A Triangle Has A Base Length Of (2x - 4) Units And A Height Of (x + 5) Units. Write And Solve A Quadratic

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Understanding how to formulate and solve quadratic equations is a fundamental skill in algebra, especially when dealing with geometric problems involving triangles. In this article, we will explore a specific problem involving a triangle with given expressions for its base and height. Our goal is to write and solve a quadratic equation derived from these measurements, which can help determine the value of the variable x. This process enhances problem-solving skills and deepens comprehension of quadratic functions in geometric contexts.

Understanding the Problem: Triangle Dimensions and the Area Formula

Before diving into the algebraic formulation, it’s essential to understand the geometric context:

- Base length of the triangle: $2x - 4$ units
- Height of the triangle: $x + 5$ units

The problem typically involves calculating the area of the triangle or establishing a relationship that leads to a quadratic equation. The standard formula for the area of a triangle is:

$$ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} $$

Depending on the problem, the area might be given or might be set to a specific value to find x. For this article, let's assume the area is a specific value—say, 30 square units—to formulate an equation and solve for x.

Formulating the Quadratic Equation

Suppose the area of the triangle is 30 square units. Using the area formula:

$$
30 = \frac{1}{2} \times (2x - 4) \times (x + 5)
$$

Our goal is to solve for x. Let’s proceed step by step:

Step 1: Eliminate the fraction

Multiply both sides of the equation by 2 to clear the denominator:

$$
2 \times 30 = (2x - 4) \times (x + 5)
$$

$$
60 = (2x - 4)(x + 5)
$$

Step 2: Expand the right side

Apply the distributive property (FOIL method):

$$
(2x - 4)(x + 5) = 2x \times x + 2x \times 5 - 4 \times x - 4 \times 5
$$

$$
= 2x^2 + 10x - 4x - 20
$$

Simplify:

$$
= 2x^2 + 6x - 20
$$

Step 3: Set up the quadratic equation

Now, rewrite the original equation:

$$
60 = 2x^2 + 6x - 20
$$

Bring all terms to one side:

$$
2x^2 + 6x - 20 - 60 = 0
$$

Simplify:

$$
2x^2 + 6x - 80 = 0
$$

Step 4: Divide through by common factor

Divide the entire equation by 2 to simplify:

$$
x^2 + 3x - 40 = 0
$$

This is a standard quadratic equation, which can be solved using factoring, the quadratic formula, or completing the square.

Solving the Quadratic Equation

The quadratic equation obtained is:

$$
x^2 + 3x - 40 = 0
$$

Method 1: Factoring

Look for two numbers that multiply to -40 and add to 3:

- Factors of -40: -8 and 5, since $-8 \times 5 = -40$ and $-8 + 5 = -3$
- Factors of 40: 8 and -5, which sum to 3

Actually, since the middle term is positive, we need factors that sum to $+3$:

- 8 and -5: $8 + (-5) = 3$

Check the product:

$$
8 \times (-5) = -40
$$

Sum:

$$
8 + (-5) = 3
$$

Since these satisfy the conditions, factor the quadratic:

$$
x^2 + 8x - 5x - 40 = 0
$$

Group:

$$
(x^2 + 8x) + (-5x - 40) = 0
$$

Factor each group:

$$
x(x + 8) -5(x + 8) = 0
$$

Factor out common binomial:

$$
(x + 8)(x - 5) = 0
$$

Set each factor equal to zero:

$$
x + 8 = 0 \quad \Rightarrow \quad x = -8
$$
$$
x - 5 = 0 \quad \Rightarrow \quad x = 5
$$

Interpreting the Solutions

The solutions to the quadratic are:

- $x = -8$
- $x = 5$

Since x represents a length in units, it must be positive. Therefore, the only valid solution is:

$$
x = 5
$$

Verifying the Solution

To verify, substitute $x = 5$ back into the expressions for base and height:

$$
\text{Base} = 2(5) - 4 = 10 - 4 = 6 \text{ units}
$$
$$
\text{Height} = 5 + 5 = 10 \text{ units}
$$

Calculate the area:

$$
\text{Area} = \frac{1}{2} \times 6 \times 10 = 3 \times 10 = 30 \text{ square units}
$$

This matches the assumed area, confirming that $x = 5$ is the correct solution.

Additional Considerations and Variations

While the above example assumes the area is 30 square units, similar problems can involve different area values or additional conditions. Here are some variations and considerations:

1. Finding x given a different area

If the area is known, simply set up the quadratic equation as above and solve accordingly.

2. Determining when the dimensions are valid

- The base length $2x - 4$ must be positive:

$$
2x - 4 > 0 \quad \Rightarrow \quad x > 2
$$

- The height $x + 5$ is positive for all real x greater than -5, which is less restrictive than the previous condition.

Since $x = 5$ satisfies both, it is a valid measurement.

3. Graphical interpretation

Plotting the quadratic function $x^2 + 3x - 40$ can help visualize the roots and understand the nature of solutions.

Conclusion and Summary

In this comprehensive analysis, we demonstrated how to write and solve a quadratic equation derived from the dimensions of a triangle with a base of $(2x - 4)$ units and a height of $(x + 5)$ units. Assuming a specific area of 30 square units, we formulated the quadratic:

$$
x^2 + 3x - 40 = 0
$$

and solved it using factoring to find:

$$
x = 5
$$

This value aligns with the geometric constraints and validates the measurements. Understanding this process enhances problem-solving skills in algebra and geometry, enabling you to approach similar problems confidently.

Key Takeaways

- Formulating quadratic equations from geometric expressions involves translating measurements into algebraic expressions.
- The quadratic formula, factoring, or completing the square are essential methods for solving quadratic equations.
- Valid solutions must satisfy the context of the problem, especially when dealing with lengths and dimensions.
- Verifying solutions by substituting back into original expressions ensures accuracy.

By mastering these steps, you can confidently tackle a wide range of algebraic and geometric problems involving quadratic equations, ultimately strengthening your mathematical proficiency.

Keywords: quadratic equation, triangle dimensions, base length, height, algebraic problem-solving, quadratic formula, geometric problem, solving quadratics, area of triangle, algebra tips

Frequently Asked Questions

How do you express the area of a triangle with base (2x - 4) and height (x + 5)?

The area is given by (1/2) base height, so it becomes (1/2) (2x - 4) (x + 5).

What is the quadratic equation formed when calculating the area of the triangle in terms of x?

Multiplying out, the area becomes (1/2)(2x - 4)(x + 5) = 0, leading to the quadratic equation $x^2 + 3x - 20 = 0$.

How do you solve the quadratic equation derived from the triangle's dimensions?

Use the quadratic formula x = [-b ± sqrt(b^2 - 4ac)] / 2a, where a=1, b=3, c=-20.

What are the solutions for x in the quadratic equation, and what do they represent geometrically?

The solutions are x = 4 and x = -5. Since dimensions must be positive, x=4 is the valid solution, representing the dimensions of the triangle.

How do you verify that the solutions for x make sense in the context of the triangle's dimensions?

Check that the base and height are positive: for x=4, base = 2(4)-4=4 and height=4+5=9, both positive, confirming valid dimensions.

What is the area of the triangle when x is 4?

Substitute x=4 into base and height: base=4 units, height=9 units. The area is (1/2)49=18 square units.

Additional Resources

A Triangle Has A Base Length Of (2x - 4) Units And A Height Of (x + 5) Units. Write And Solve A Quadratic

Understanding how to translate word problems into algebraic expressions and then solve them is a fundamental skill in mathematics. The problem involving a triangle's base and height, expressed in terms of a variable \(x\), presents a classic example of algebraic modeling and quadratic equations. This type of problem not only enhances algebraic manipulation skills but also deepens comprehension of geometric concepts such as area calculation. In this article, we will explore how to approach such problems systematically, develop the corresponding quadratic equations, and solve them step-by-step, highlighting key strategies and common pitfalls along the way.

Understanding the Problem

Before jumping into algebra, it’s essential to interpret the problem carefully. The problem states:

“A triangle has a base length of (2x - 4) units and a height of (x + 5) units. Write and solve a quadratic.”

This indicates that the dimensions of the triangle are expressed in terms of a variable x, and the task involves deriving an algebraic equation—specifically quadratic—that relates these dimensions, likely through the area formula or some other geometric property.

Key points to identify:

- The base length: $2x - 4$ units
- The height: $x + 5$ units
- The goal: Write an algebraic expression involving these dimensions and solve for x

While the problem statement does not specify what exactly to solve for, a natural assumption is that we are asked to find the value of x when the area of the triangle meets a certain criterion or to derive an expression for the area.

Formulating the Quadratic Equation

Step 1: Recall the area formula for a triangle

The area A of a triangle is given by:

$$
A = \frac{1}{2} \times \text{base} \times \text{height}
$$

Step 2: Substitute the given expressions

Using the given expressions for base and height:

$$
A = \frac{1}{2} \times (2x - 4) \times (x + 5)
$$

\]

Step 3: Expand the expression

First, expand the product:

\[
(2x - 4)(x + 5) = 2x \times x + 2x \times 5 - 4 \times x - 4 \times 5
\]
\[
= 2x^2 + 10x - 4x - 20
\]
\[
= 2x^2 + 6x - 20
\]

Step 4: Write the expression for the area

Now, multiply by \(\frac{1}{2}\):

\[
A = \frac{1}{2} (2x^2 + 6x - 20)
\]

\[
A = x^2 + 3x - 10
\]

This is the algebraic expression for the area in terms of \(x\).

Setting Up the Equation to Solve

To write and solve a quadratic, we need a specific condition. Commonly, such problems might ask:

- For what value of \(x\) does the area equal a certain number?
- Or, to find the value of \(x\) that makes the area be a particular value, say 0, 20, or any given number.

Example:

Suppose the problem asks: "Find the value of \(x\) when the area of the triangle is 0."

This is a typical approach to generate a quadratic equation, leading to:

\[
x^2 + 3x - 10 = 0

\]

Alternatively, if the problem states: "Find x when the area is 15 units", then:

$$
x^2 + 3x - 10 = 15
$$
$$
x^2 + 3x - 25 = 0
$$

In both cases, we arrive at a quadratic equation.

For demonstration purposes, let's proceed with the problem:

Find all values of x such that the area equals zero.

Solving the Quadratic Equation

Given the quadratic:

$$
x^2 + 3x - 10 = 0
$$

We will use the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=3$, and $c=-10$.

Step 1: Calculate the discriminant

$$
D = b^2 - 4ac = 3^2 - 4 \times 1 \times (-10) = 9 + 40 = 49
$$

Step 2: Compute the roots

$$
x = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2}
$$

Step 3: Find the two solutions

$$
x_1 = \frac{-3 + 7}{2} = \frac{4}{2} = 2
$$
$$
x_2 = \frac{-3 - 7}{2} = \frac{-10}{2} = -5
$$

Interpreting the Solutions

While the algebraic solutions are straightforward, interpreting them in the context of the problem is equally important.

- For $x=2$:

Calculate the base and height:

$$
\text{Base} = 2(2) - 4 = 4 - 4 = 0
$$
$$
\text{Height} = 2 + 5 = 7
$$

A base of zero units implies the triangle collapses into a line, resulting in zero area, which aligns with the solution $x=2$.

- For $x=-5$:

$$
\text{Base} = 2(-5) - 4 = -10 - 4 = -14
$$
$$
\text{Height} = -5 + 5 = 0
$$

A negative base length isn't geometrically meaningful, and a height of zero indicates no triangle area. Hence, in practical terms, $x=-5$ does not produce a valid triangle.

Conclusion: The only physically meaningful solution is $x=2$, which yields a degenerate (collapsed) triangle with zero area.

Extending the Problem: If the Area Is Given

Suppose the problem states: "Find x such that the area of the triangle is 20 units."

Given:

$$
A = x^2 + 3x - 10
$$
Set $A = 20$:

$$
x^2 + 3x - 10 = 20
$$
$$
x^2 + 3x - 30 = 0
$$

Solve using the quadratic formula:

$$
D = 3^2 - 4 \times 1 \times (-30) = 9 + 120 = 129
$$
$$
x = \frac{-3 \pm \sqrt{129}}{2}
$$

Since $\sqrt{129}$ is irrational (~11.36), the solutions are:

$$
x = \frac{-3 \pm 11.36}{2}
$$

- $x \approx \frac{-3 + 11.36}{2} \approx \frac{8.36}{2} \approx 4.18$
- $x \approx \frac{-3 - 11.36}{2} \approx \frac{-14.36}{2} \approx -7.18$

Again, check for the validity:

- For $x \approx 4.18$:

$$
\text{Base} = 2(4.18) - 4 \approx 8.36 - 4 = 4.36 >0
$$
$$
\text{Height} = 4.18 + 5 = 9.18 > 0
$$

Valid dimensions.

- For $x \approx -7.18$:

$$
\text{Base} = 2(-7.18) - 4 \approx -14.36 - 4 = -18.36 <0
$$

Negative base length; invalid physically.

Thus, only $x \approx 4.18$ makes sense geometrically in this context.

Features, Pros and Cons of the Approach

Features:

- Translates geometric problems into algebraic equations.
- Uses quadratic formulas to find solutions efficiently.
- Demonstrates the importance of checking the validity of solutions in context.

Pros:

- Provides a systematic method to handle similar problems.
- Enhances understanding of the relationship between algebra and geometry.
- Clear step-by-step approach makes complex problems manageable.

Cons:

- Assumes familiarity with quadratic formulas and algebraic expansion.
- Sometimes solutions may be extraneous (not physically meaningful), requiring interpretation.
- Without explicit problem conditions (like a specific area), the solutions may lack complete context.

Summary and Final Remarks

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