{kg·m/s}$
- Particle mass $M \approx 1.37 \times 10^{-26}\, \text{kg}$
- Velocity $v \approx 48\, \text{m/s}$

- Implications:
- Mass similar to proton
- Non-relativistic speeds
- Atomic-scale relevance

Closing Remarks

The detailed analysis of a non-relativistic particle with specific energy and wavelength parameters not only deepens our understanding of quantum behavior but also highlights the interconnectedness of physical properties at microscopic scales. Such insights are fundamental to advancing technology and fundamental physics alike, making this a compelling study for scientists and engineers dedicated to exploring the quantum world.

Note: For precise applications or experimental design, always consider potential relativistic effects at higher energies and incorporate quantum corrections where necessary.

[A Non Relativistic Particle Of Mass M With Kinetic Energy 0 01 Ev Has Wavelength Of 0 1 Nm If The Energy]

A Non Relativistic Particle Of Mass M With Kinetic Energy 0 01 Ev Has Wavelength Of 0 1 Nm If The Energy

Related Articles

- A Board Member Is Curious About The Progress Of Your Team On The New Corporate Intranet. The Board Member
- A 0.500-kg Glider, Attached To The End Of An Ideal Spring With Force Constant K=450 N/m, Undergoes Simple
- Conflicts May Occur Between The Stockholder-elected Board Of Directors And The ____ About How The Company

Back to Home